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## 1. Introduction

"Study on Local Non-Linear Effects in Ship Waves" is the theme of an international joint research program supported by the Japan Society for Promotion of Science. Within this framework the authors perform an assessment of some current approaches for improving the linearized thin-ship theory with regard to their merits and to their mutual interrelations.

One dominant trend of investigations may be traced back to the pioneering work of R. Guilloton. Resulting from a series of earlier publications where he had been working towards the idea, Guilloton [15] described a transformation by which the flow around a real ship should be obtained to a good approximation from calculating the Michell type flow connected with a related "linearized hull". The semi-graphical mapping procedure between these two hull forms, though entirely practicable as amply demonstrated by Emerson [10] , appeared to be somewhat lengthy and tedious (e.g., for any Froude number a separate mapping had to be performed). However, Gadd [12] succeeded to translate Guilloton's basic ideas into a standard computer program, and a considerable number of practical hull forms have been investigated with this program. In all cases the predictions of wave resistance compared reasonably with experimentally measured residuary resistance. It should be mentioned here that Gadd did not follow Guilloton literally, but only within the first-order level of approximation. Thus he calculated first-order flow components in terms of a source-sink distribution over the ship's centerplane rather than from the flow due to a system of elementary wedges as favored by Guilloton.

In spite of these remarkable results it must be admitted that the original derivation of Guilloton's method is basically intuitive rather than systematic. This was obviously felt as a challenge by F. Noblesse, who strived to mould this approach to a basis of a formal perturbation procedure in the spirit of thin-ship theory, under inclusion of a spatial coordinate straining in the sense of Poincaré-Lighthill-Kuo, which again depended on the perturbation parameter. Noblesse [25] showed that a first-order coordinate displacement could be defined as the gradient of a displacement potential, which is generated by a source distribution over the ship's centerplane obtained by lengthwise integration of the sources which generate the first-order velocity potential. He further showed that by including this displacement and the square of the first-order velocity the free surface boundary condition could be satisfied to such degree that no second-order singularity distribution is required there. However, this could only be achieved at the expense of violating the field equations (irrotationality and continuity) at the second-order level. Due to Lagally forces resulting therefrom in the fluid domain, second-order resistance evaluated from far-field momentum flux then must in general come out different from the component obtained by pressure integration over the hull, in contrast to the case of the consistent second-order approach. *)

Noblesse's original coordinate transformation had also been derived by Wehausen [38] through an independent approach to thin-ship theory within the framework of a Lagrangian representation of the flow.

The procedure was subjected to modifications and new interpretations in a series of subsequent papers of Noblesse ( [26], [27] ), concurrently by Dagan ( [3], [4] ), and jointly by Noblesse and Dagan [30]. In particular, a justification was sought from Joseph's [18] arguments connected with parametric mapping of the fluid domain in two-dimensional periodic wave flow to a reference domain bounded above by a horizontal plane. Special emphasis was attributed to the fact that such mapping connecting reference domain and physical space can to a large extent be chosen arbitrary (except for points on the boundary), since Joseph was able to show that the result does not depend on the particular choice of the mapping. This arbitrariness was exploited in Noblesse and Dagan [30], aiming at an optimal choice of

*) See Appendix C, where a derivation of the consistent second-order approach is given for comparison.

the first-order displacement.

Our present investigation originated from the desire to provide information on the numerical implications of these inconsistent methods with regard to ship wave patterns;-so far the argumentation had been on a somewhat speculative level only. *) We have developed in the past a computer program for determining first-order flow components due to continuous center plane source distributions of polynomial type together with the complete consistent first- and second-order wave resistance. We succeeded in extending this program to include the first-order displacement in the sense of Noblesse's original approach.

However, within the course of our work we felt confronted with an inevitable need to re-examine the internal systematic of the procedure. Hence our report is now divided into an analysis of the underlying theoretical foundations and an interpretation of our numerical findings.

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*) Of course, this does not apply to the report of Kajitani [20], showing the first numerical results.

# 2. Theory

## 2.1 Mapping methods and perturbation expansions

Free boundary value problems are usually considered too difficult to be solved directly. A common procedure for obtaining approximate solutions is a perturbation expansion with respect to a parameter $\varepsilon$ . Other methods, as e.g. iteration schemes, will not be discussed here (cf. Wehausen [38] ). With regard to Noblesse's work, we are mainly interested in methods that have been applied to thin-ship theory, where $\varepsilon$ is chosen as the beam/length ratio; but we shall also have a glance at the case of plane periodic progressive waves, where Joseph [18] connects $\varepsilon$ with the wave amplitude.

The starting point is always a family $(P_\varepsilon)$ of boundary value problems defined on domains $\Omega_\varepsilon$ contained in 2- or 3-dimensional Euclidean space $\mathbb{R}^n$ . Part of the solution of the problem will also be the determination of the boundary $\partial\Omega_\varepsilon$ of $\Omega_\varepsilon$ . It is only for the special case $\varepsilon = 0$ that the domain $\Omega_0$ is known (but the corresponding problem $P_0$ is - at least relatively - trivial).

All approximation schemes seem to be motivated by the hope that the solution depends nicely on $\varepsilon$ . This hope can be expressed in several ways, two of which we shall compare in the sequel. According to the aim of our study, we shall restrict our attention to the case of thin-ship theory, summarizing first the classical procedure and then turning to Joseph's method of domain perturbations.

From now on, space variables in $\Omega_\varepsilon$ will be denoted by $x = (x_1,\dots,x_n)$, in $\Omega_0$ by $\xi = (\xi_1,\dots,\xi_n)$. The differential operators $\Delta$ and $\nabla$ will be indexed by the independent variable with respect to which they are applied ( $\nabla_x$ , $\nabla_\xi$ , $\Delta_x$ , $\Delta_\xi$ ). In the context of thin-ship theory, it is understood that $x_1$, $\xi_1$ have the direction of the ship's motion, whereas $x_2$, $\xi_2$ are the sidewise and $x_3$, $\xi_3$ are the vertical coordinates.

### 2.1.1 The mapping-free classical method

The classical procedure (Peters and Stoker [32] ) in thin-ship theory, where the solution consists of the velocity potential $\Phi$ and the wave elevation Z (neglecting for the moment variations of sinkage and trim), assumes expansions

$$\Phi = \sum_{i=0}^{\infty} \Phi_i \, \varepsilon^i \tag{2.1.1}$$

$$Z = \sum_{i=1}^{\infty} Z_i \, \varepsilon^i \tag{2.1.2}$$

with $\Phi_i$ and $Z_i$ independent of $\varepsilon$.

The next step is a Taylor expansion of $\Phi_i$ about points of the undisturbed boundary $\partial\Omega_0$. These expansions, together with (2.1.1) and (2.1.2), are substituted into the original boundary conditions for $\Phi$ and Z. Gathering terms with equal factor $\varepsilon^K$ and using the identity principle for power series, we arrive at the well-known first-, second- etc. order boundary value problems of classical thin-ship theory. Solving these problems leads to functions $\Phi_i$ <u>defined on $\Omega_0$</u> ; the theory will be complete only if

1. The series for Z converges (in some sense)
2. The $\Phi_i$ can be extended to the corresponding $\Omega_\varepsilon$
3. The series for $\Phi$ converges.

(conditions 1–3: for $\varepsilon$ small enough)

In their original article, Peters and Stoker are optimistic: "...the authors believe that the series converge for sufficiently small values of the parameter ß " - this is our $\varepsilon$ - "and, in fact, it seems rather likely that the existence question thus posed belongs in the same category as the similar question resolved by Levi-Cività for the case of a plane periodic progressive wave" ( [32], p. 427).

One should, however, keep in mind that the method of Levi-Cività [21] is <u>genuinely</u> two-dimensional and does not admit extension to three-dimensional problems. Furthermore, it can be shown *) that

*) See Appendix A

in the corresponding three-dimensional case the first-order problem admits solutions, consisting of a velocity potential and a wave elevation, which allow an arbitrary degree of non-differentiability in the wave elevation, such that the velocity potential can nowhere find an extension to points above the undisturbed free surface.

### 2.1.2 Joseph's Method

In view of the unresolved extension problem, Joseph [18] *), for the case of plane periodic progressive waves, introduces a more rational expansion. He assumes the existence of a mapping

$$T_\varepsilon : \Omega_0 \longrightarrow \Omega_\varepsilon \quad ,$$

sufficiently regular and invertible, such that $T_0$ is the identity mapping of $\Omega_0$ . Then he assumes that the velocity potential can be written as

$$\Phi\left(T_\varepsilon(\xi), \varepsilon\right) = \sum_{i=0}^{\infty} \frac{1}{i!} \Phi^{[i]}(\xi)\, \varepsilon^i \quad . \tag{2.1.3}$$

The analogous equation for the wave elevation would read

$$Z\left(T_\varepsilon(\bar{\xi}), \varepsilon\right) = \sum_{i=0}^{\infty} \frac{1}{i!} Z^{[i]}(\bar{\xi})\, \varepsilon^i \tag{2.1.4}$$

with $\bar{\xi}$ denoting a boundary point.
If we define

$$\varphi(\xi, \varepsilon) = \Phi\left(T_\varepsilon(\xi), \varepsilon\right) \quad , \tag{2.1.5}$$

we find, following Joseph, that

$$\Phi^{[i]} = \lim_{\varepsilon \to 0} \frac{\partial^i}{\partial \varepsilon^i} \varphi \quad . \tag{2.1.6}$$

Generally, for functions E of the variables $x \in \Omega_\varepsilon$ and $\varepsilon$ , we set

$$e(\xi, \varepsilon) = E\left(T_\varepsilon(\xi), \varepsilon\right) \tag{2.1.7}$$

---

*) cf. especially his remarks on p. 302.

and define the i-th "substantial derivative" of E with respect to $\varepsilon$ at $\varepsilon = 0$ by

$$E^{[i]}(\xi) = \lim_{\varepsilon \to 0} \frac{\partial^i}{\partial \varepsilon^i} e(\xi, \varepsilon) . \tag{2.1.8}$$

This has to be distinguished from the i-th "partial derivative" of E with respect to $\varepsilon$ at $\varepsilon = 0$ defined by

$$E^{\langle i \rangle}(\xi) = \lim_{\varepsilon \to 0} \frac{\partial^i}{\partial \varepsilon^i} E(T_\varepsilon(\xi), \varepsilon) \tag{2.1.9}$$

(The notation "partial derivative" for the function defined by (2.1.9) is not quite correct, but coincides with the usual definition in case of continuously differentiable functions). On the right-hand side of (2.1.9), it is understood that we first differentiate with respect to $\varepsilon$, then substitute the argument ( $T_\varepsilon(\xi)$, $\varepsilon$ ) and afterwards let $\varepsilon$ tend to zero.

The fact that $\lim_{\varepsilon \to 0} T_\varepsilon(\xi) = \xi$ implies the commutation rule

$$\frac{\partial}{\partial \xi_j} \left( E^{\langle i \rangle} \right) = \left( \frac{\partial}{\partial x_j} E \right)^{\langle i \rangle} \tag{2.1.10}$$

for arbitrary but sufficiently regular functions E.

(2.1.10) and the chain rule allow us to express the $E^{[i]}$ in terms of the $E^{\langle i \rangle}$ and $\varepsilon$ derivatives of $T_\varepsilon$ at $\varepsilon = 0$:

(2.1.11) $$E^{[0]} = E^{\langle 0 \rangle}$$

(2.1.12) $$E^{[1]} = E^{\langle 1 \rangle} + \nabla_\xi E^{\langle 0 \rangle} \left. \frac{\partial x}{\partial \varepsilon} \right|_{\varepsilon=0}$$

(2.1.13) $$E^{[2]} = E^{\langle 2 \rangle} + 2 \nabla_\xi E^{\langle 1 \rangle} \left. \frac{\partial x}{\partial \varepsilon} \right|_{\varepsilon=0} + \nabla_\xi E^{\langle 0 \rangle} \left. \frac{\partial^2 x}{\partial \varepsilon^2} \right|_{\varepsilon=0} + \left. \frac{\partial x}{\partial \varepsilon} \right|_{\varepsilon=0} \nabla_\xi \nabla_\xi E^{\langle 0 \rangle} \left. \frac{\partial x}{\partial \varepsilon} \right|_{\varepsilon=0}$$

etc.

Here

$$x = x(\xi, \varepsilon) = T_\varepsilon(\xi) \quad . \tag{2.1.14}$$

With the aid of (2.1.11) - (2.1.13) it is now possible to generate linear boundary value problems for the $\Phi^{\langle i \rangle}$ . For example, from

$$\Delta_x \; \Phi(x, \varepsilon) = 0 \tag{2.1.15}$$

and (2.1.10) it follows by <u>partial</u> differentiation that

$$\Delta_\xi \; \Phi^{\langle i \rangle}(\xi) = 0 \quad . \tag{2.1.16}$$

Since the boundary of $\Omega_\varepsilon$ depends on $\varepsilon$ , the boundary conditions are found by <u>substantial</u> differentiation of the original boundary conditions and subsequent application of (2.1.11) - (2.1.13) etc. This has been performed for the first steps of the scheme by Joseph [18] (plane periodic progressive waves) and Noblesse and Dagan [30] (thin-ship theory).
It turned out that, in both cases, the corresponding boundary conditions for $\frac{1}{i!} \Phi^{\langle i \rangle}$ were identical with those delivered by the classical perturbation scheme for $\Phi_i$ . In particular, the boundary conditions were independent of the particular choice of the mapping $T_\varepsilon$ (they only contain consequences of the natural condition that boundary points of $\Omega_0$ are mapped to boundary points of $\Omega_\varepsilon$ ).

### 2.1.3 Relation between Joseph's and the classical scheme

Let us now assume that both (2.1.1) and (2.1.3) are valid. At points x interior to $\Omega_0$ - which are therefore also interior to $\Omega_\varepsilon$ for $\varepsilon$ small enough -, we can differentiate (2.1.1) with respect to $\varepsilon$ and find

$$\Phi_i(x) = \frac{1}{i!} \lim_{\varepsilon \to 0} \frac{\partial^i}{\partial \varepsilon^i} \Phi(x,\varepsilon) \quad . \qquad (2.1.17)$$

But, by continuity of $\frac{\partial^i}{\partial \varepsilon^i} \Phi$ and as $T_\varepsilon(x) \to x$ for $\varepsilon \to 0$,

$$\lim_{\varepsilon \to 0} \frac{\partial^i}{\partial \varepsilon^i} \Phi(x,\varepsilon) = \lim_{\varepsilon \to 0} \frac{\partial^i}{\partial \varepsilon^i} \Phi\left(T_\varepsilon(x), \varepsilon\right) \; , \qquad (2.1.18)$$

and hence

$$\Phi_i(x) = \frac{1}{i!} \Phi^{\langle i \rangle}(x) \qquad \text{for all } i, \qquad (2.1.19)$$

which gives the fundamental relation between the two schemes.

Equation (2.1.19) was first derived by Joseph ( [18] , equ. (6.1) ) for plane periodic progressive waves and by Noblesse and Dagan ( [30] , equ. (4.2)) for thin-ship theory.
Let us summarize some implications of formula (2.1.19), partly due already to Joseph.

1. Joseph's method is just another - maybe a more rational- approach to derive classical thin-ship theory. *)
2. As $\Phi_i$ and $\frac{1}{i!} \Phi^{\langle i \rangle}$ are identical in $\Omega_0$, they obey the same boundary conditions. Hence the boundary conditions for the $\Phi^{\langle i \rangle}$ are <u>necessarily</u> (not just by chance or only for the first few i) mapping - independent (or can at least be made mapping - independent), since the boundary conditions for the $\Phi_i$ are.

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*) Joseph writes: "The traditional derivation of the higher order wave theory is rational, given the understandings achieved in this note ..." ( [18] , p. 302). Rationality here refers to the treatment of the problem of extending the $\Phi_i$ harmonically to $\Omega_\varepsilon$ .

3. The idea that classical first-order thin-ship theory delivers quite good a flow field, only for the wrong hull and at wrong points - which is fundamental to Guilloton's method - is <u>not</u> backed by Joseph's results.

We finally mention that the problem of harmonic extension of the $\Phi_i$ (or, what amounts now to the same, the $\Phi^{\langle i \rangle}$ ) beyond the undisturbed free surface is still open. In Joseph's formulation it has been replaced by the general problem of convergence in (2.1.3) and (2.1.4).

### 2.1.4 The work of Noblesse and Dagan

The idea of mapping the complicated domain around the ship and below the unknown free surface onto a simpler reference domain and expanding the functions now defined on the reference domain, including the coordinate mapping, into power series with respect to $\varepsilon$ was first persued thoroughly by Noblesse ( [25] , [26] ). In these early papers he formulated boundary value problems for the vector functions

$$V^{[i]} \; , \quad i = 1,2,3 \; , \qquad \text{(see (2.1.7), (2.1.8) )},$$

where $V = \nabla_x \Phi$ . In contrast to the case of partial $\varepsilon$ derivatives (see section 2.1.2), the higher ( $i \geq 2$ ) order problems for the substantial derivatives were mapping-dependent. Noblesse used the mapping-dependence to decompose the second-order problem into one problem with inhomogeneous boundary data which has an explicit solution $W_2$ when the first-order displacement, i.e. $\left.\frac{\partial x}{\partial \varepsilon}\right|_{\varepsilon=0}$, is suitable chosen, and another one with homogeneous boundary data but inhomogeneous field equations. He then dropped the second of these problems and called the (perturbation) velocity field

$$V\left(\xi + \varepsilon \left.\frac{\partial x}{\partial \varepsilon}\right|_{\varepsilon=0}\right) = \varepsilon V^{[1]}(\xi) + \varepsilon^2 W_2(\xi) \qquad (2.1.20)$$

an "inconsistent second-order approximation": it "satisfies the boundary conditions to second order, but the field equations are satisfied to first order only" ( [26], p. 144; cf. [25] , p. 28).*) **)

*) We shall see in the next section that satisfying the hull boundary condition to second order was only possible by violating vital principles of perturbation expansions.

**) Actually, this is the solution proposed in ( [25], p. 27). In [26] , Noblesse later included some additional second-order term in the $\xi_1$ displacement. We consider this a minor change not affecting the structure of the theory, and therefore keep dealing with (2.1.20) as a proposal of [26] , even though this is not strictly true.

The main aim of this approach was a re-derivation of Guilloton's method in the spirit of Lighthill's method of coordinate straining (the relation with Joseph's work was not yet known).

Solution (2.1.20) is complete only if the first $\varepsilon$-derivative of $T_\varepsilon$ or first-order displacement is defined everywhere in $\Omega_0$. Now to render the simple function $W_2$ a solution of the first problem with inhomogeneous boundary conditions, the first-order displacement had been chosen as (U denoting the ship's velocity)

$$\frac{\partial x_i}{\partial \varepsilon}(\xi_1,\xi_2,\xi_3)\bigg|_{\varepsilon=0} = \frac{1}{U}\int_{\xi_1}^{\infty} \frac{\partial}{\partial \xi_i} \Phi^{[1]}(\xi_1',\xi_2,\xi_3)\, d\xi_1' \qquad (2.1.21)$$

for $\xi$ on the undisturbed free surface ( $\xi_3 = 0$ ) and on the ship's centerplane ( $\xi_2 = 0$ ). It therefore seemed natural to define

$$\frac{\partial x}{\partial \varepsilon}\bigg|_{\varepsilon=0} = \nabla_\xi \Psi^{[1]} \qquad (2.1.22)$$

with

$$\Psi^{[1]}(\xi_1,\xi_2,\xi_3) = \frac{1}{U}\int_{\xi_1}^{\infty} \Phi(\xi_1',\xi_2,\xi_3)\, d\xi_1' \qquad (2.1.23)$$

<u>everywhere</u> in $\Omega_0$ (Noblesse [26], p. 144/45).

As we have

$$\Phi^{[1]}(\xi_1,\xi_2,\xi_3) = U\iint_S G(\xi_1,\xi_2,\xi_3,\xi_1',0,\xi_3')\, F^{[1]}_{\xi_1'}(\xi_1',\xi_3')\, d\xi_1'\, d\xi_3' \quad , \qquad (2.1.24)$$

with S: ship's centerplane, G: Havelock-Green function and $F^{[1]}$ being the first-order ship's hull,

$$\Psi^{[1]}(\xi_1,\xi_2,\xi_3) = \iint_S G(\xi_1,\xi_2,\xi_3,\xi_1',0,\xi_3')\, F^{[1]}(\xi_1',\xi_3')\, d\xi_1\, d\xi_3 \qquad (2.1.25)$$

gives the first-order displacement potential in terms of the first-order ship hull.

Equation (2.1.21) for a first-order displacement at points of $\partial\Omega_0$

was also derived by Dagan ( [3], equ. (30), (31), (35), (36), (38), (39)), in a less systematic way. The whole mapping

$$\text{identity} + \varepsilon \nabla_\xi \psi^{[1]} \qquad (2.1.26)$$

which is applied on the left-hand side of (2.1.20), and with $\psi^{[1]}$ defined by (2.1.25), also appears in Wehausen's Lagrangian theory ( [38] , equations (71), (72) ) as the first step of an iteration scheme.

Should a numerical determination of the entire flow field of the classical second-order thin-ship theory be considered too much effort, the principal advantage of the solution (2.1.20) is that it can be computed with essentially the same expenses (and same computer program) as a pure first-order solution. For instance, dropping the problem with inhomogeneous field equations also implies that the (inconsistent) second-order wave elevation vanishes ( [26] , equ. (48) ). Noblesse weighed the disadvantage of being inconsistent against having a closer relation to Guilloton's successful method: "The improvement in results obtained with the method of Guilloton suggests, then, that it may be more important (in the present problem) to satisfy the boundary conditions than the field equations to higher approximations" ( [26], p. 141).

In a subsequent joint paper, Noblesse and Dagan [30] studied again mapping methods in thin-ship theory and their relation to Guilloton's method. Using Joseph's results, they simplified the problem for $V^{[2]}$ and finally reduced it to the problem for $\vec{\Phi}^{\langle 2\rangle}$ . They observed the mapping-independence of the first- and second-order problem ( $\vec{\Phi}^{\langle 1\rangle}$ and $\vec{\Phi}^{\langle 2\rangle}$ ) and gave a derivation of (2.1.19). Then they turned again to "mapping-dependent solutions" and proposed a new class of first-order displacements, with some formal advantages against (2.1.22), (2.1.25), but also more difficult to compute.

Further aspects of this approach (not discussed here) can be found in the contributions of Noblesse and Dagan at the Tokyo Seminar 1976 ( [4], [27] , [28] ). In this context we should not miss to quote the survey lecture of Ogilvie's and its discussion [31] .

### 2.1.5 The "linearized hull"

We can observe two different aims in the articles of Noblesse and Dagan under discussion: firstly, to develop a systematic perturbation method in the spirit of Poincaré-Lighthill-Kuo, or later, of Joseph; secondly, to re-derive (and later, generalize) the method of Guilloton. The latter has as its central concept the "linearized hull", in contrast to the classical theory, which uses the "real hull" exclusively. We want to show here that the PLK or Joseph's method cannot in essence lead to results similar to those of Guilloton's and that in fact the concept of a linearized hull is incompatible with the systematic perturbation procedure outlined in section 2.1.2.

Let us again neglect the influence of sinkage and trim, in accordance e.g. with Guilloton's [15] approach. Then the (real) ship's hull

$$x_2 = F(x_1, x_3, \varepsilon) = \varepsilon F_1(x_1, x_3)$$

is a known function of $x_1$ and $x_3$ (and $\varepsilon$). Therefore, if we write

$$F(x_1, x_3, \varepsilon) = \sum_{i=0}^{\infty} \frac{\varepsilon^i}{i!} F^{[i]}(\xi_1, \xi_3) \quad , \qquad (2.1.27)$$

the functions $F^{[i]}$ appearing on the right hand side are already determined by $F_1$ and boundary values of $T_\varepsilon$. It is easy to compute $F^{[o]}$, $F^{[1]}$, and $F^{[2]}$, and so we shall write them down, using (2.1.11) - (2.1.13).

Firstly, it is clear that

$$F^{\langle o \rangle}(\xi_1, \xi_3) = \lim_{\varepsilon \to 0} F(x_1, x_3, \varepsilon) = 0 \quad . \qquad (2.1.28)$$

Then

$$\begin{aligned} F^{\langle 1 \rangle}(\xi_1, \xi_3) &= \lim_{\varepsilon \to 0} \frac{\partial}{\partial \varepsilon} F(x_1, x_3, \varepsilon) \\ &= \lim_{\varepsilon \to 0} F_1\left(x_1(\xi_1, 0, \xi_3, \varepsilon), x_3(\xi_1, 0, \xi_3, \varepsilon)\right) \\ &= F_1(\xi_1, \xi_3) \end{aligned} \qquad (2.1.29)$$

and
$$F^{\langle i \rangle}(\xi_1, \xi_3) = 0 \qquad \text{for } i > 1 \quad . \tag{2.1.30}$$

Therefore
$$F^{[0]} = 0 \quad , \tag{2.1.31}$$
$$F^{[1]} = F_1 \quad , \tag{2.1.32}$$
$$F^{[2]} = 2\left( \frac{\partial F_1}{\partial x_1} \cdot \frac{\partial x_1}{\partial \varepsilon}\bigg|_{\varepsilon=0} + \frac{\partial F_1}{\partial x_3} \cdot \frac{\partial x_3}{\partial \varepsilon}\bigg|_{\varepsilon=0} \right) \quad . \tag{2.1.33}$$

That is, the first-order hull function is identical with the real hull. Furthermore, the r.h.s. of (2.1.33) is in general different from zero; it therefore does not seem correct to choose $F^{[2]} = 0$ (equ. (44) in [26] ). This implies that the inconsistent second-order solution (2.1.20) does not satisfy the hull boundary condition to second order, if we evaluate $F^{[1]}$ and $F^{[2]}$ properly.

The dilemma is shortly mentioned in Noblesse and Dagan [30] (p. 352). Interpreting an equation of the type (2.1.27), the authors admit two possibilities of solving for $F^{[i]}$ : firstly, via a Taylor expansion (this leads to (2.1.31) - (2.1.33), as we have seen), and secondly, "implicitly via an iterative procedure" (this would correspond to the approach followed by Noblesse earlier). In a footnote the authors concede that in the second case the $F^{[i]}$ would depend on $\varepsilon$ - a fundamental violation of the idea of a perturbation expansion, which relies essentially on the possibility of separating the influence of $\varepsilon$ and the other variables.

In this context it is also important to recall that it is not in the spirit of Lighthill's method of strained coordinates *) to choose the first-order displacement and first-order source distribution such that the ship's centerplane is mapped onto the

*) so that the claim: "The present perturbation analysis is in many ways similar to the method of strained coordinates and may indeed be regarded as an application of this method to a regular perturbation problem" ( [26] , p. 140/41) does not seem to be pertinent.

"real hull". In fact, Lighthill's first-order potential is identically equal to the potential delivered by the classical first-order theory, and his first-order displacement even vanishes (only the second-order displacement is non-zero, viz. a simple translation; see [22], [37] ): in the context of thin aerofoil theory, it was obviously never intended by Lighthill to map the mid section onto the surface of the airfoil.

It is not clear from their statements which of the two aims mentioned in the introduction to this section would be preferred by Noblesse and Dagan in case of a conflict. The authors of this report decided to follow the systematic approach, i.e. we really chose

$$F^{[1]} = F_1 \quad {}^{*)} \quad . \qquad (2.1.34)$$

In this respect we differ e.g. from the interpretation of the displacement (or PLK, as he terms it) method underlying Kajitani's paper [20], and certainly from the genuine method of Guilloton [15] (or Gadd's [12] modification of it). This cannot be helped, as we have the fundamental ambiguity in the original papers of Noblesse and Dagan outlined above. Of course, if the reader prefers the Guilloton-like interpretation, some of the statements we shall make in the sequel will have to be modified somewhat, and, in particular, the computed and measured wave patterns should not be compared directly. But most of the qualitative features that can be observed still carry over to this interpretation. There remain at least the inversion problem and the decay of far-field displacement with all its consequences.

*) This choice is also in agreement with Wehausen [38] and Dagan [3], and it is the only choice for which (2.1.19) is true.

## 2.2 The inversion problem in mapping methods

The mapping

$$T_\varepsilon : \Omega_0 \to \Omega_\varepsilon$$

which allows us to transfer the problem $P_\varepsilon$ from $\Omega_\varepsilon$ to $\Omega_0$ could in some sense be chosen arbitrary (see (2.1.19)!). But one property of $T_\varepsilon$ seems to be indispensable for the equivalence of the original and the transported problem: $T_\varepsilon$ must be invertible, i.e. both one-to-one and onto. *) An equation like

$$\Phi\left(T_\varepsilon(\xi), \varepsilon\right) = \sum_{i=0}^{\infty} \frac{1}{i!} \Phi^{[i]}(\xi)\, \varepsilon^i \qquad 2.2.1)$$

would otherwise not be suitable to define $\Phi$ on $\Omega_\varepsilon$.

In Joseph's original paper [18] the invertibility of $T_\varepsilon$ was demanded explicitly. The problem of proving this property, which we want to call the <u>inversion problem</u>, did not really arise, because Joseph could easily write down a particular mapping which obviously was invertible for arbitrary $\varepsilon$ (as long as the wave elevation was defined).

In the three-dimensional problem of thin-ship theory the situation is much more difficult. As in the case of plane progressive waves, only boundary values of the displacement are prescribed. But now there is no easy choice for the whole displacement field making it an obviously invertible mapping from $\Omega_0$ to $\Omega_\varepsilon$. In fact, only Noblesse ( [25], [26] ) proposed a concrete extension of the first-order displacement potential $\psi^{[1]}$ from the boundary of $\Omega_0$ to the interior. In the other articles, the displacement field at interior points was left unspecified - and the inversion problem was not mentioned at all. The general belief seemed to be that as $T_\varepsilon$ is an $\varepsilon$ analytic perturbation of the identity mapping it must be invertible for $\varepsilon$ sufficiently small. Moreover, this should even hold when $T_\varepsilon$ is replaced by its first- (see (2.1.26) ), second-etc. order approximation, and as the result is <u>independent</u> of the particular choice <u>of $T_\varepsilon$</u> (see (2.1.19); of course, this is true only if $T_\varepsilon$ is invertible), these first- etc. order <u>approximations</u> can be chosen practically

---

*) Of course, for the transfer of the differential equations we also require some differentiability of $T_\varepsilon$., but for the moment we are not interested in this point.

arbitrary.

Whereas proving convergence or nonconvergence of the r.h.s. in (2.2.1) for some positive $\varepsilon$ is yet impossible, the local invertibility (necessary for global invertibility) of a given differentiable mapping such as (2.1.26) depends on a property that can easily be checked: the Jacobian (or functional determinant) must never change sign. For the mapping (2.1.26) this means that

$$J = \begin{vmatrix} 1+\varepsilon\psi^{[1]}_{\xi_1\xi_1} & \varepsilon\psi^{[1]}_{\xi_1\xi_2} & \varepsilon\psi^{[1]}_{\xi_1\xi_3} \\ \varepsilon\psi^{[1]}_{\xi_2\xi_1} & 1+\varepsilon\psi^{[1]}_{\xi_2\xi_2} & \varepsilon\psi^{[1]}_{\xi_2\xi_3} \\ \varepsilon\psi^{[1]}_{\xi_3\xi_1} & \varepsilon\psi^{[1]}_{\xi_3\xi_2} & 1+\varepsilon\psi^{[1]}_{\xi_3\xi_3} \end{vmatrix}$$

$$= 1 + \varepsilon\Delta_\xi\psi^{[1]} + \varepsilon^2 D_2 + \varepsilon^3 D_3 \qquad (2.2.2)$$

where

$$D_2 = \psi^{[1]}_{\xi_1\xi_1}\psi^{[1]}_{\xi_2\xi_2} + \psi^{[1]}_{\xi_2\xi_2}\psi^{[1]}_{\xi_3\xi_3} + \psi^{[1]}_{\xi_3\xi_3}\psi^{[1]}_{\xi_1\xi_1} - \left(\psi^{[1]}_{\xi_1\xi_2}\right)^2 - \left(\psi^{[1]}_{\xi_2\xi_3}\right)^2 - \left(\psi^{[1]}_{\xi_3\xi_1}\right)^2 \qquad (2.2.3)$$

and

$$D_3 = \psi^{[1]}_{\xi_1\xi_1}\psi^{[1]}_{\xi_2\xi_2}\psi^{[1]}_{\xi_3\xi_3} + 2\,\psi^{[1]}_{\xi_1\xi_2}\psi^{[1]}_{\xi_2\xi_3}\psi^{[1]}_{\xi_3\xi_1}$$
$$- \psi^{[1]}_{\xi_1\xi_1}\left(\psi^{[1]}_{\xi_2\xi_3}\right)^2 - \psi^{[1]}_{\xi_2\xi_2}\left(\psi^{[1]}_{\xi_3\xi_1}\right)^2 - \psi^{[1]}_{\xi_3\xi_3}\left(\psi^{[1]}_{\xi_1\xi_2}\right)^2 \qquad (2.2.4)$$

has to be positive throughout $\Omega_0$ , since it tends to 1 for $\xi$ approaching infinity. Now by (2.1.25) we have

$$\Delta_\xi\psi^{[1]} = 0 \qquad \text{in } \Omega_0 \qquad (2.2.5)$$

which implies that the second term on the r.h.s. of (2.2.2) vanishes, and furthermore that

$$\psi^{[1]}_{\xi_1\xi_1}\,\psi^{[1]}_{\xi_2\xi_2} + \psi^{[1]}_{\xi_2\xi_2}\,\psi^{[1]}_{\xi_3\xi_3} + \psi^{[1]}_{\xi_3\xi_3}\,\psi^{[1]}_{\xi_1\xi_1} = -\frac{1}{2}\left[\left(\psi^{[1]}_{\xi_1\xi_1}\right)^2 + \left(\psi^{[1]}_{\xi_2\xi_2}\right)^2 + \left(\psi^{[1]}_{\xi_3\xi_3}\right)^2\right] \tag{2.2.6}$$

so that $D_2$ is negative definite ($D_3$ may take both positive and negative values).

We conclude that a positive Jacobian in this case can only be guaranteed, for all $\xi \in \Omega_0$ and $\varepsilon$ sufficiently small, if the second derivatives of $\psi^{[1]}$ are bounded. This is never true for ships with a flat bottom (the $\xi_2$ or sidewise displacement has a jump then, and therefore $\psi^{[1]}_{\xi_2\xi_3}$ cannot be bounded), and so one has to expect difficulties with the displacement mapping for ships with nearly flat bottom, i.e. practically all merchant ships.

We became confronted with the inversion problem in the course of our computations. We observed that the first-order wave patterns were not only distorted and pressed together, but sometimes even folded by the first-order displacement mapping: the mapping was not one-to-one. We stress the fact that this is not just a particular property of the extension proposed in [26], which we are mainly interested in; it occurs on the boundary (here: on the free surface), and hence is a phenomenon basically independent from further extension to $\Omega_0$ , which means that it is also a property of Dagan's proposal. For numerical details see section 3.3.

## 2.3 General properties of the first- order displacement

Before entering into a discussion of the effects of the first-order displacement, it is necessary to specify which interpretation of the theory is chosen. In section 2.1.5 we made already clear that for our computations we decided to choose

$$F^{[1]}(\xi_1,\xi_3) = F_1(\xi_1,\xi_3) \qquad (2.3.1)$$

as the first-order hull function, because we consider this a necessary consequence of the assumptions. The other possibility allowed in the papers of Noblesse and Dagan,

$$F^{[1]}(\xi_1,\xi_3) = F_1\left(\xi_1+\varepsilon\psi^{[1]}_{\xi_1},\xi_3+\varepsilon\psi^{[1]}_{\xi_3}\right) \qquad (2.3.2)$$

as chosen by Kajitani [20], makes $F^{[1]}$ dependent of $\varepsilon$, similar to Guilloton's method.
Of course, (2.3.2) can only be solved, for a given hull $\varepsilon F_1$, by iteration. This makes it much more difficult to state general properties of the corresponding first-order displacement, whereas the interpretation (2.3.1) allows us to give quite precise information about the effect of the displacement in advance. We shall therefore mainly consider (2.3.1) and only at the end of this section mention some consequences for (2.3.2).
Let us recall that the inconsistent second order solution proposed in [26] can be written as

$$V\left(\xi+\varepsilon\nabla\psi^{[1]}(\xi)\right) = \varepsilon V^{[1]}(\xi)+\varepsilon^2 W_2(\xi) \qquad (2.3.3)$$

with $\psi^{[1]}$ given by (2.1.25). The corresponding wave elevation is

$$Z\left(\bar{\xi}+\varepsilon\nabla\psi^{[1]}(\bar{\xi})\right) = \varepsilon Z^{[1]}(\bar{\xi}) \qquad (2.3.4)$$

(see (2.1.4) ). This means that by taking into account the first-order displacement we simply distort the classical first-order wave pattern. Thus, e.g., maxima and minima of wave elevation can only suffer a shift, but cannot alter their magnitude or even disappear. We conclude that

a) the interference effects between bow and stern wave system remain essentially unchanged;

b) the exponent of decay of wave amplitude along the ship must be the same as in first-order theory.

This follows from the most natural properties of a displacement mapping, namely, that it be invertible and map boundary points to boundary points. Our conclusions a) and b) are therefore also valid for more arbitrary mappings, not given by the gradient of a potential as in (2.1.26).

But let us now return to the proposal (2.3.3), (2.3.4). It follows from the representation (2.1.25) and known properties of the Havelock-Green function that the displacement potential $\psi^{[1]}$ tends to zero, together with all its derivatives, as $\xi$ approaches infinity. Therefore

c) in the far-field, there can be no forward and outward shift of the first-order bow wave system;

d) the far-field wave spectrum, and hence wave resistance derived therefrom, remains unchanged.

These observations already show that we can hardly expect progress from including the first-order displacement if we apply interpretation (2.3.1). On the other hand, interpretation (2.3.2) delivers results similar to those of Guilloton's method, as has been shown by Kajitani [20] (but then the derivation is no more systematic than Guilloton's *) ): in general the predicted wave resistance and the wave elevation along the ship's waterline were in better agreement with experiment than the classical results.

Observations a) - d) made above do not directly apply to case (2.3.2); they only show that the final application of the displacement mapping on the l.h.s. of (2.3.4) is no real further

*) cf. Gadd [10].

improvement (improvement can only result from (2.3.2), i.e. modification of the hull function). But we can say that the forward and outward shift of the bow wave system even in the far-field that can be observed in experiment will not be predicted by this method. We rather have to expect an opposite effect, since the $\xi_1$ component of the displacement is in general positive at the bow, which implies that the ordinate $\xi_{1b}$ of the "linearized hull's" bow is smaller than the ordinate of the real hull's bow, $X_{1b}$ *). Near the bow, this undesired effect largely cancels against the corresponding forward displacement (l.h.s. of (2.3.4) ), but it must be present in the far-field.

*) see figures 5,8,11 in Hong [17]

# 3. Numerical results

## 3.1 General remarks

To our knowledge the methods proposed by Noblesse and Dagan have not yet been tested numerically in a systematic way. The only publication known to us devoted to this topic, Kajitani's paper [20] , does not study the mapping of the flow domain itself, but mainly wave resistance and the "linearized hull". Some material relevant to our theme is also contained in the paper of Hong [17] on Wehausen's Lagrangian theory. A germ of the present investigation can be found in Eggers and Choi [9].

The aim of our numerical study was the comparison of wave patterns produced by classical first-order theory and Noblesse's inconsistent second-order theory with experimental records. Since we could neither carry out experiments for this purpose nor develop a new program, we had to concentrate our efforts on those cases in which mathematical models representable in our existing program (see [6], [7], [9] for details) had already been tested. It turned out that the following records could be used:

a) the M6/M7/M8 family at $\gamma_0 = 6$; *)

b) the Wigley (S) and (D) models at $\gamma_0 = 5$ and $\gamma_0 = 6$;

c) the ES-201 model at $\gamma_0 = 53.7$

The measured wave patterns for a) and b) were kindly contributed by Inui and Kajitani from Tokyo University; the last model has been examined by Adachi from Ship Research Institute in Tokyo, to whom we feel obliged for letting us use this measurement. We would like to point out here that we used <u>all</u> the material available to us, and that we present our results <u>completely</u>, i.e. we did not select results to support whatever conclusions we wanted to draw. The discussion in the next section does not aim at completeness, and the reader is invited to study the wave patterns for himself.

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*) $\gamma_0 = \dfrac{1}{2F_n^2}$

After the wave patterns we investigate the folding effect more closely and comment a series of graphs displaying regions of nonuniqueness of the mapping (2.1.26).

For all our computations we used the underlying polynomial source distribution. *) Therefore, in the strict sense of classical (Michell) ship wave theory, we can compare the theoretical with the experimental wave patterns only for the Wigley models. Readers hesitating to accept the Inuid as an appropriate materialization of a given source distribution should restrict their attention in the other cases (Inuids M6, M7, M8, Adachi's model) to the comparison of the two theoretical wave patterns (which we consider interesting enough). The ship's hull according to the source distribution is given in the lower half of the figures showing the theoretical wave patterns. This hull differs of course from the hull found by streamline tracing for the Inuids and the "mixed" hull of the ES-201 model.

*) See Appendix D for details. In the case of the "elongated Inuid" ES-201, an InuidS-201 of lenth 3.5 m with inserted parallel middle body of length 20 m, we used the polynomial source distribution underlying the Inuid and inserted there a "parallel middle body", i.e. vanishing source distribution, of corresponding length.

## 3.2 Wave patterns

The most obvious effect of the first-order displacement is of course an outward shift of the whole first-order wave system which has the magnitude of the beam at the line $\xi_2 = 0$ and rapidly decays with $\xi_2$. By this rapid decay the contour lines delivered by classical first-order theory are pressed together along the ship; see figures 2,8,9,11,12. This does not seem to model the experimental findings very well. With increasing beam/length ratio, it leads to the nonphysical folding behavior mentioned in section 2.2 (see figures 4 and 6; less prominently in figures 14,16,18).

Much smaller, but still observable, is the shift in $\xi_1$ direction, which generally is positive at the bow and negative at the stern *) (behind the ship, the $\xi_1$ displacement has oscillatory character). Taken together, the $\xi_1$ and $\xi_2$ displacements move the bow wave system "into the right direction": near the bow, the maxima and minima of the displaced bow wave system are between linear theory and experiment. This has already been observed by Standing [34] for Guilloton's mapping. With regard to wave patterns, it is the most satisfactory result we obtained with the displacement method. For the far-field, however, the remarks of section 2.3 apply.

Another phenomenon that can be observed, well-known in the theory of plane periodic progressive waves, is a sharpening of the wave crests and flattening of the troughs in $\xi_1$ direction. This effect of the first-order displacement has already been predicted by Dagan [3].

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*) Conceptually misleading are Dagan's [3] words "forward virtual displacement of bow singularities". He even claims that "waves are apparently generated by singularities located upstream of the bow" ([3], p. 153). The reader should observe that according to displacement theory we first have to compute the flow in the reference space, from the given singularity distribution, and then map this flow to the physical space (including boundary values). This is very different from first mapping the singularities and then computing the waves generated by the mapped singularities. Only the latter would produce a forward shift of the whole bow wave system, an effect suggested by Dagan's formulations.

## 3.3 Folding effect

To study the invertibility of the mapping (2.1.26) more thoroughly, we investigated the magnitude of the associated Jacobian (2.2.2) around the "ship", i.e. that part of the plane $\xi_2 = 0$ of $\partial\Omega_0$ where the first order hull function $F^{[1]}$ (= $F_1$, see section 2.1.5) does not vanish. This was carried out paradigmatically for the model family M6/M7/M8, since we were also interested in the influence of the beam/length ratio $\varepsilon$ . Computations were made at Froude numbers

$$F_n = 0.354,\ 0.289,\ 0.236,\ 0.204$$

corresponding to

$$\gamma_0 = 4,\ 6,\ 9,\ 12,$$

and at the relative depths

$$\frac{2\xi_3}{L} = -0.0001,\ -0.05,\ -0.10$$

(since the (relative) "draught" of the three source distributions is

$$2\ T/L = 0.08,$$

the plane $2\xi_3/L = -0.10$ is already <u>below</u> the "ship").

Figures 19-22 show the results of these computations.
In each of the graphs, the long vertical line indicates the ship's centerplane on $\xi_2 = 0$, the one, two, or three lines to the right of it represent the values $J = 0.5$, $J = 0.0$, $J = -0.5$ of the Jacobian (2.2.2) (starting from the right). For reasons of economy we did not always compute until the point where all contour lines have already reached the line $\xi_2 = 0$.

Our program delivered first derivatives of the first-order displacement potential (2.1.25). The second derivatives necessary for the computation of (2.2.2) were obtained by numerical differentiation.

The result can be summarized as follows: there will always be a region of negative Jacobian - which implies nonuniqueness of the mapping (2.1.26) - around the ship, even for the smallest family member, M6. Significant variations with Froude number cannot be observed. The region of negative Jacobian need not reach the plane $2\xi_3/L = -0.0001$ - therefore fig. 2 shows no folds -, but the intersection with the plane $2\xi_3/L = -0.05$ is never void. In general, even below the ship there is a region of negative Jacobian.

For the sake of completeness we have also added some results for the limiting cases $\gamma_0 = 0$ and $\gamma_0 = \infty$, showing the same tendency (fig. 23, 24).

## 4. Conclusion

The result of the present investigation seems to be that in a certain sense we are thrown back to the situation of 1973. The fundamental dilemma of wave resistance prediction, namely insufficiency of rational first-order thin-ship theory on one hand and irrationality of the successful [12] method of Guilloton on the other, is still awaiting resolution. In this respect our findings are primarily of a negative character.

But we hope that our contribution may also be a starting point for a refined analysis of the whole ship wave problem. By the mapping approach along Joseph's [18] lines it is possible to re-derive classical thin-ship theory in a much more satisfactory way than before, and the possibility cannot be excluded that there exists an invertible mapping such that an easily computable inconsistent second order solution gives results similar to those of Guilloton's method. All we have shown here is that this approach requires more considerations than have been incorporated in the proposals known so far.

Even the <u>linear</u> boundary value problems of thin-ship theory are widely felt to be somewhat different in character from the familiar boundary value problems of mathematical physics such as the Dirichlet or Neumann problem. The latter belong to the large class of elliptic or coercive boundary value problems (to be distinguished from elliptic differential equations), for which quite general theorems concerning the behaviour of solutions exist (see [2] for the most general formulation, [14] for a more recent survey). The feeling that the linear boundary value problems of our field are different in character can be expressed more precisely by the statement that they are <u>not elliptic</u> (see Appendix B). We add this information to support our conviction that mathematically the theory of ship waves and wave resistance is not yet sufficiently well understood.

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## Appendix A Irregular solutions of the first-order problem in the theory of three-dimensional periodic progressive waves

We shall consider properties of solutions of the following problem *) :

$$\Omega = \{(x_1,x_2,x_3) \in \mathbb{R}^3 \mid x_3 < 0\} \quad , \qquad \text{(A.1)}$$

$$\Delta u = 0 \quad \text{in } \Omega \quad , \qquad \text{(A.2)}$$

$$(\partial^2/\partial x_1^2 + \partial/\partial x_3)\, u = 0 \quad \text{on } \partial\Omega \quad , \qquad \text{(A.3)}$$

$$u(x_1 + 2\pi, x_2,x_3) = u(x_1,x_2,x_3) \quad \text{in } \Omega \cup \partial\Omega \quad . \qquad \text{(A.4)}$$

It is relatively easy to show

<u>THEOREM 1</u> Given an arbitrary function f, which is square integrable over the interval $[0,2\pi]$ and satisfies the linear constraint

$$\int_0^{2\pi} f(x_1)dx_1 = 0 \quad , \qquad \text{(A.5)}$$

there is a solution of problem (A.1) - (A.4) such that

$$u(x_1,0,0) = f(x_1) \quad \text{almost everywhere in } [0,2\pi] \;{}^{**)}. \qquad \text{(A.6)}$$

Theorem 1 already implies that we may construct solutions u for our problem such that the corresponding wave elevation

$$\zeta(x_1,x_2) = \partial/\partial x_1 \; u(x_1,x_2,0)$$

---

*) cf. Joseph [18] for the two-dimensional case.

**) In case (A.3) is replaced by

$$(\partial^2/\partial x_1^2 + \gamma_0 \; \partial/\partial x_3)\, u = 0$$

with $\gamma_0$ an arbitrary positive real number, in addition to (A.5) we have to exclude components $\cos nx_1$, $\sin nx_1$ with $\gamma_0^{-2}n^4 - n^2 < 0$. These are however always only a <u>finite</u> number of frequencies, so that differentiability properties are not affected.

is not differentiable on the line $x_2 = 0$. If we choose f as a continuous but nowhere differentiable function *), we see that u cannot be extended beyond the line $x_3 = x_2 = 0$ as a harmonic function, since harmonic functions are always analytic at interior points (see Treves [36], p. 22).

A little bit more elaborate is the proof that irregularity does not only occur on $x_2 = 0$, but in the neighborhood of any point on $x_3 = 0$:

THEOREM 2 If the function f mentioned in Theorem 1 is not differentiable on $[0,2\pi]$ , then there is an integer k such that $\partial^k/\partial x_1{}^k u$ is not differentiable on $x_3 = 0$ for a dense set of numbers $x_2$.

Proof of Theorem 1

If we write

$$f(x_1) = \sum_{n=0}^{\infty} (a_n \sin nx_1 + b_n \cos nx_1) \quad , \tag{A.7}$$

we know that $f \in L^2(0,2\pi)$ is equivalent to

$$\sum_{n=0}^{\infty} (a_n{}^2 + b_n{}^2) < \infty \quad . \tag{A.8}$$

Condition (A.5) simply states that we can omit $a_o$, $b_o$ and start the summation with n=1.

Let us introduce the abbreviation

$$m = m(n) = n\sqrt{n^2 - 1} \quad .$$

Then

$$u(x_1,x_2,x_3) = \sum_{n=1}^{\infty} (a_n \sin nx_1 + b_n \cos nx_1) \cos mx_2 \exp(n^2 x_3) \tag{A.9}$$

has the properties we desire.

We must say some words about the convergence of the series on the r.h.s. of (A.9). The simple estimates

*) For an example see Dieudonné [5], section 8.4, problem 1. To meet condition (A.5), we possibly have to subtract a linear function with the same mean value.

$$| a_n \cos m x_2 \exp n^2 x_3 | \leq | a_n |$$
$$| b_n \cos m x_2 \exp n^2 x_3 | \leq | b_n | \qquad (A.10)$$

(remember $x_3 \leq 0$) show that the series converges in the sense of $L^2[0,2\pi]$ with respect to $x_1$ uniformly with respect to $x_2$ and $x_3$.+) On $x_3 = 0$, estimates (A.10) are sharp, and therefore the convergence has exactly the character of uniform $L^2$ convergence. But for $x_3 < 0$, the stronger estimates

$$| n^k a_n \cos m x_2 \exp n^2 x_3 | \leq n^k | a_n | \exp n^2 x_3$$
$$| n^k b_n \cos m x_2 \exp n^2 x_3 | \leq n^k | b_n | \exp n^2 x_3 \qquad (A.11)$$

hold. (A.8) together with (A.11) proves that also the derivatives of u of arbitrary order converge in this uniform $L^2$ sense, since for given k and $x_3$ there is always $N_0$ such that

$$n^k \exp n^2 x_3 < 1 \qquad \text{for all } n \geq N_0 . \qquad (A.12)$$

Therefore, the series on the r.h.s. of (A.9) indeed represents an infinitely differentiable function in the open lower half-space. This ends the proof of Theorem 1.

For the proof of Theorem 2 we need a more refined tool, known as Sobolev spaces. Unfortunately, we know of no treatment where the reader might find exactly the information we need for this rather simple case of one-dimensional periodic non-constant functions and distributions. Proofs of analogous statements can be found e.g. in Chapter 31 of Treves [35].
For real s, let $H^s_{2\pi}$ denote the space of $2\pi$ periodic distributions f (Schwartz [33], p. 229) on the real line such that

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+) That is, if we define the partial sums
$$f_N = \sum_{n=1}^{N} (a_n \sin n x_1 + b_n \cos n x_1)$$
and similarly $u_N$, then
$$\int_0^{2\pi} | f - f_N |^2 \, dx_1 \leq \varepsilon_N$$
implies
$$\int_0^{2\pi} | u(x_1,x_2,x_3) - u_N(x_1,x_2,x_3) |^2 \, dx_1 \leq \varepsilon_N$$
for all $x_2$ and $x_3$ .

$$f(x_1) = \sum_{n=0}^{\infty} (a_n \sin nx_1 + b_n \cos nx_1) \tag{A.13}$$

with $a_o = b_o = 0$ and

$$\sum_{n=1}^{\infty} (a_n^2 + b_n^2) n^{2s} < \infty \tag{A.14}$$

(cf. (A.7), (A.8) ). For $s \geq 0$, $H^s_{2\pi}$ is contained in $L^2(0,2\pi)$, and of course $H^o_{2\pi}$ is the space of functions admissible in Theorem 1. For $s < 0$, to each $f \in H^s_{2\pi}$ there corresponds at least a continuous ($2\pi$ periodic) function g such that

$$\partial^k/\partial x_1^k \; g = f \tag{A.15}$$

for some integer k (derivatives are always taken in the sense of distributions), see Schwartz [33], p. 230.
We introduce a norm on $H^s_{2\pi}$ by

$$\left\| \sum_{n=1}^{\infty} (a_n \sin nx_1 + b_n \cos nx_1) \right\|_s = \left( \sum_{n=1}^{\infty} (a_n^2 + b_n^2) n^{2s} \right)^{1/2} \tag{A.16}$$

An immediate consequence is that for $f \in H^{s+1}_{2\pi}$

$$\| \partial/\partial x_1 \, f \|_s = \| f \|_{s+1} \tag{A.17}$$

We are now ready to formulate precisely what we are going to prove.

<u>THEOREM 2'</u> If $f \in H^s_{2\pi}$, but $f \notin H^t_{2\pi}$ for some $t > s$, the function u constructed as in the proof of theorem 1 has the following property: for any $x_2 \in \mathbb{R}$ that can be written as a rational multiple of $\pi$, i.e.

$$x_2 = (p/q)\pi \qquad , \text{ p,q integers,} \tag{A.18}$$

the function g of $x_1$, defined by

$$g(x_1) = u(x_1, x_2, 0) \quad , \tag{A.19}$$

does not belong to $H_{2\pi}^{t+2}$.

Remark 1. g belongs to $H_{2\pi}^{s}$ , by generalization of the proof of Theorem 1.

Remark 2 The word "differentiable" used in the formulation of Theorem 2 is interpreted here in the sense of the $H_{2\pi}^{s}$ spaces, i.e. f differentiable means $f \in H_{2\pi}^{1}$ . In this case, the integer k of Theorem 2 is just 2.

The classical case of continuously differentiable functions follows from the inclusions

$$L^2(0,2\pi) \supset C[0,2\pi] \supset H_{2\pi}^{1} \qquad (A.20)$$

and their higher order analogues.+) Namely, if f is continuous but not continuously differentiable, then $f \in H_{2\pi}^{0}$ but $f \notin H_{2\pi}^{2}$ , which implies $g \notin H_{2\pi}^{4}$ by Theorem 2', showing that $\partial^3 g/\partial x_1^{\,3}$ cannot be continuously differentiable; i.e. here we have k = 3.

Proof of Theorem 2'

We know that by $f \notin H_{2\pi}^{t}$ for each N and M > N

$$\sum_{n=N}^{M} (a_n^2 + b_n^2)\, n^{2t}$$

is unbounded as $M \to \infty$ . By definition of u , g has coefficients

$$g(x_1) = \sum_{n=1}^{\infty} (a_n' \sin x_1 n + b_n' \cos n x_1) \qquad (A.21)$$

given by

$$\begin{aligned} a_n' &= a_n \cos m x_2 \\ b_n' &= b_n \cos m x_2 \end{aligned} \qquad (A.22)$$

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+) C [0,2π] is the space of continuous function on [0,2π]. The first inclusion is well known, while the second is a special case of the celebrated "Sobolev embedding theorems" (see Treves [35] , Prop. 31.11)

If we could prove an estimate

$$|\cos m x_2| \geq C\, n^{-2} , \qquad C > 0 , \tag{A.22}$$

for n large enough (C depending only on $x_2$, but not on n), the theorem would follow, since then

$$\begin{aligned} \sum_{n=N}^{M} (a_n'^2 + b_n'^2)\, n^{2(t+2)} &= \sum_{n=N}^{M} (a_n^2 + b_n^2) \cos^2 m x_2\, n^{2(t+2)} \\ &\geq \sum_{n=N}^{M} (a_n^2 + b_n^2)\, C^2 n^{-4} n^{2(t+2)} \\ &= C^2 \sum_{n=N}^{M} (a_n^2 + b_n^2)\, n^{2t} , \end{aligned} \tag{A.23}$$

and the r.h.s. being unbounded, this implies $g \notin H_{2\pi}^{t+2}$ .

The zeroes of cos m $x_2$ occur at

$$m x_2 = (\ell + 1/2)\pi , \qquad \ell \in \mathbb{Z} . \tag{A.24}$$

As

$$x_2 = (p/q)\pi \tag{A.25}$$

we have to ask for

$$\inf_{\ell \in \mathbb{Z}} |m p/q - (\ell + 1/2)| = \frac{1}{q} \inf_{\ell \in \mathbb{Z}} |m p - q(\ell + 1/2)| . \tag{A.26}$$

We can estimate this from below by

$$\frac{1}{q} \inf_{\ell \in \mathbb{Z}} |m p - \ell/2| \tag{A.27}$$

Now for large n

$$1 - \frac{1}{2n^2} - \frac{1}{4n^4} \leq \sqrt{1 - \frac{1}{n^2}} \leq 1 - \frac{1}{2n^2} - \frac{1}{8n^4} , \tag{A.28}$$

and therefore

$$n^2 - \frac{1}{2} - \frac{1}{4n^2} \leq m(n) \leq n^2 - \frac{1}{2} - \frac{1}{8n^2} \tag{A.29}$$

and furthermore

$$(n^2 - \tfrac{1}{2})|p| - \frac{1}{4} \leq m\,|p| \leq (n^2 - \tfrac{1}{2})|p| - \frac{|p|}{8n^2} \tag{A.30}$$

The inf thus occurs at $\ell = (2n^2-1)\,p$ and can be estimated as

$$\inf_{\ell \in \mathbb{Z}} |mp - \ell/2| \geqslant \frac{|p|}{8n^2} \quad . \tag{A.31}$$

From this we can estimate the cosine as

$$|\cos m x_2| \geqslant \frac{|x_2|}{16\, n^2} \quad , \tag{A.32}$$

which ends the proof of Theorem 2'.

## Appendix B Non-ellipticity of the linear boundary value problems of thin-ship theory

Let $\Omega$ denote an open set in $\mathbb{R}^n$, with boundary $\partial\Omega$ possibly consisting of several parts:

$$\partial\Omega = \Gamma = \Gamma_1 \cup \Gamma_2 \ldots\ldots \cup \Gamma_k \ .$$

The operator "restriction to $\Gamma_j$" will be written $\gamma_j$. Then the linear boundary value problems of mathematical physics can mostly be written in the form

$$Lu = f \qquad \text{in } \Omega$$

$$\gamma_j B_{ij} u = g_{ij} \text{ on } \Gamma_j \qquad i = 1,2,..m; \ j = 1,2,..k$$

with differential operators L (of order 2m) and $B_{ij}$ and given function f, $g_{ij}$, with u to be determined.

An important step in the solution of these problems consists in proving inequalities of the type

$$\|u\|_{A_1} \le C \left( \|Lu\|_{A_2} + \sum_{\substack{1\le i\le m\\ 1\le j\le k}} \|\gamma_j B_{ij} u\|_{C_{ij}} \right)$$

which are called regularity theorems.

Here $\|\cdot\|_{A_1}$ and $\|\cdot\|_{A_2}$ are appropriate norms for functions defined on $\Omega$ or $\bar{\Omega} = \Omega \cup \partial\Omega$, and $\|\cdot\|_{C_{ij}}$ norms for functions defined on $\Gamma_j$; C is a positive constant.

The general problem of proving optimal regularity estimates for variable-coefficient linear elliptic differential operators L was solved by Agmon, Douglis and Nirenberg [2]. It turned out that the boundary operators $B_{ij}$ had to satisfy a certain algebraic condition in order to make these estimates possible. This so-called complementing condition +) is shown in [2] to be both necessary and sufficient for extending the classical results known from

+) It is also customary to say that the $B_{ij}$ cover L or that the boundary value problem is elliptic or coercive or of the Lopatinski-Shapiro type; see [16], [23], [36].

the study of the Dirichlet and Neumann problem for the Laplacian to situations with more general elliptic operators and more general boundary conditions.

The original complementing condition as formulated in [2] looks a bit complicated, but in our case it is quite simple to check. We consider the following situation familiar in thin-ship theory:

$$\Omega = \{(x_1,x_2,x_3) \in \mathbb{R}^3 \mid x_2 > 0,\ x_3 < 0\} \quad ,$$

$$k = 2 \quad ,$$

$$\Gamma_1 = \{(x_1,x_2,x_3) \in \mathbb{R}^3 \mid x_2 \geq 0,\ x_3 = 0\} \quad ,$$

$$\Gamma_2 = \{(x_1,x_2,x_3) \in \mathbb{R}^3 \mid x_2 = 0,\ x_3 \leq 0\} \quad ,$$

$$m = 1 \quad ,$$

$$L = \Delta = \sum_{i=1}^{3} \frac{\partial^2}{\partial x_i^2} \quad ,$$

$$B_{11} = \frac{\partial^2}{\partial x_1^2} + \frac{\partial}{\partial x_3} \quad ,$$

$$B_{12} = \frac{\partial}{\partial x_2} \quad .$$

We ask for the boundary condition on $\Gamma_1$ only (a Neumann boundary condition always satisfies the complementing condition with respect to any second order elliptic operator).

The differential operators L and $B_{ij}$ are now viewed as polynomials $L(D)$, $B_{ij}(D)$ in the vector variable $D = (\partial/\partial x_1, \partial/\partial x_2, \partial/\partial x_3)$. In the place of this variable we shall subsequently substitute real vectors $\xi = (\xi_1, \xi_2, \xi_3)$, and then write $L(\xi)$, $B_{ij}(\xi)$. Furthermore, all that really matters is not $B_{ij}$, but its <u>highest</u>

order part $B_{ij}'$. In our example,

$$B'_{11}(\xi) = \xi_1^{\,2} \quad ;$$

of course, $L = L'$ for the Laplacian.

Complementing Condition +) Let $\vec{n}$ denote the normal to $\Gamma_j$ and $\xi \neq 0$ any real vector parallel to $\Gamma_j$. We require that the polynomial, in $\tau$, $B_{1j}'(\xi + \tau\vec{n})$, be linearly independent modulo the polynomial $\tau - \tau^+(\xi)$, where $\tau^+(\xi)$ is the root of $L'(\xi + \tau\vec{n})$ with positive imaginary part.

Let us show that this condition is not satisfied on $\Gamma_1$. We have

$$\vec{n} = (0,0,1) \quad ,$$

vectors $\xi \neq 0$ parallel to $\Gamma_1$ have the form

$$\xi = (\xi_1,\xi_2,0) \quad ,$$

with $\xi_1^2 + \xi_2^2 \neq 0$. Thus

$$L'(\xi + \tau\vec{n}) = L'(\xi_1,\xi_2,\tau) = \xi_1^2 + \xi_2^2 + \tau^2 \quad ,$$

and therefore

$$\tau^+(\xi) = i\sqrt{\xi_1^2 + \xi_2^2} \qquad (\text{here } i = \sqrt{-1})$$

Finally

$$B_{11}'(\xi + \tau\vec{n}) = \xi_1^2 \quad ,$$

as noted above. If we now choose ++)

$$\xi = (0,1,0) \quad ,$$

we have

$$B_{11}'(\xi + \tau\vec{n}) = 0 \quad ,$$

i.e. the identically vanishing polynomial, which certainly is not linearly independent modulo the polynomial

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+) See [2] , p. 626. We slightly simplified the formulation.

++) Here we exploit the three-dimensionality of the problem; in the two-dimensional case, the complementing condition is met !

$$\tau - \tau^{+}(\xi) = \tau - i \quad .$$

It is not quite clear how we should interpret this result. In some respect it reflects the nonuniqueness of the problem due to the presence of free waves for which we just derived strong irregularity properties (Appendix A), and in fact one of the main purposes of introducing the complementing condition in [2] is that it implies uniqueness. Nevertheless, the fact that it is also a necessary condition already indicates that it can hardly be replaced by another conditon which simply guarantees uniqueness.

It is also of possible importance to note that if we define

$$B_{11}^{\varepsilon}(D) = \frac{\partial^2}{\partial x_1^2} + \varepsilon \frac{\partial^2}{\partial x_2^2} + \frac{\partial}{\partial x_3} \quad ,$$

the complementary condition will be satisfied for $B_{11}^{\varepsilon}$ with $\varepsilon > 0$. Thus the situation is radically different whether $\varepsilon \leq 0$ or $\varepsilon > 0$, which may be a hint that linearization of the free boundary conditions must be re-examined.

## Appendix C Derivation of resistance expressions and their invariance within the framework of first- and second-order thin-ship theory

Let $\phi$ stand for a disturbance velocity potential solving the "exact" boundary value problem of wave resistance theory in an inviscid fluid, and let Z be the associated wave elevation above the plane $x_3 = 0$, such that

$$g x_3 + \frac{1}{2}(\nabla\phi \cdot \nabla\phi) - U\frac{\partial}{\partial x_1}\phi = 0 \tag{C.1}$$

for $x_3 = Z$. Now imagine a bounded closed surface $S'$ in the (whole) three-dimensional space containing the ship in its interior, fixed with respect to the ship. For simplicity, let us assume that $S'$ is vertical near the plane $x_3 = 0$, and that the intersection C with this plane is a simple closed curve. The part of $S'$ below $x_3 = Z$ will be denoted by $S_\varepsilon$ , below $x_3 = 0$ by S. Both the (outer) normal vector to $S'$ and the normal vector to C (in the plane $x_3 = 0$) will be called $\vec{n} = (n_1, n_2, n_3)$ (in the latter case, $n_3 = 0$). Then it is well-known that wave resistance can be written as

$$R = -\frac{1}{2}\rho\Big[\iint_{S_\varepsilon}\Big\{(\nabla\phi\cdot\nabla\phi)n_1 - 2\frac{\partial}{\partial x_1}\phi\frac{\partial}{\partial n}\phi\Big\}dS' + g\int_C Z^2 n_1 dC\Big] \tag{C.2}$$

(cf. Newman [24] , equ. (16) ). As the index of $S_\varepsilon$ already indicates, we are again interested not in solving one particular problem but in approximating a whole class of problems parametrized by the real number $\varepsilon$ near $\varepsilon = 0$.

If we now assume

$$\begin{aligned} \phi &\sim \varepsilon\phi_1 + \varepsilon^2\phi_2 \\ Z &\sim \varepsilon Z_1 + \varepsilon^2 Z_2 \end{aligned} \tag{C.3}$$

then clearly, by (C.1) and a Taylor expansion about $x_3 = 0$,

$$Z_1(x_1, x_2) = \frac{U}{g}\frac{\partial}{\partial x_1}\phi_1(x_1, x_2, 0) \tag{C.4}$$

and

$$Z_2 = \frac{U}{g}\frac{\partial}{\partial x_1}\phi_2 - \frac{1}{2g}(\nabla\phi_1\cdot\nabla\phi_1) + \frac{U^2}{g^2}\frac{\partial}{\partial x_1}\phi_1\frac{\partial^2}{\partial x_1\partial x_3}\phi_1 \quad . \tag{C.5}$$

It is then pertinent to approximate R by

$$R \sim \rho\varepsilon^2 R_2 + \rho\varepsilon^3 R_3$$

with

$$R_2 = -\frac{1}{2}\Big[\iint_S \Big\{(\nabla\phi_1\cdot\nabla\phi_1)n_1 - 2\frac{\partial}{\partial x_1}\phi_1\frac{\partial}{\partial n}\phi_1\Big\}dS' + \frac{1}{k_0}\int_C \Big(\frac{\partial}{\partial x_1}\phi_1\Big)^2 n_1\, dC\Big] \tag{C.6}$$

and

$$\begin{aligned} R_3 = -\Big[&\iint_S \Big\{(\nabla\phi_1\cdot\nabla\phi_2)n_1 - \frac{\partial}{\partial x_1}\phi_1\frac{\partial}{\partial n}\phi_2 - \frac{\partial}{\partial n}\phi_1\frac{\partial}{\partial x_1}\phi_2\Big\}dS' \\ &+ \frac{1}{k_0}\int_C \Big\{\frac{\partial}{\partial x_1}\phi_1\frac{\partial}{\partial x_1}\phi_2 + \frac{U}{g}\Big(\frac{\partial}{\partial x_1}\phi_1\Big)^2\frac{\partial^2}{\partial x_1\partial x_3}\phi_1\Big\}n_1\, dC \\ &- \frac{U}{g}\int_C \Big(\frac{\partial}{\partial x_1}\phi_1\Big)^2\Big\{\frac{\partial}{\partial x_1}\phi_1 n_1 - \frac{\partial}{\partial x_2}\phi_1 n_2\Big\}dC \quad . \end{aligned} \tag{C.7}$$

($k_o = \frac{g}{U^2}$). For reasons of economy, let us indicate partial derivatives from now on by indices, e.g.

$$\frac{\partial}{\partial x_1}\phi_1 = \phi_{1,1} \;,\; \frac{\partial}{\partial n}\phi_2 = \phi_{2,n} \quad \text{and} \quad \frac{\partial^2}{\partial x_1\partial x_3}\phi_1 = \phi_{1,13}$$

We shall derive some consequences for the potentials $\phi_1$ and $\phi_2$ from the requirement that $R_2$ and $R_3$ must come out independent from choice of $S'$.

Therefore let $S_1'$ and $S_2'$ be two different control surfaces (with $S_{1\varepsilon}$, $S_1$, $C_1$, $S_{2\varepsilon}$, $S_2$, $C_2$ defined as above), and let $S_1'$ enclose $S_2'$. First we require that $R_2$ should not depend on the choice of control surface:

$$\begin{aligned} 0 = R_2(S_1') - R_2(S_2') = \frac{1}{2}\Big[&\Big\{\iint_{S_2} - \iint_{S_1}\Big\}\Big\{(\nabla\phi_1\cdot\nabla\phi_1)n_1 - 2\phi_{1,1}\phi_{1,n}\Big\}dS' \\ &+ \frac{1}{k_0}\Big\{\int_{C_2} - \int_{C_1}\Big\}\Big\{\phi_{1,1}^2 n_1\Big\}dC\Big] \quad . \end{aligned} \tag{C.8}$$

If we now denote by $S_{12}$ the surface area between $C_1$ and $C_2$ and by $V_{12}$ the volume bounded by $S_1$, $S_2$, and $S_{12}$, we find by partial integration

$$\int_{V_{12}} \phi_{1,1} \Delta \phi_1 \, dV + \int_{S_{12}} \phi_{1,1} (\phi_{1,3} + \frac{1}{k_0} \phi_{1,11}) \, dS = 0 \tag{C.9}$$

Using a standard argument of the calculus of variations, this implies, under mild assumptions about $\phi_{1,1}$ and $\Delta\phi_1$, the classical first-order conditions

$$\Delta \phi_1 = (\frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2} + \frac{\partial^2}{\partial x_3^2}) \phi_1 = 0 \qquad \text{for } x_3 < 0 \tag{C.10}$$

and

$$\phi_{1,3} + \frac{1}{k_0} \phi_{1,11} = 0 \qquad \text{for } x_3 = 0 \tag{C.11}$$

away from the ship.
For invariance of $R_3$ the requirement is

$$\begin{aligned} 0 = R_3(S_1') - R_3(S_2') = & \left\{\int_{S_2} - \int_{S_1}\right\} \left\{(\nabla\phi_1 \cdot \nabla\phi_2) n_1 - \phi_{1,1}\phi_{2,n} - \phi_{1,n}\phi_{2,1}\right\} dS' \\ & + \frac{1}{k_0} \left\{\int_{C_2} - \int_{C_1}\right\} \left\{\phi_{1,1}\phi_{2,1} + \frac{U}{g} \phi_{1,1}^2 \phi_{1,13}\right\} n_1 \, dC \\ & - \frac{U}{g} \left\{\int_{C_2} - \int_{C_1}\right\} \left\{\phi_{1,1}^2 (\phi_{1,1} n_1 - \phi_{1,2} n_2)\right\} dC . \end{aligned} \tag{C.12}$$

Partial integration here yields

$$\begin{aligned} & \int_{V_{12}} \left\{\phi_{1,1} \Delta\phi_2 + \phi_{2,1} \Delta\phi_1\right\} dV \\ & + \int_{S_{12}} \left\{\phi_{1,1} (\phi_{2,3} + \frac{1}{k_0} \phi_{2,11}) + \phi_{2,1} (\phi_{1,3} + \frac{1}{k_0} \phi_{1,11})\right\} dS \\ & + \frac{U}{g} \int_{S_{12}} \left\{(\frac{1}{k_0} \phi_{1,1}^2 \phi_{1,13} - \phi_{1,1}^3)_{,1} - (\phi_{1,1}^2 \phi_{1,2})_{,2}\right\} dS = 0 . \end{aligned} \tag{C.13}$$

By the same reasons as above, we have

$$\Delta \phi_2 = 0 \qquad \text{for } x_3 < 0 \qquad (C.14)$$

and

$$\phi_{2,3} + \frac{1}{k_0}\phi_{2,11} = \frac{U}{g\,\phi_{1,1}}\left\{\left(\frac{1}{k_0}\phi_{1,1}^2\phi_{1,13} - \phi_{1,1}^3\right)_{,1} - \left(\phi_{1,1}^2\phi_{1,2}\right)_{,2}\right\}$$

$$\text{for } x_3 = 0 \qquad (C.15)$$

Equation (C.15) is a special case of the general second-order free surface condition (10.12) in Wehausen and Laitone [39] if we are allowed to apply the Laplace equation (C.10) for $\phi_1$ also on the plane $x_3$=0.

We see that for invariance of resistance expressions it is essential not only to satisfy free surface boundary conditions but also Laplace's equation.
Let us now turn to body boundary conditions. So far we have not made any specification of the parameter $\varepsilon$ nor of the first approximation $\varepsilon\phi_1$ to the velocity potential. In particular, our analysis up to now is valid whether the ship is flat, slender, or thin, submarine, surface-piercing or a hovercraft. For thin-ship theory we even could modify our approach so that $\phi_1$ is a potential generated by first-order sources on the hull rather than the centerplane, in order to prevent a hypothetical deterioration of the short-wave components of the free wave part of the potential. However, down to Froude numbers $\sim$ 0.2 such modification did not show significant effects in the first author's calculations of second-order wave resistance of some mathematical hull forms.

One way of taking care of local non-uniformities near bow and stern occurring in the formal thin-ship expansion is to substitute for $\varepsilon\phi_1$ the double body disturbance potential $\psi_0$ modified by the wavy potential due to a free surface distribution of intensity $U^2\partial^2/\partial x_1^2\psi_0$ in order to satisfy (C.11). Though such approach resembles the basic procedure of slow-ship theory, it is not essentially different from Michell's approach in so far as in both cases the first-order flow is generated from singularities pertinent to the case of zero Froude number. Even the difference in next-higher-order free surface correction is not fundamental: in fact, if on the r.h.s. of (C.15), omitting terms subject to a single differentiation with respect to $x_3$, we replace $\varepsilon\phi_1$ by $\psi_0$ and add $U^2\partial^2/\partial x_1^2\psi_0$, this is essentially the inhomogeneous part of the free surface condition investigated by Inui and Kajitani +) if we neglect triple products of double-body disturbance flow components.

We shall restrict ourselves in the sequel to the consistent thin-ship perturbation expansion in the sense of Peters and Stoker. Thus we assume that the ship's hull may be represented as

$$x_2 = \pm\varepsilon F_1(x_1, x_3) \tag{C.16}$$

and that all flow components may be expanded as Taylor series about the longitudinal centerplane $x_2 = 0$. The ship's hull itself will be called $T'_\varepsilon$, the part of $T'_\varepsilon$ below $x_3 = Z$ is termed $T_\varepsilon$, and T is the name for the region on the plane $x_2 = 0$ for which $x_3 \leq 0$ and $F_1(x_1,x_3) \neq 0$. D is the line common to T and the plane $x_3 = 0$.

In view of the symmetry with respect to $x_2 = 0$, we shall integrate from now on only for $x_2 \geqslant +0$ and multiply the results by 2.++)

We want to compare resistance expressions resulting from contracting the control surface $S_\varepsilon$ in (C.2) to the plane $x_2 = 0$ with resistance obtained from pressure integration over the hull.

For simplicity, let $S'$ be built from planes $x_1 = \pm c_1, x_2 = \pm c_2$, $x_3 = \pm c_3$. Then (C.6) yields, with $c_2 \rightarrow 0$,

$$R_2 = 2\int_T \phi_{1,1}\phi_{1,2}\, dT \quad , \tag{C.17}$$

and from (C.7) we find

$$R_3 = 2\int_T (\phi_{1,1}\phi_{2,2} + \phi_{1,2}\phi_{2,1})dT + \frac{2U}{g}\int_D \phi_{1,1}^2\phi_{1,2}\, dx_1 \quad . \tag{C.18}$$

+) See Schiffstechnik 24 (1977),178-213,equ. (55). Here even the associated resistance is calculated, without any low-Fn simplifications.

++) It is assumed in the sequel that $\lim_{x_2 \to +0}$ and $\lim_{x_3 \to -0}$ commute.

If we approximate the resistance from pressure integration

$$R^P = -2 \int_{T_\varepsilon} p n_1 \, dT_\varepsilon' \qquad (C.19)$$

in the same way as we have done above, i.e.

$$R^P \sim \rho \varepsilon^2 R_2^P + \rho \varepsilon^3 R_3^P \quad , \qquad (C.20)$$

we find after some manipulation

$$R_2^P = -2 \int_T U \phi_{1,1} F_{1,1} \, dT \qquad (C.21)$$

$$R_3^P = \int_T \{(\nabla\phi_1 \cdot \nabla\phi_1) F_{1,1} - 2U F_{1,1} \phi_{2,1} - 2U F_{1,1} F_1 \phi_{1,12}\} dT - \frac{1}{K_0} \int_D \phi_{1,1}^2 F_{1,1} \, dx_1 \qquad (C.22)$$

To achieve equivalence of the resistance expressions (C.17) and (C.21), the usual first-order thin-ship centerplane boundary condition

$$\phi_{1,2} = -U F_{1,1} \qquad \text{for} \quad x_2 = 0 \, , \quad x_3 < 0 \qquad (C.23)$$

is sufficient. To compare (C.18) with (C.22), we note that

$$\int_T \phi_{1,1}^2 F_{1,1} \, dT = -2 \int_T \phi_{1,1} \phi_{1,11} F_1 \, dT = 2 \int_T (F_1 \phi_{1,1})_{,1} \phi_{1,1} \, dT \quad , \qquad (C.24)$$

$$\int_T \phi_{1,2}^2 F_{1,1} \, dT = -2 \int_T \phi_{1,2} \phi_{1,12} F_1 \, dT = 2 \int_T U F_{1,1} F_1 \phi_{1,12} \, dT \quad , \qquad (C.25)$$

$$\int_T \phi_{1,3}^2 F_{1,1} \, dT = -2 \int_T \phi_{1,3} \phi_{1,13} F_1 \, dT = 2 \int_T (F_1 \phi_{1,3})_{,3} \phi_{1,1} \, dT \qquad (C.26)$$

$$-2 \int_D F_1 \phi_{1,3} \phi_{1,1} \, dx_1 \quad ,$$

Thus

$$R_3^P = R_3^{P,1} + R_3^{P,2} \tag{C.27}$$

with

$$R_3^{P,2} = -2\,U \int_T F_{1,1}\,\Phi_{2,1}\,dT \tag{C.28}$$

and

$$R_3^{P,1} = 2\iint_T \{(F_1\,\Phi_{1,1})_{,1} + (F_1\,\Phi_{1,3})_{,3}\}\,\Phi_{1,1}\,dT - \frac{1}{k_0}\int_D F_{1,1}\,\Phi_{1,1}^2\,dx_1 - \int_D 2F_1\,\Phi_{1,3}\,\Phi_{1,1}\,dx_1$$

(Note that two line integrals are equal due to (C.11) !)

$$= -2\int_T F_1(\Phi_{1,1}\Phi_{1,11} + \Phi_{1,3}\Phi_{1,13})\,dT - 2\int_D F_1\,\Phi_{1,3}\Phi_{1,1}\,dx_1 \;\;^{+)} \;. \tag{C.29}$$

Now if (C.11) and (C.23) also hold on $x_2 = x_3 = 0$, a sufficient condition for equivalence of (C.22) and (C.18) is

$$\Phi_{2,2} = (F_1\,\Phi_{1,1})_{,1} + (F_1\Phi_{1,3})_{,3} \qquad \text{for} \quad x_2 = 0 \quad , \tag{C.30}$$

which is again equivalent to the second-order boundary condition found by Taylor expansion

$$\Phi_{2,2} = F_{1,1}\,\Phi_{1,1} + F_{1,3}\,\Phi_{1,3} - F_1\,\Phi_{1,22} \tag{C.31}$$

if we can use Laplace's equation (C.10) on $x_2 = 0$.

We observe that $R_3^{P,1}$ represents a correction of first-order resistance $R_2^P$ of linear thin-ship theory accounting for

a) change of wetted surface
b) quadratic pressure terms
c) sidewise variation of linearized pressure

under exploit of first-order flow information only. As was pointed out by Gadd [11], there is a conceptional relation between Guilloton's introduction of a linearized hull with improving the Michell source distribution by a term corresponding to the r.h.s. of (C.31). Hence one may say that all essential features of Guilloton's approach are incorporated in the consistent second-

---

+) For the descent of (C.29) from components of $\Phi_2$ one may consult [9] equ. (B.17).

order analysis.

The line integral term in (C.18) has been investigated in earlier publications (in [8] under notation $R_M^{(o)}$, in [9] as twice the quantity called $R^{(\text{pressure profile})}$+)). The authors would not contradict a claim that this term includes an essential part of second-order wave resistance, as it could be shown that it covers largely the discrepancy between Michell's theory and the experimental results for the Weinblum-Kendrick-Todd plank in the area of the second hump (see [8], [9] ). In a formal sense, this term could be interpreted as a Lagally force on a line distribution of sources along D, of output $2UF_{1,1}\, Z_1$, if the flow extends over an entire vicinity of D. Hence this resistance component could alternatively be derived from inclusion of a line integral term $I_M$ generated by these sources in the second-order velocity potential. $I_M$ was - somewhat intuitively - included already in Sisov's pioneering work, it can be recognized as one of Wehausen's line integrals, and it appears in Yim's work [40] in a modified version. A similar contribution to the velocity potential, but of opposite sign, occurs from transfer of Green's theorem contributions from the free surface to the plane $x_3 = 0$. Hence it does not seem appropriate to include both this line integral and a singularity distribution over $x_3 = 0$ into a second-order potential, and the underlying idea of condensing singularities over the wave profile area to one line may cause irregularities of the flow near D. However, for certain approximations to second-order wave resistance inclusion of such a term (without additional free surface singularities) may turn out to be efficient.

One should observe that a source line integral of just the opposite sign as $I_M$ is characteristic for the Neumann-Kelvin approach to the determination of the velocity potential. This found ample discussion in the last decade.

The term $R_3^{P,1}$ needs only first-order flow components on the centerplane T for its numerical evaluation. If $\Phi_{1,22}$ is approximated by finite differences in $\Phi_{1,2}$, its evaluation is straight forward once $\nabla\Phi_1$ is calculated. It is only for the second term, i.e. $R_3^{P,2}$, that an explicit evaluation of second-order flow seems to be un-

+) See (C.22) and (C.23)

avoidable; however, for ships with fore-and-aft symmetry even this can be circumvented by using a reverse flow principle (see [9] ). In any case, it may be observed that $\phi_2$ flow components from areas on the plane $x_3 = 0$ behind the ship have strong decay within the domain of integration, i.e. at the ship's centerplane. This is an advantage of the present approach against calculation of second-order wave resistance from far field waves via evaluation of a Kochin function.

Let us have a brief comment on the second-order singularity distribution over the plane $x_3 = 0$. In the discussion of equ. (C.15) we have already mentioned that the r.h.s. of this formula can be written as

$$\frac{U}{g}\left[(\nabla\phi_1\cdot\nabla\phi_1)_{,1} - \frac{1}{k_0}\,\phi_{1,1}(k_0\phi_{1,3}+\phi_{1,11})_{,3}\right] \qquad (C.32)$$

Note that for a source influence function G which is singular like 1/r we have

$$G_{,11} + k_0\,G_{,3} = \left(\frac{1}{r}-\frac{1}{r_1}\right)_{,11} + k_0\left(\frac{1}{r}+\frac{1}{r_1}\right)_{,3} \qquad (C.33)$$

if r is the distance from source point to flow point and $r_1$ the distance from image source point to flow point. Then if

$$\phi_1(x_1,x_2,x_3) = \frac{1}{2\pi}\int_T U f_{1,1}(x_1',x_3')\,G(x_1',0,x_3';x_1,x_2,x_3)\,dx_1'\,dx_3' \; , \qquad (C.34)$$

we have

$$(\phi_{1,11}+k_0\phi_{1,3})_{,3} = \frac{1}{2\pi}\int_T U f_{1,1}\left\{\left(\frac{1}{r}-\frac{1}{r_1}\right)_{,113} + k_0\left(\frac{1}{r}+\frac{1}{r_1}\right)_{,33}\right\}dx_1'\,dx_3' \qquad (C.35)$$

This term is obviously singular near bow and stern, hence one should expect significant contributions from the second-order free surface distribution which so far have not been investigated numerically.

Concluding our digression to second-order wave resistance theory, we should mention that the validity of (C.23) even for $x_3 = 0$ was realized by the authors not earlier than under recent numerical investigation of the folding effect in mapping methods. With $\frac{1}{r}$ minus $\frac{1}{r_1}$ as leading terms in the source influence potential, one should expect that the step in $x_2$ velocity expressed by (C.23) should change sign for $x_3$ positive and hence should vanish for $x_3 = 0$. This led to a long-lasting consternation+) culminating in the question whether or not the line integral term in (C.18) must be included in second-order wave resistance, as it seems to vanish when C is contracted to the centerplane.

But, calculating the free wave component of $\phi_{1,2}$, we observed poor convergence of its $x_2$ Fourier representation unless a term

$$-\frac{1}{\pi} U F_{1,1}(x_1,0) \int_{-\infty}^{+\infty} \sin x_2 v \, \frac{dv}{v} = -U F_{1,1}(x_1,0) \operatorname{sgn} x_2 \qquad \text{(C.36)}$$

was split off and evaluated in closed form.

Thus we had to learn that the non-oscillatory wave component termed "free-local" in Kajitani's early investigations (see [19]), present only for $x_1$ values in the range of the ship's length, provides the step in $\phi_{1,2}$ to satisfy (C.23) for $x_3 = 0$, as an essential link for proving the equivalence of pressure integral and far-field momentum flux integral at second-order level.

+) See e.g. [8], [13] and the discussion to [7].

Appendix D Hull functions used for computations

Ship hulls (in the Michell sense) were represented in the form

$$F(\xi_1,\xi_3) = \varepsilon\, p_k(2\xi_3/L)\cdot q(2\xi_1/L) \quad .$$

with

$$p_0(t) = \begin{cases} 1 & , \quad 0 \geq t \geq -2T/L \\ 0 & , \quad -2T/L > t \end{cases}$$

$$p_2(t) = \begin{cases} 1-(t/(2T/L))^2 & , \quad 0 \geq t \geq -2T/L \\ 0 & , \quad -2T/L > t \end{cases}$$

and

$$q(t) = \begin{cases} 0 , & |t| > 1 \\ 1-\sum_{i=1}^{4} \frac{c_i}{i}\left(\frac{|t|-t_0}{1-t_0}\right)^i & , \quad 1 \geq |t| \geq t_0 \\ 1 , & t_0 > |t| \end{cases}$$

Thus F is characterized by the numbers $\varepsilon$, k, 2T/L, $t_o$, $c_1,c_2,c_3,c_4$. In our cases $c_1 = c_3 = 0$. The other values were chosen as follows:

| Model | $\varepsilon$ | k | 2 T/L | $t_o$ | $c_2$ | $c_4$ |
|---|---|---|---|---|---|---|
| M6 | 0.11333 | 0 | 0.08 | 0. | 3.678 | -3.356 |
| M7 | 0.22666 | 0 | 0.08 | 0. | 3.678 | -3.356 |
| M8 | 0.33999 | 0 | 0.08 | 0. | 3.678 | -3.356 |
| Wigley (D) | 0.1 | 2 | 0.2 | 0. | 2. | 0. |
| Wigley (S) | 0.1 | 2 | 0.125 | 0. | 2. | 0. |
| ES-201 | 0.0298 | 0 | 0.0149 | 0.851 | 2. | 0. |

List of figures

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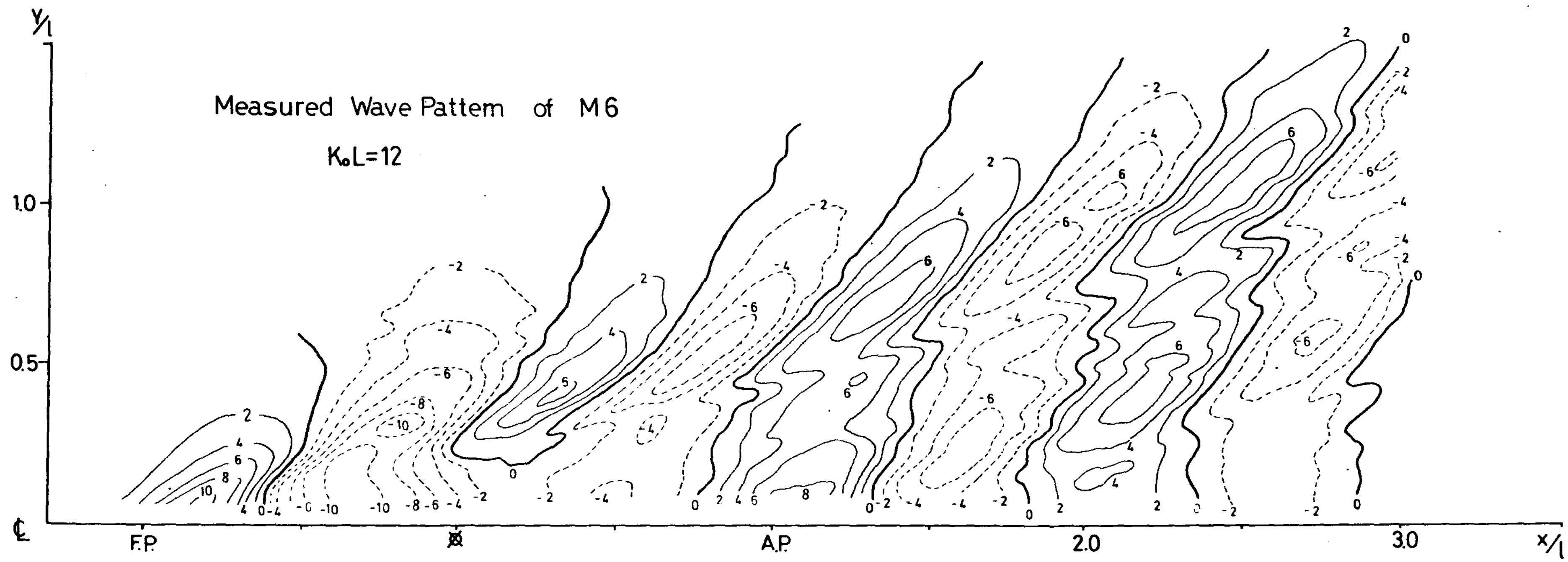
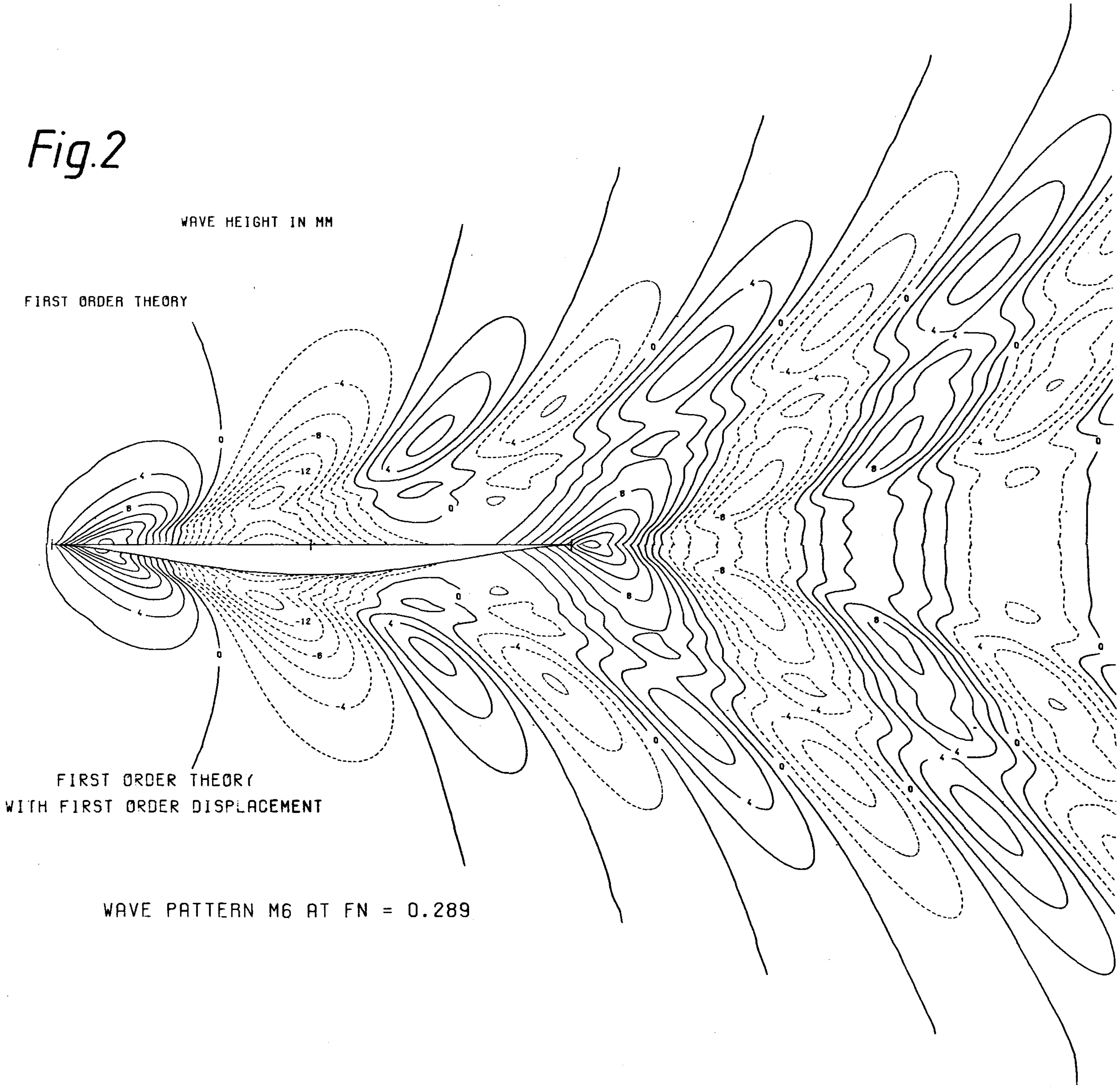



*Fig.1*

Fig.2


WAVE HEIGHT IN MM
FIRST ORDER THEORY
FIRST ORDER THEORY
WITH FIRST ORDER DISPLACEMENT
WAVE PATTERN M6 AT FN = 0.289

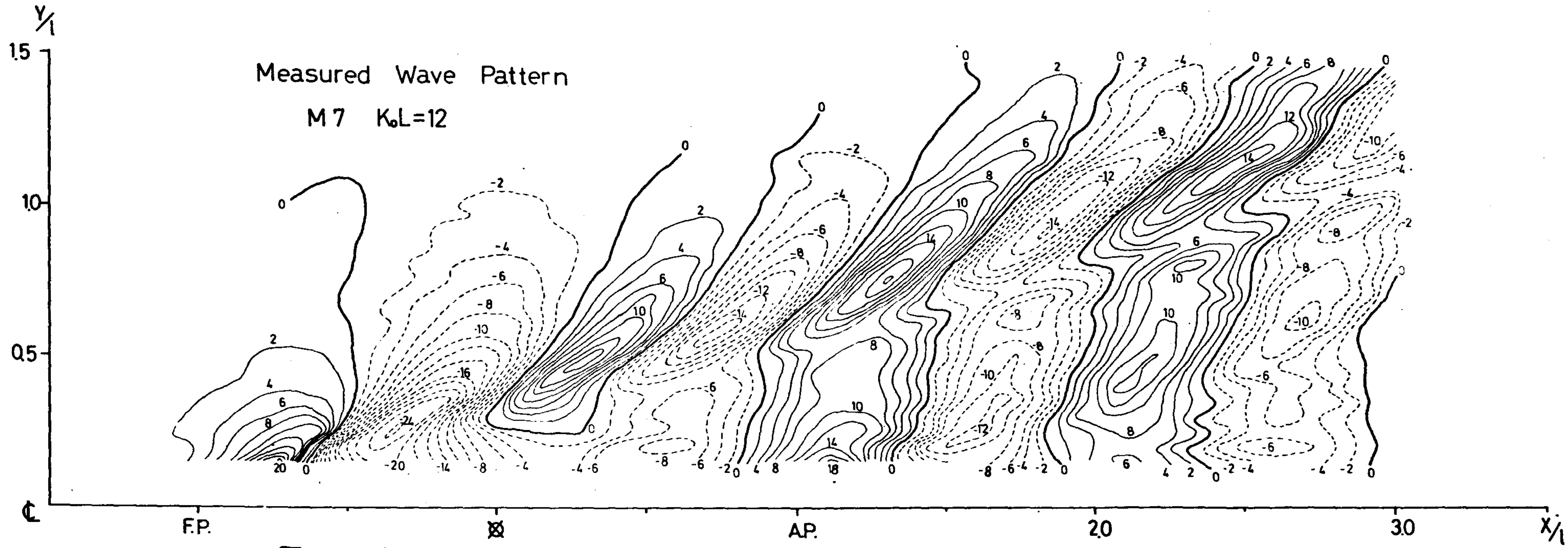
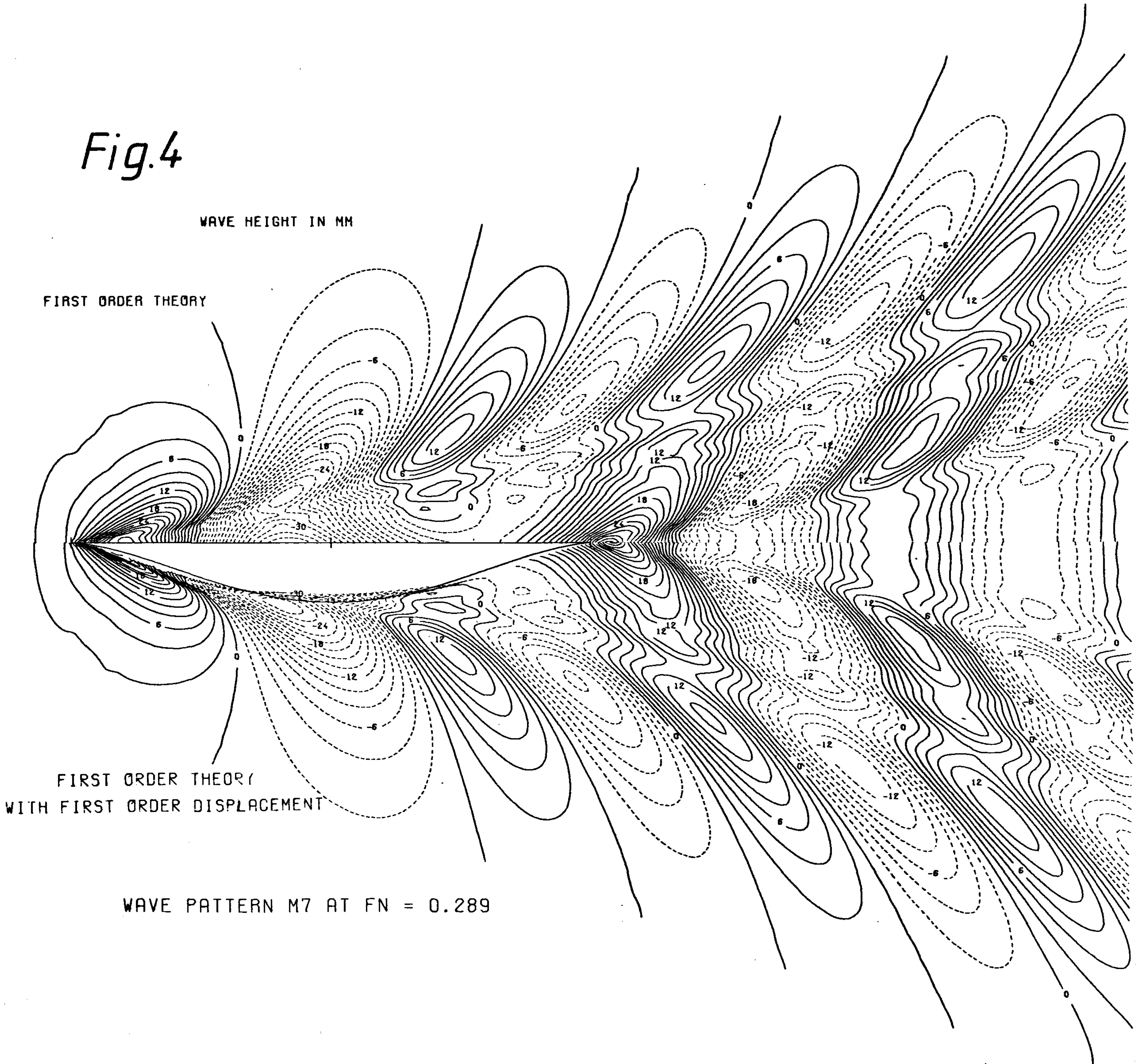



Fig. 3

Fig.4


WAVE HEIGHT IN MM
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FIRST ORDER THEORY
WITH FIRST ORDER DISPLACEMENT
WAVE PATTERN M7 AT FN = 0.289

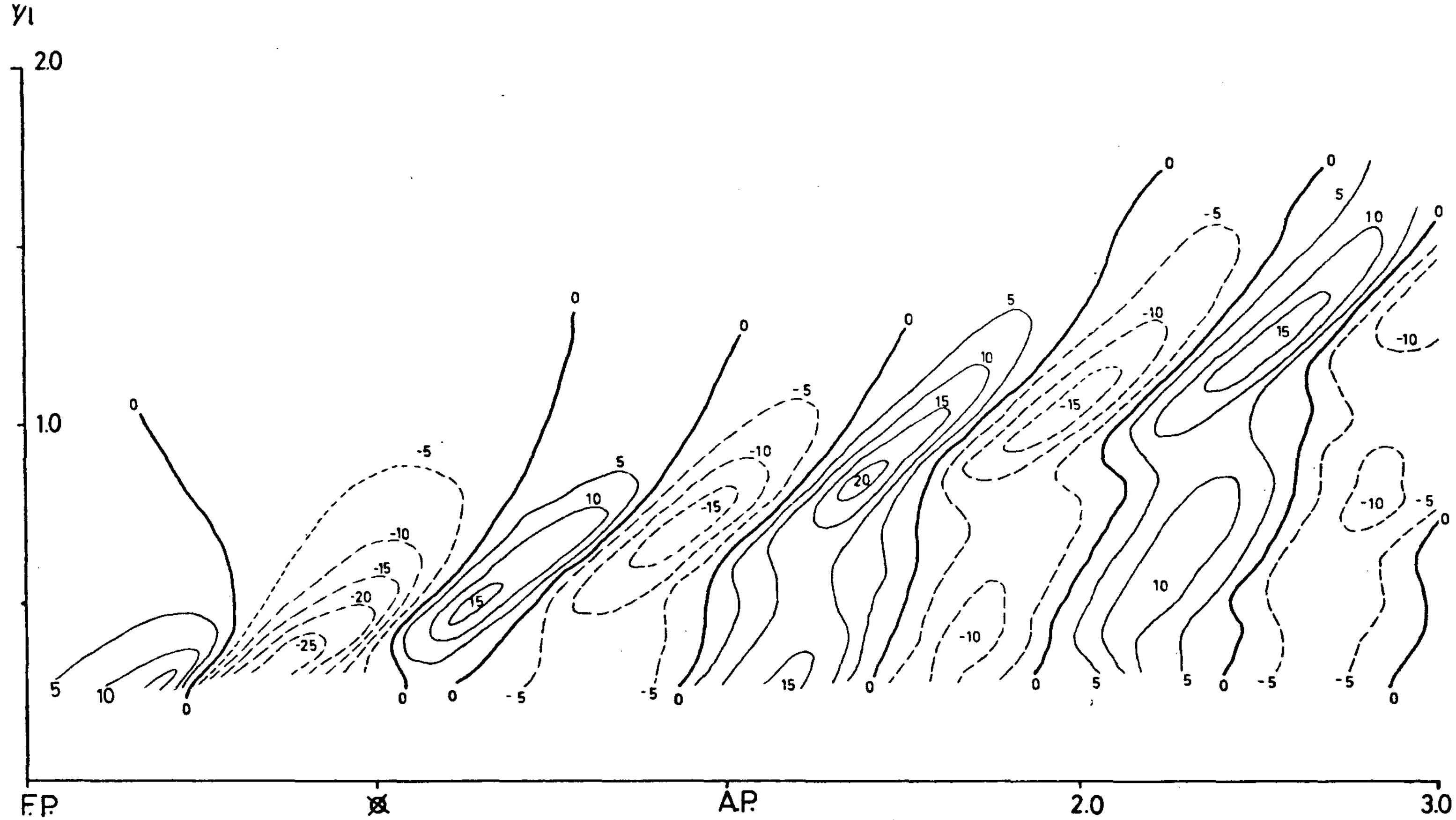
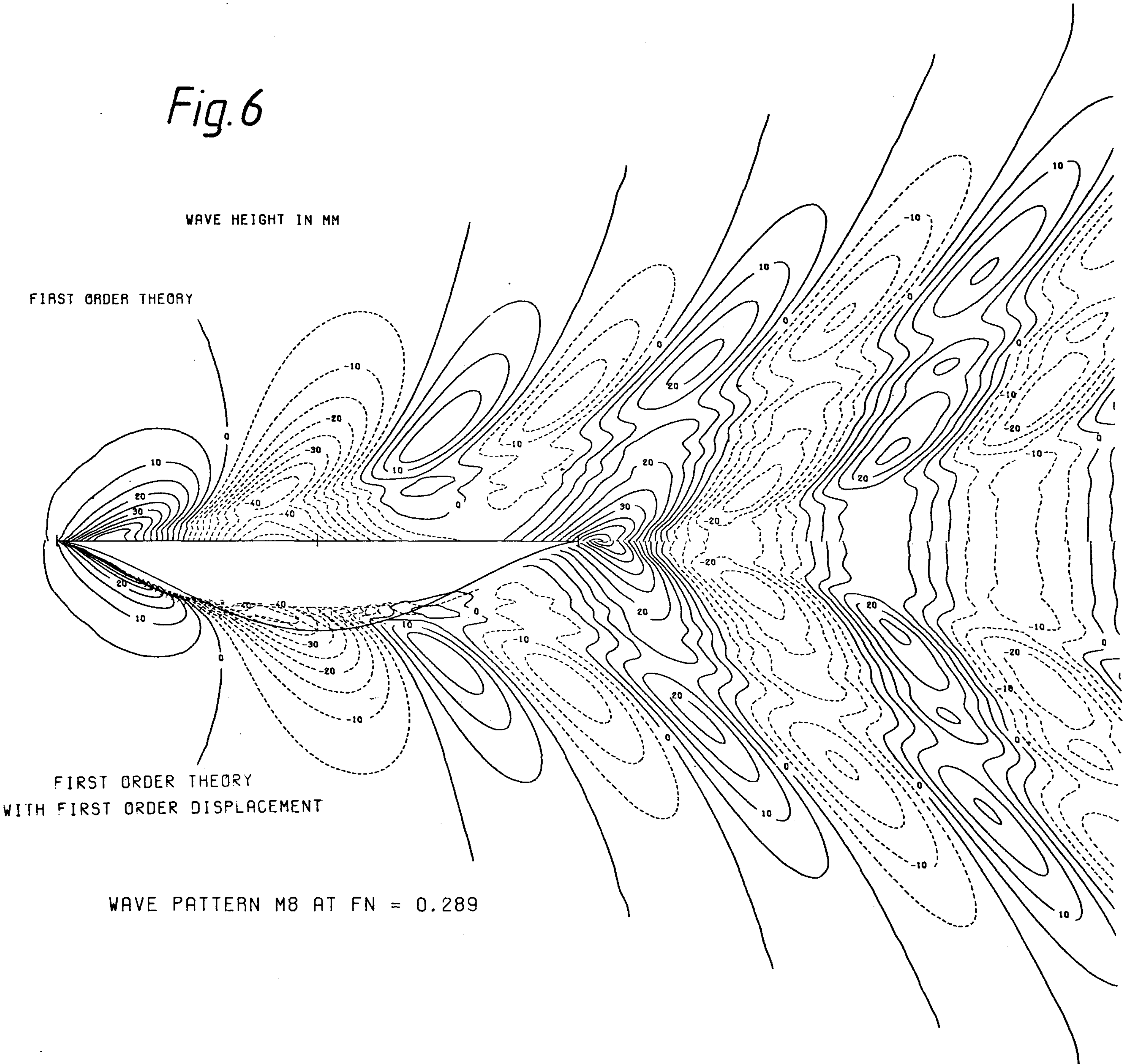



*Fig.5*

Fig. 1c Measured Wave Pattern of M8 ($K_oL=12$)

Fig.6




WAVE PATTERN M8 AT FN = 0.289

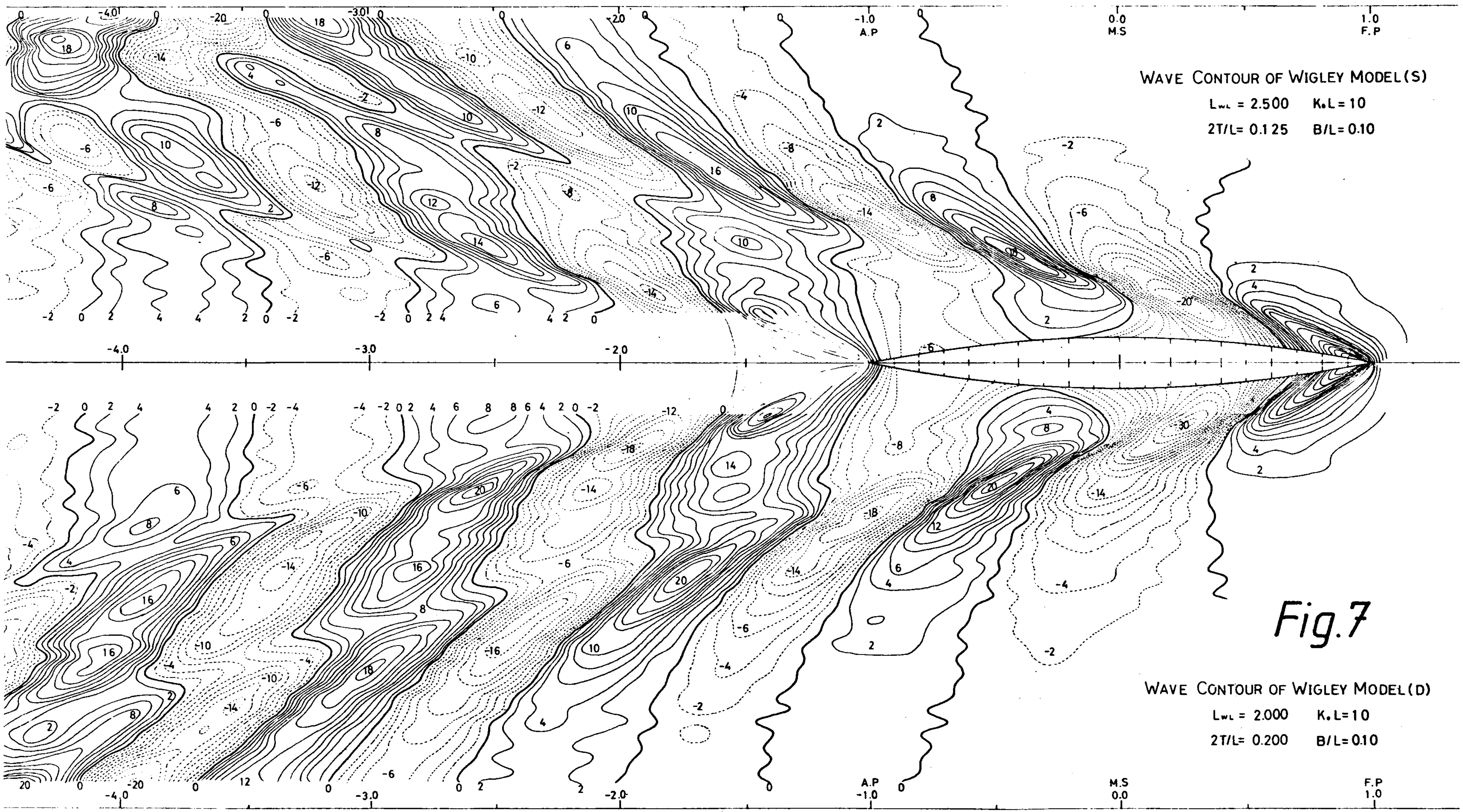
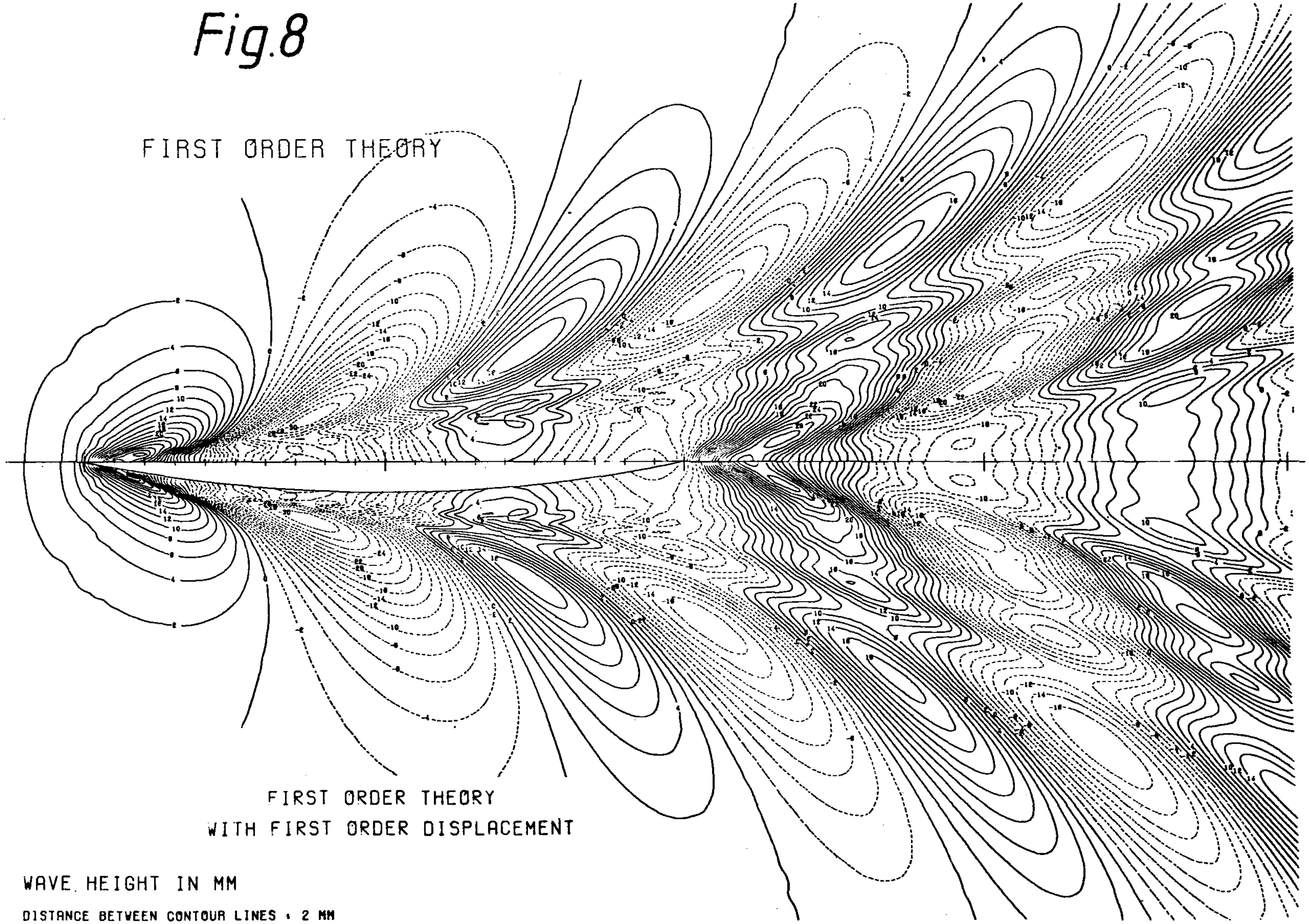

WAVE CONTOUR OF WIGLEY MODEL(S)
Lwl = 2.500 K.L = 10
2T/L= 0.125 B/L= 0.10
WAVE CONTOUR OF WIGLEY MODEL(D)
Lwl = 2.000 K.L=10
2T/L= 0.200 B/L=0.10
1.0 F.P
0.0 M.S
-1.0 A.P
-2.0
-3.0
-4.0

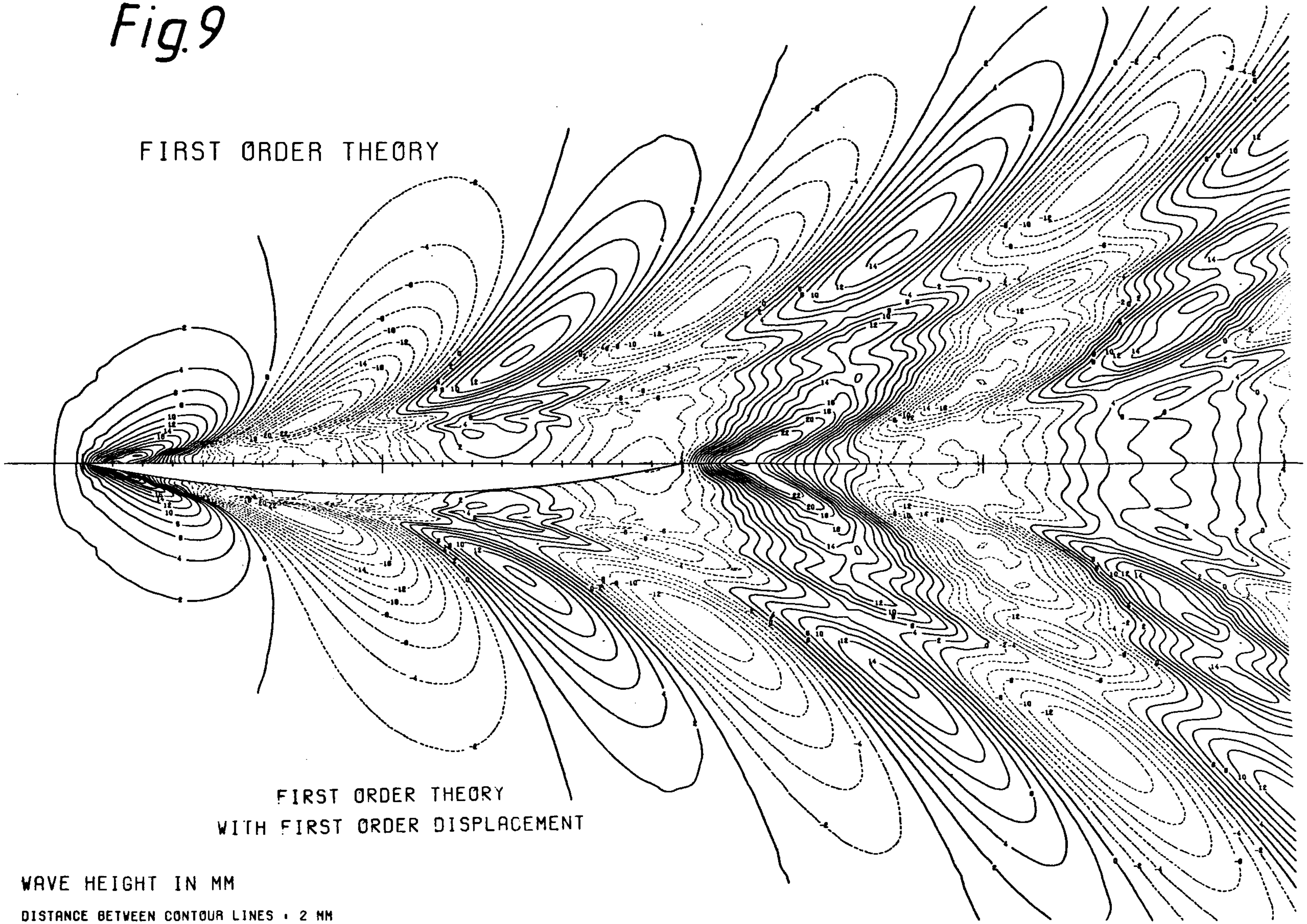
Fig.7

Fig.8




WAVE PATTERN WIGLEY MODEL (D) AT FN = 0.316

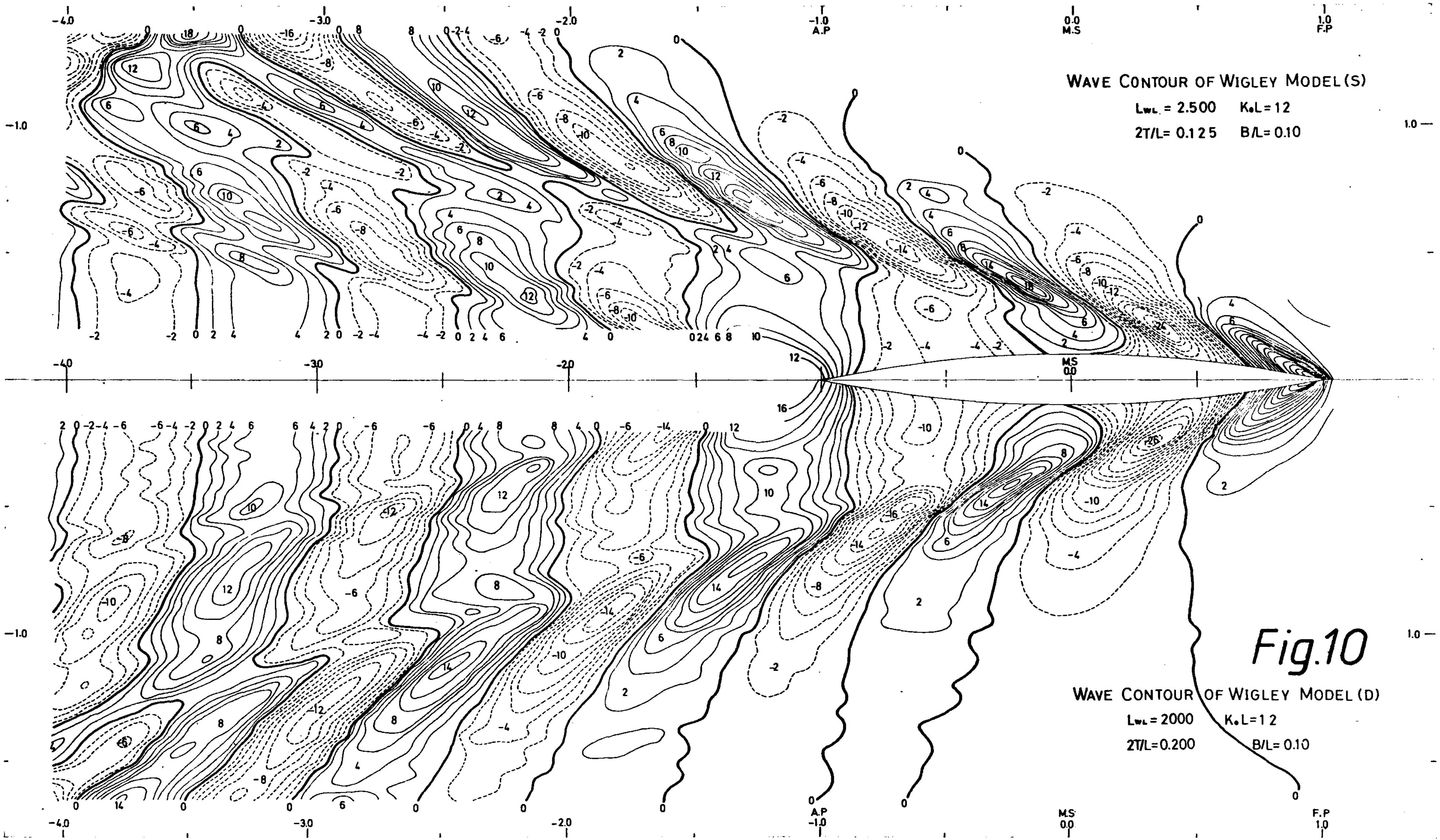
Fig.9




WAVE HEIGHT IN MM

DISTANCE BETWEEN CONTOUR LINES : 2 MM

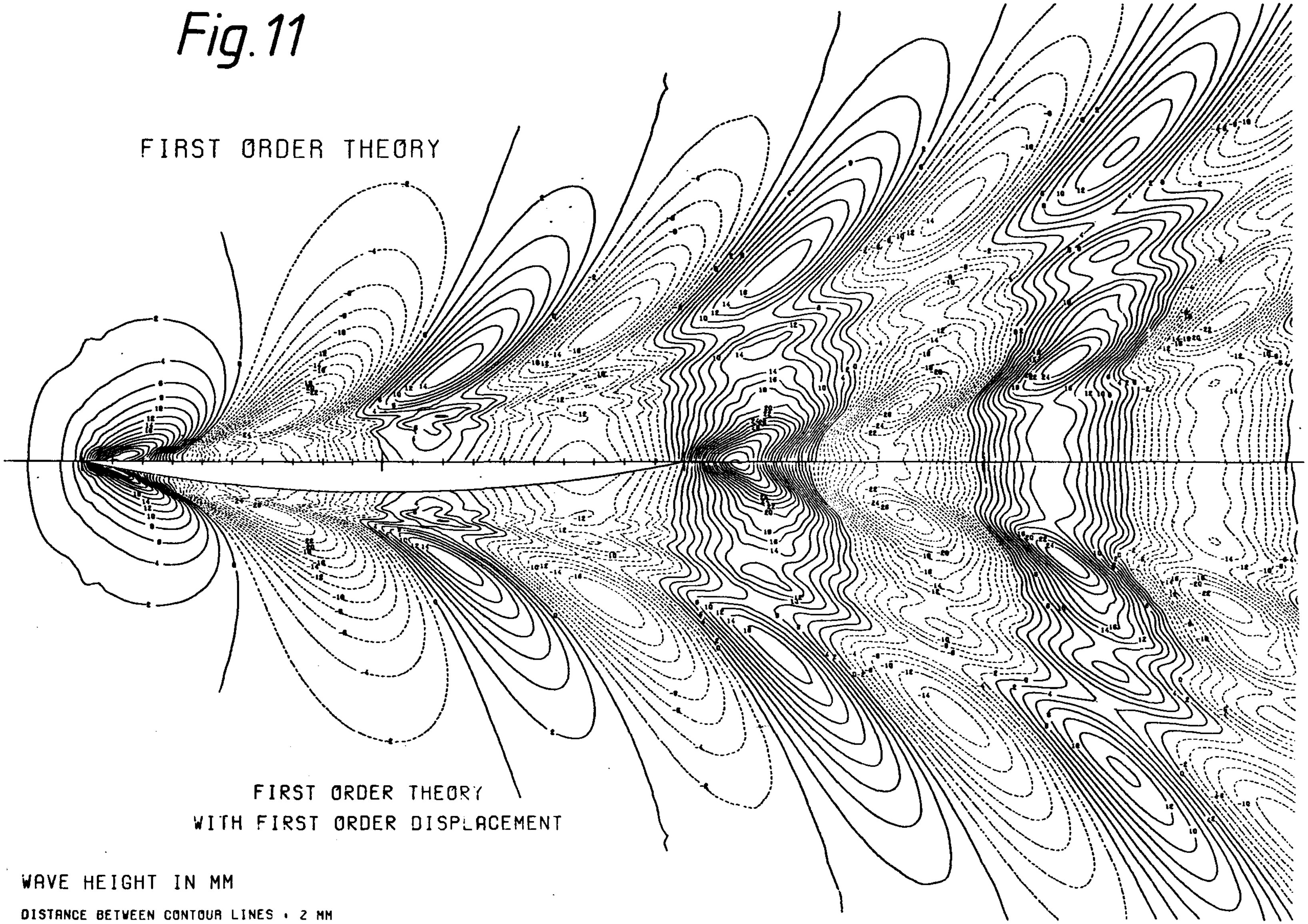
WAVE PATTERN WIGLEY MØDEL (S) AT FN = 0.316



WAVE CONTOUR OF WIGLEY MODEL (S)
$L_{WL}$ = 2.500 $K_0L$ = 12
2T/L = 0.125 B/L = 0.10
WAVE CONTOUR OF WIGLEY MODEL (D)
$L_{WL}$ = 2000 $K_0L$ = 12
2T/L = 0.200 B/L = 0.10
A.P
M.S
F.P

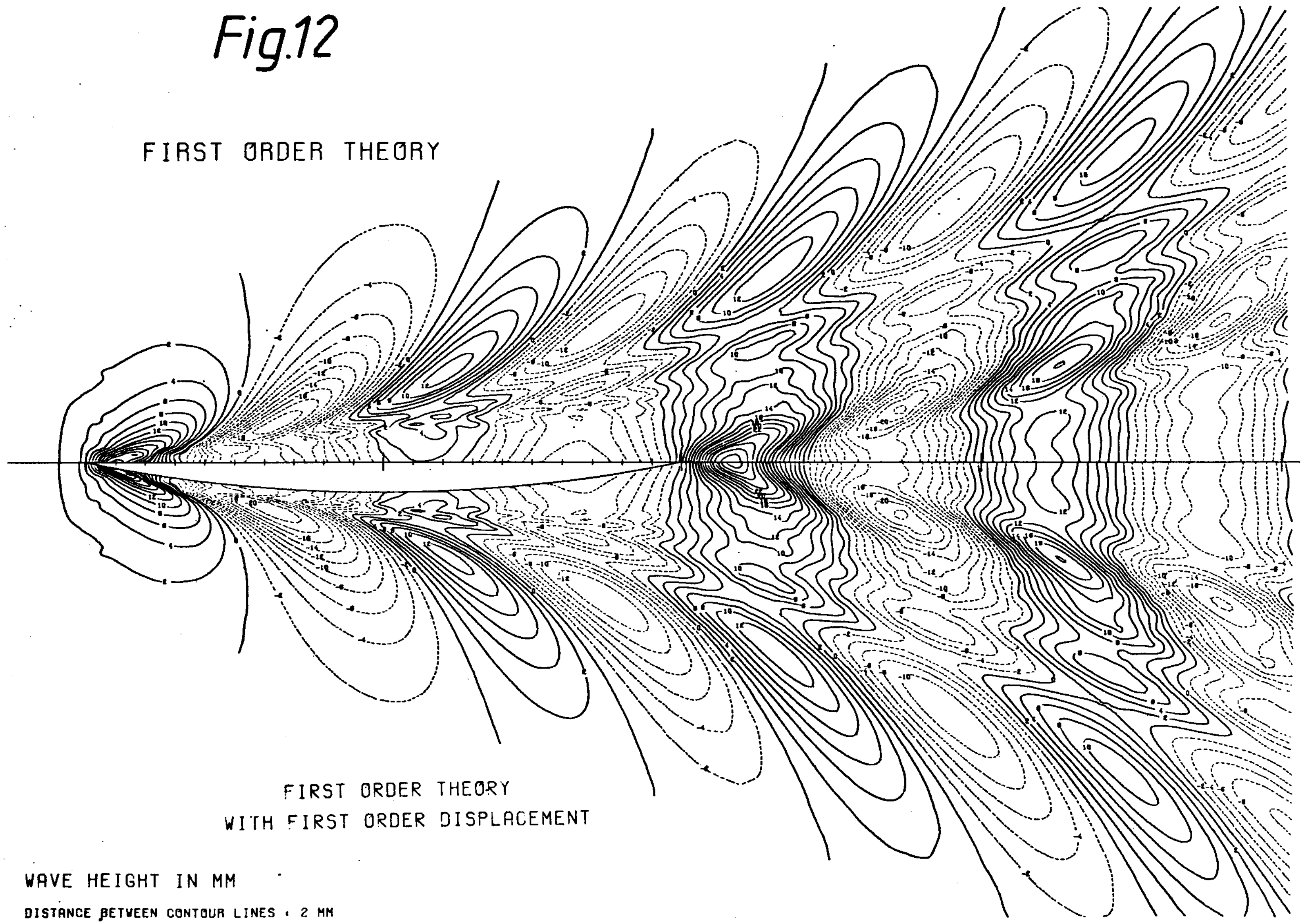

Fig.10

*Fig. 11*




WAVE PATTERN WIGLEY MODEL (D) AT FN = 0.289

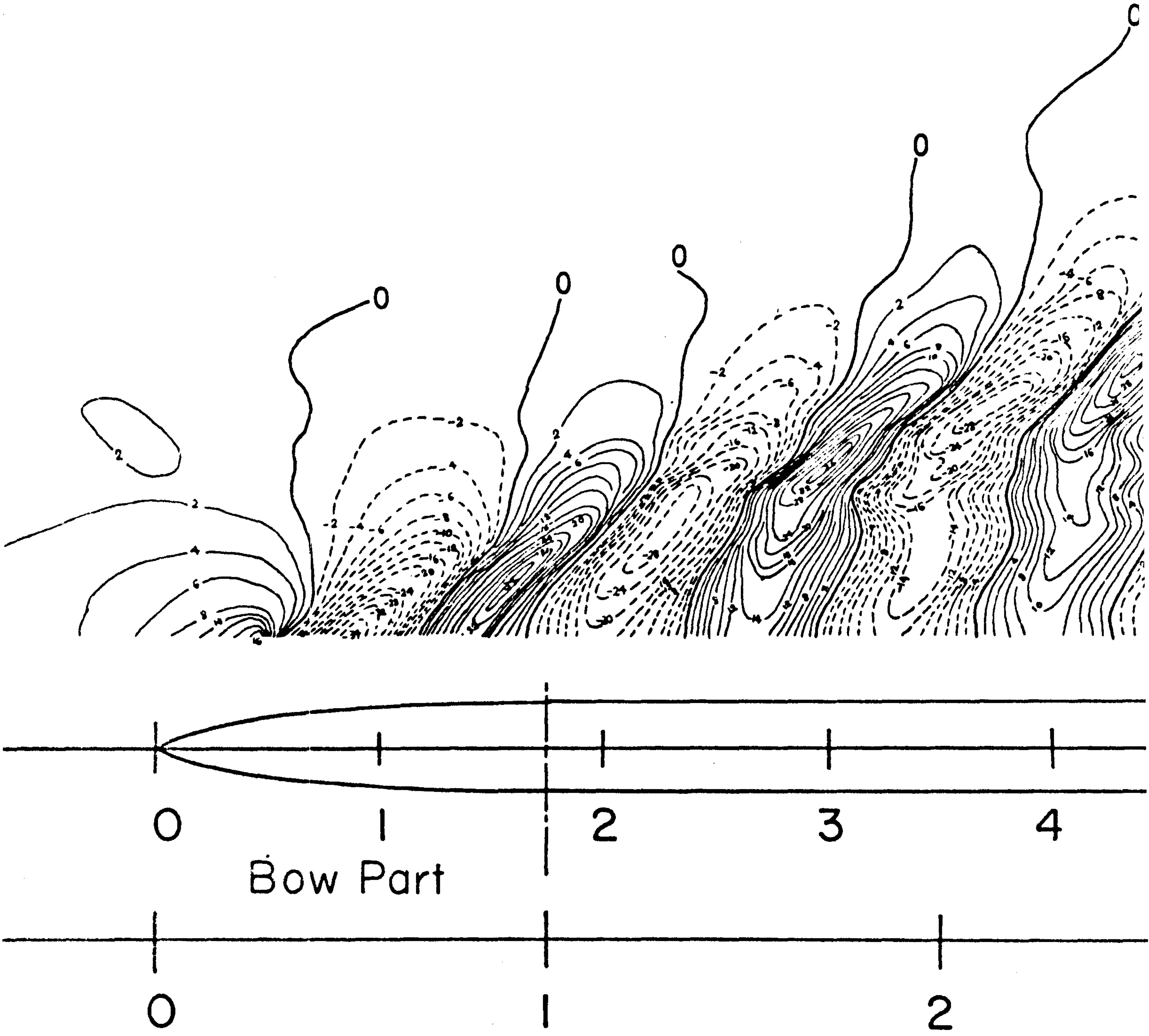
Fig.12




WAVE PATTERN WIGLEY MØDEL (S) AT FN = 0.289

$V = 1.464^{M/S}$

*Fig. 13*

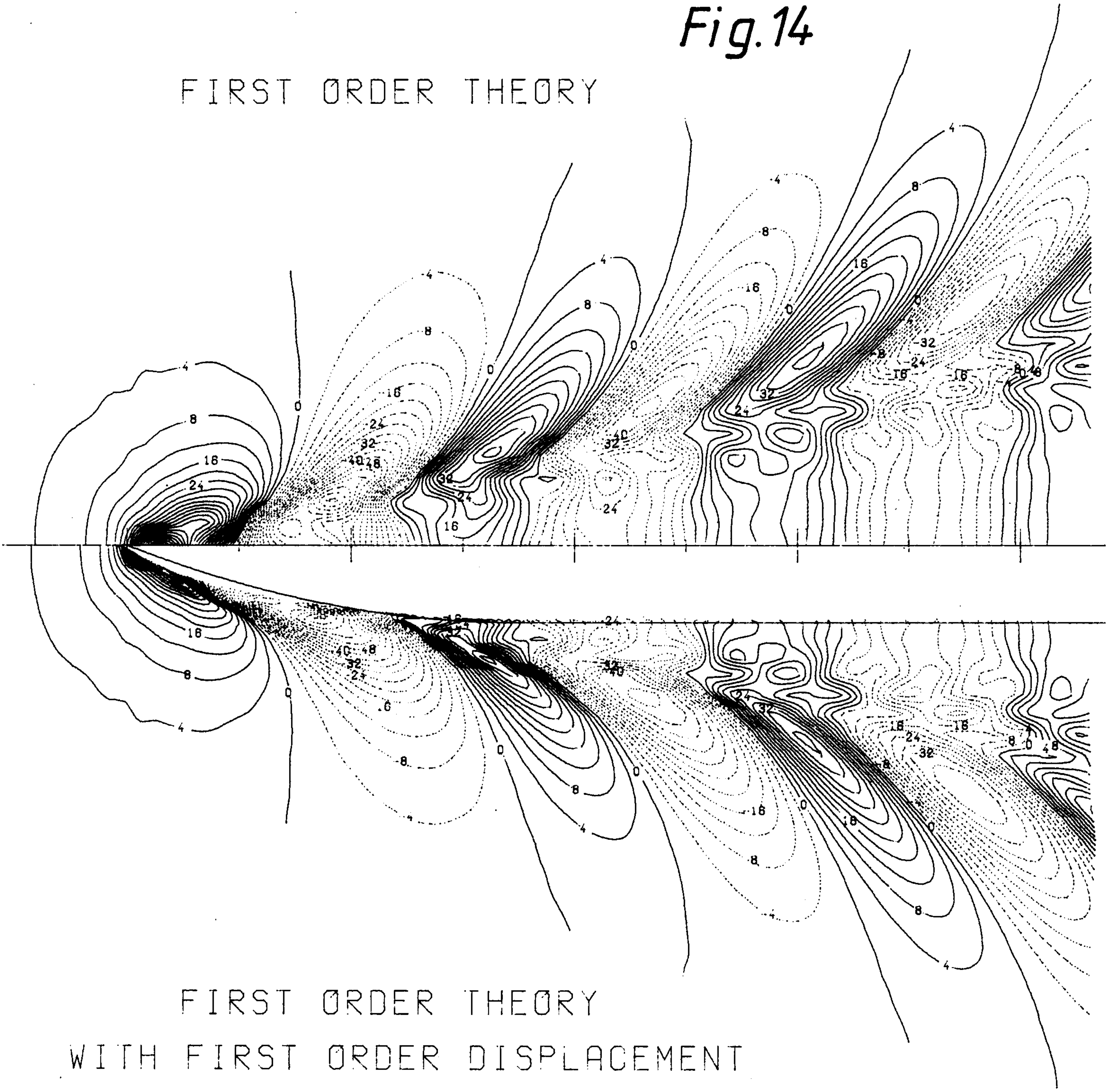


WAVE HEIGHT IN MM

DISTANCE BETWEEN CONTOUR LINES : 4 MM

*Fig.14*

FIRST ORDER THEORY



FIRST ORDER THEORY
WITH FIRST ORDER DISPLACEMENT

WAVE PATTERN ES-201 (ADACHI'S MODEL)

BOW WAVE SYSTEM

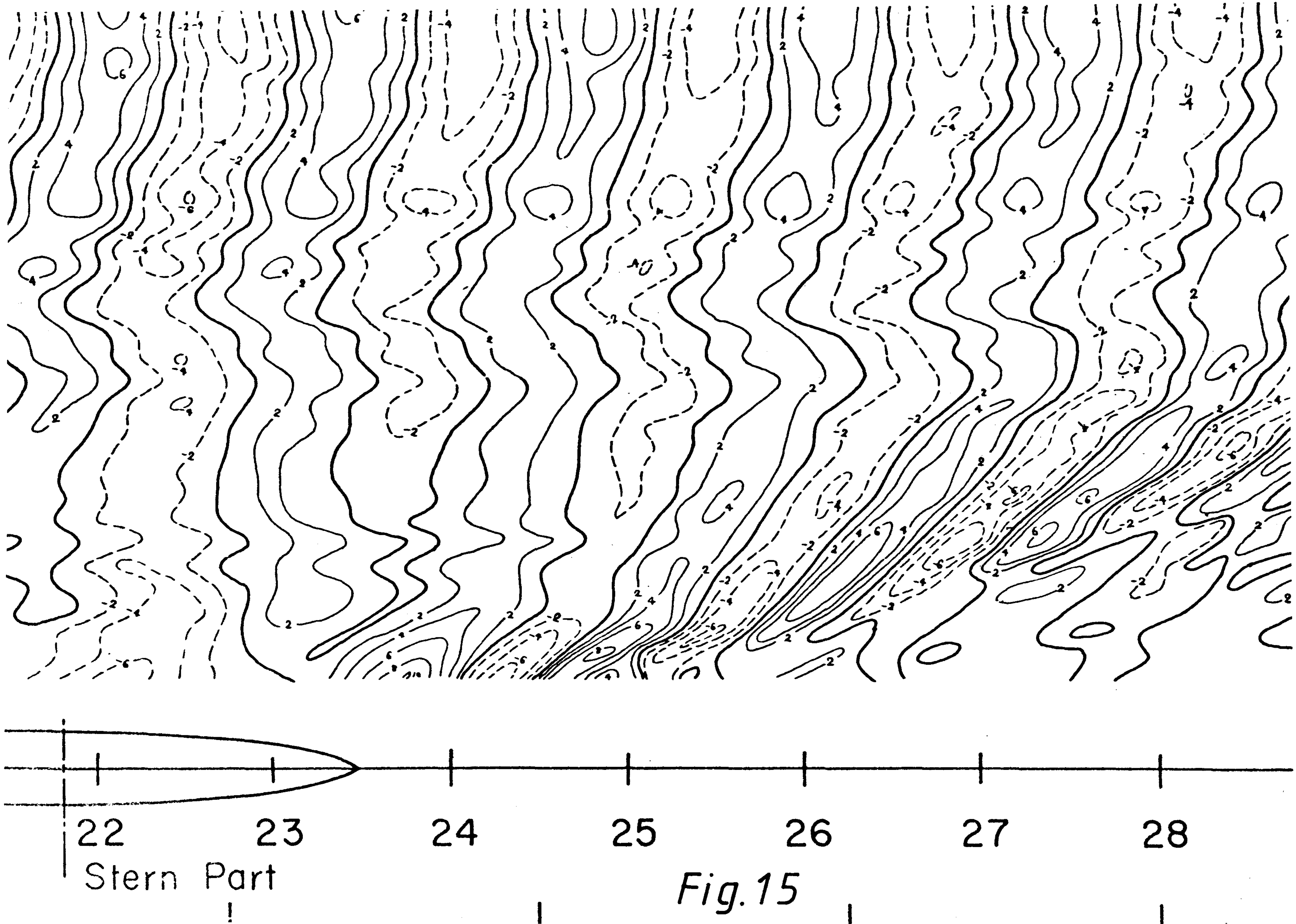



Fig. 15

Fig.16

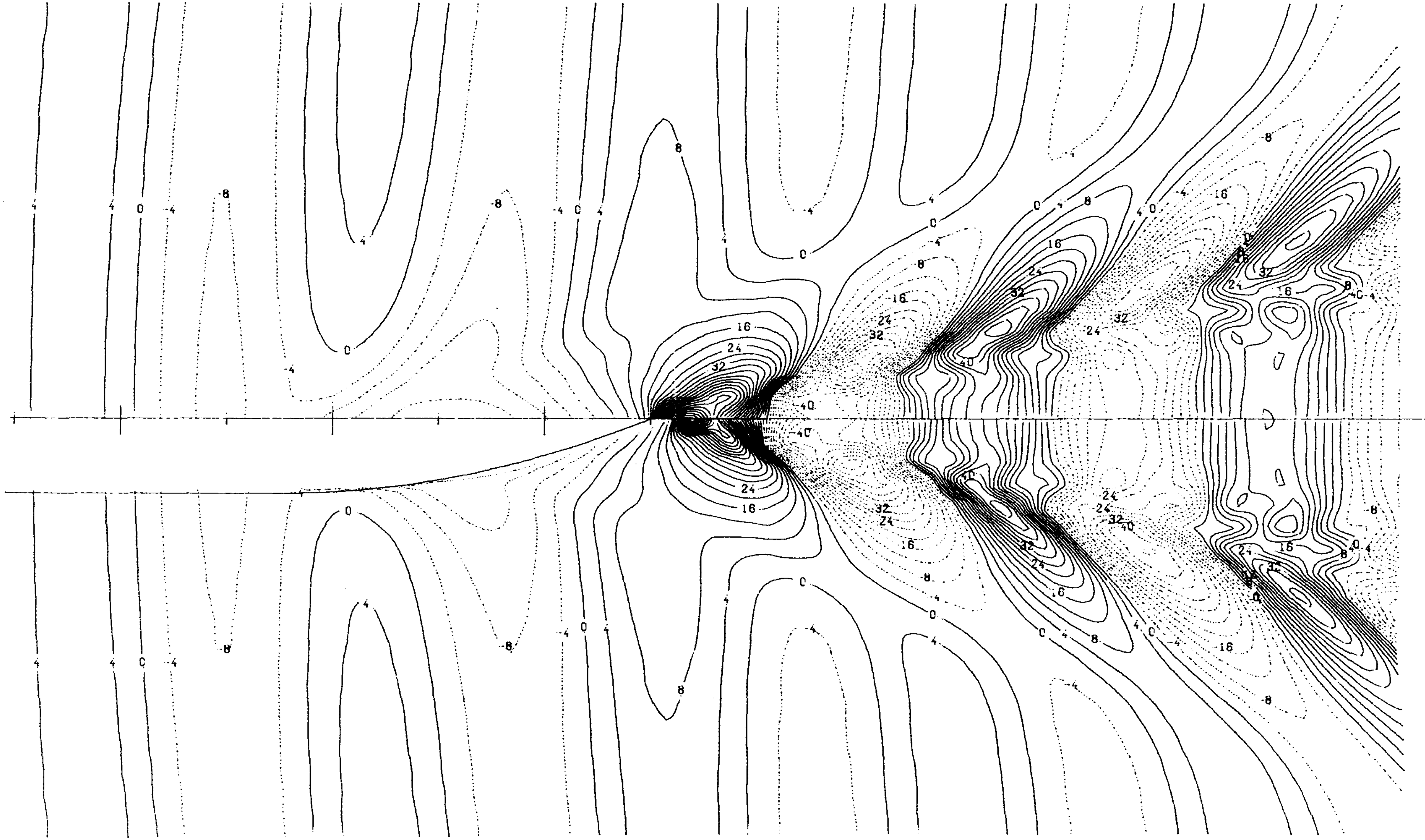
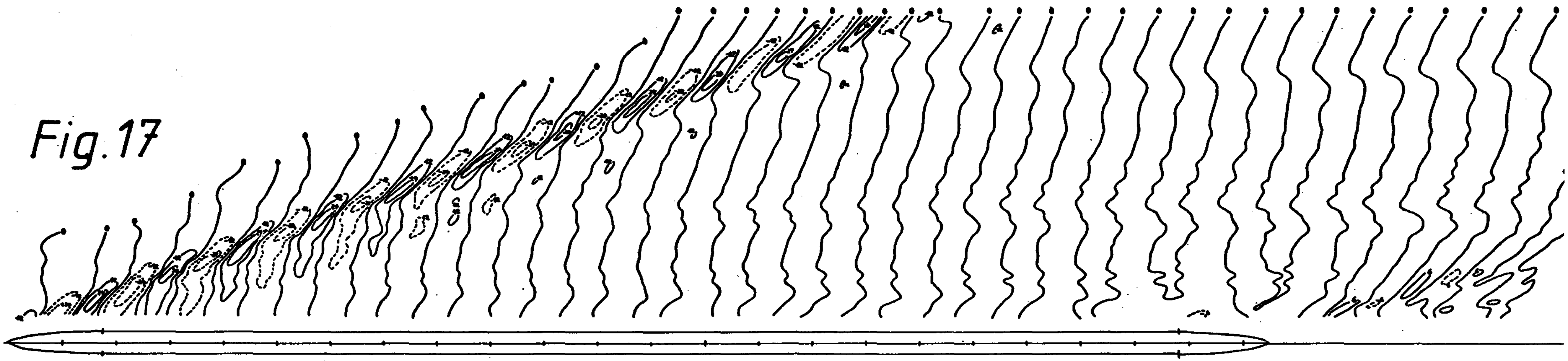


Fig. 17

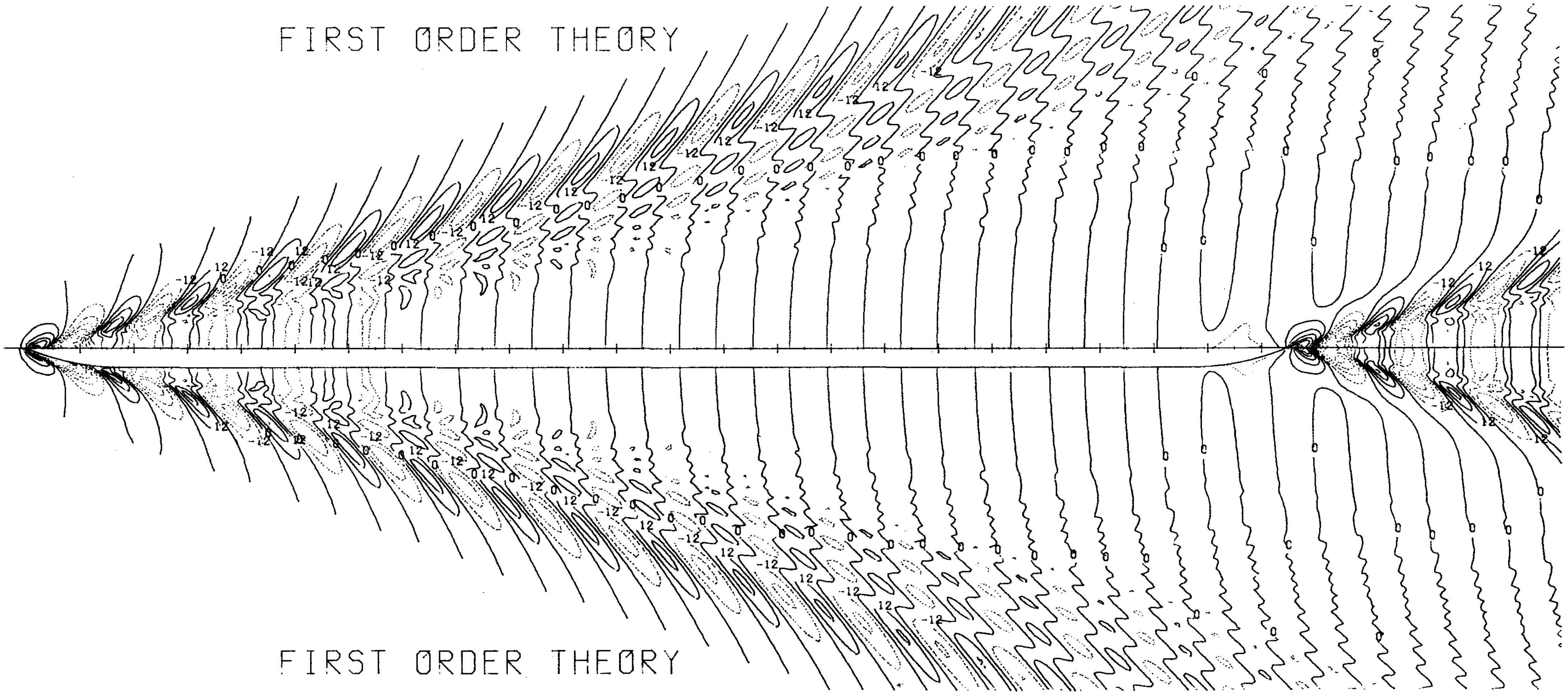


WAVE HEIGHT IN MM

DISTANCE BETWEEN CONTOUR LINES : 12 MM

## Fig.18

FIRST ORDER THEORY



FIRST ORDER THEORY

WITH FIRST ORDER DISPLACEMENT

WAVE PATTERN ES-201 (ADACHI'S MODEL)

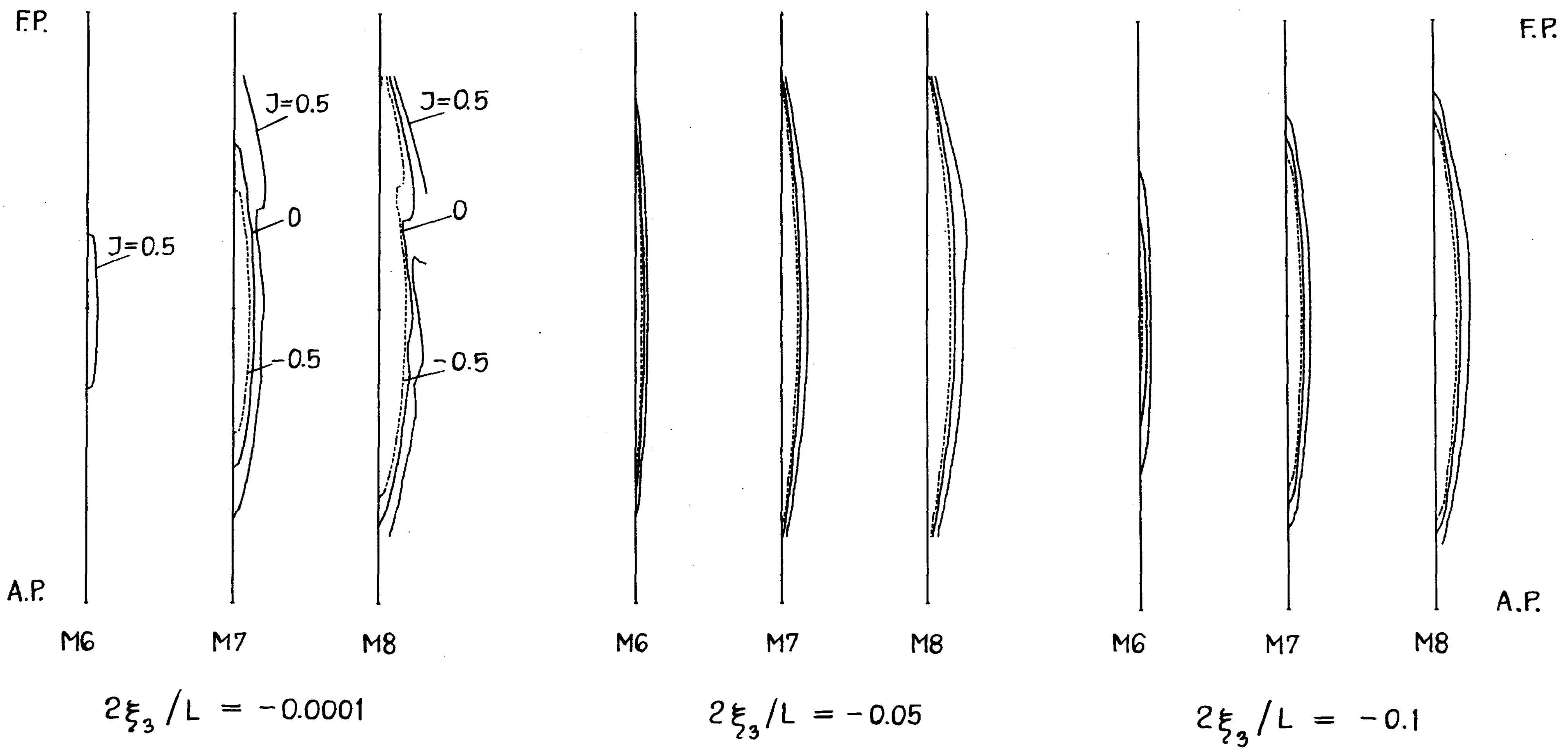



Fig. 19 Contour lines of Jacobian (2.2.2) for M6/M7/M8 at $\gamma_0 = 4$

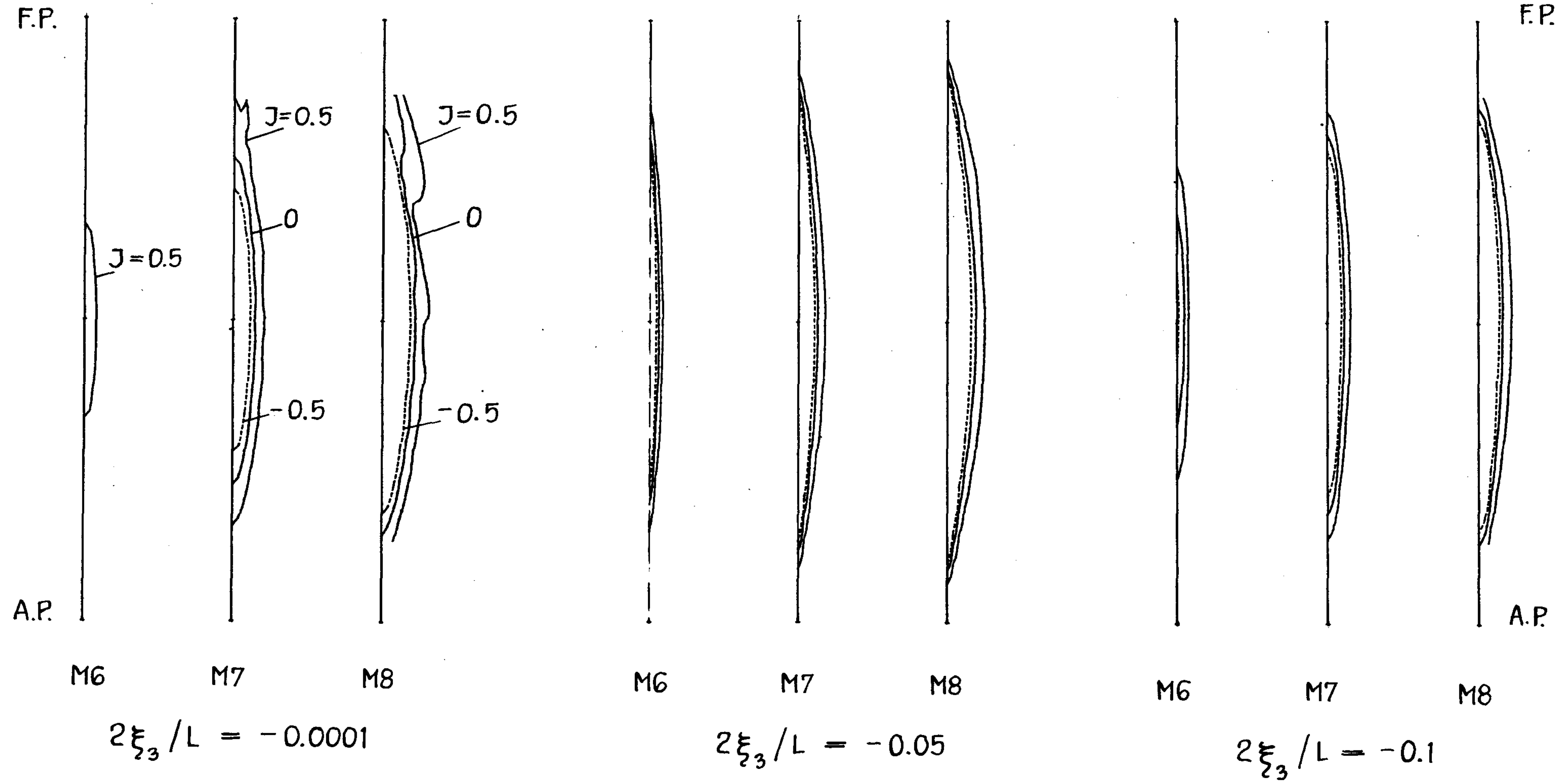



Fig. 20 Contour lines of Jacobian (2.2.2) for M6/M7/M8 at $\gamma_0 = 6$

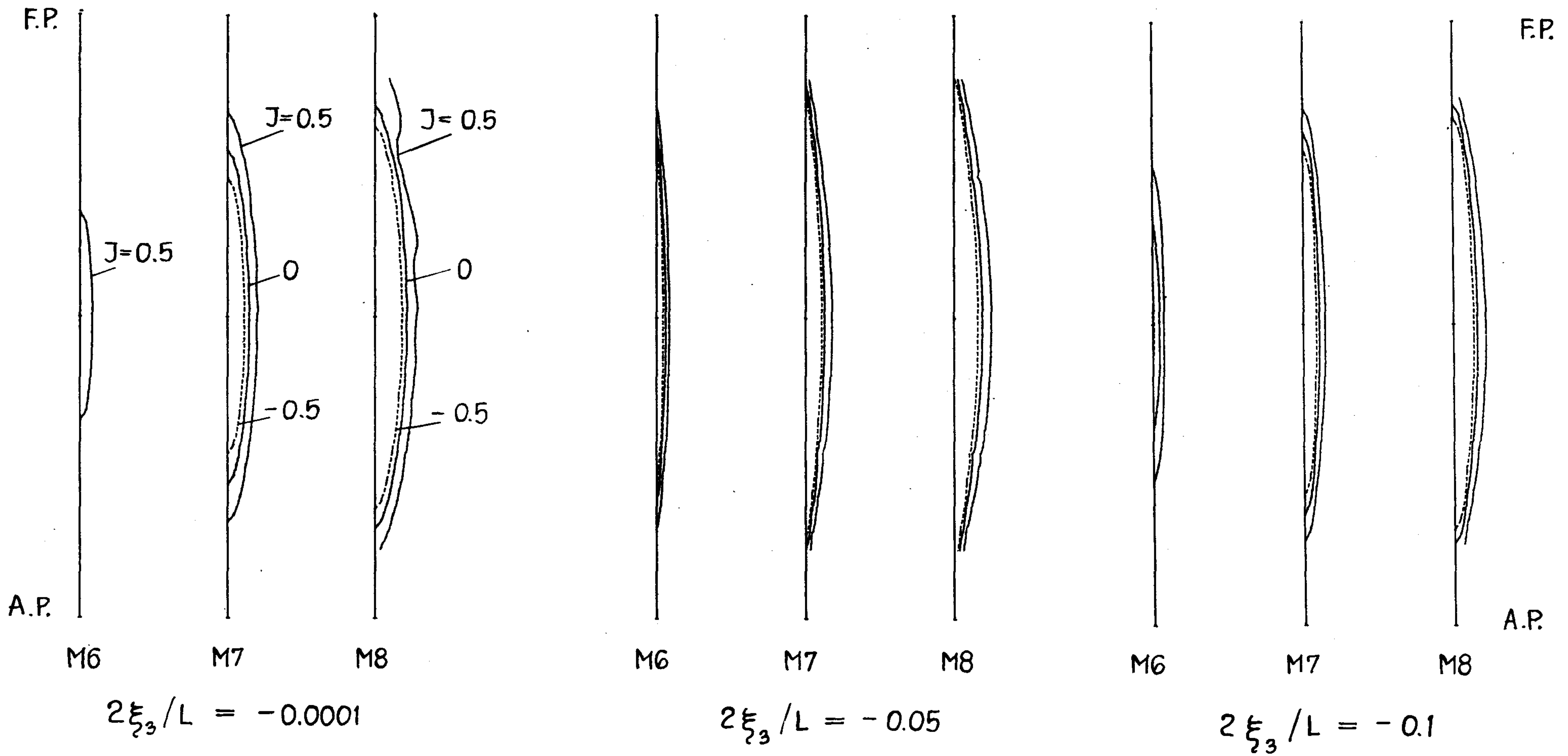



Fig. 21 Contour lines of Jacobian (2.2.2.) for M6/M7/M8 at $\gamma_0 = 9$

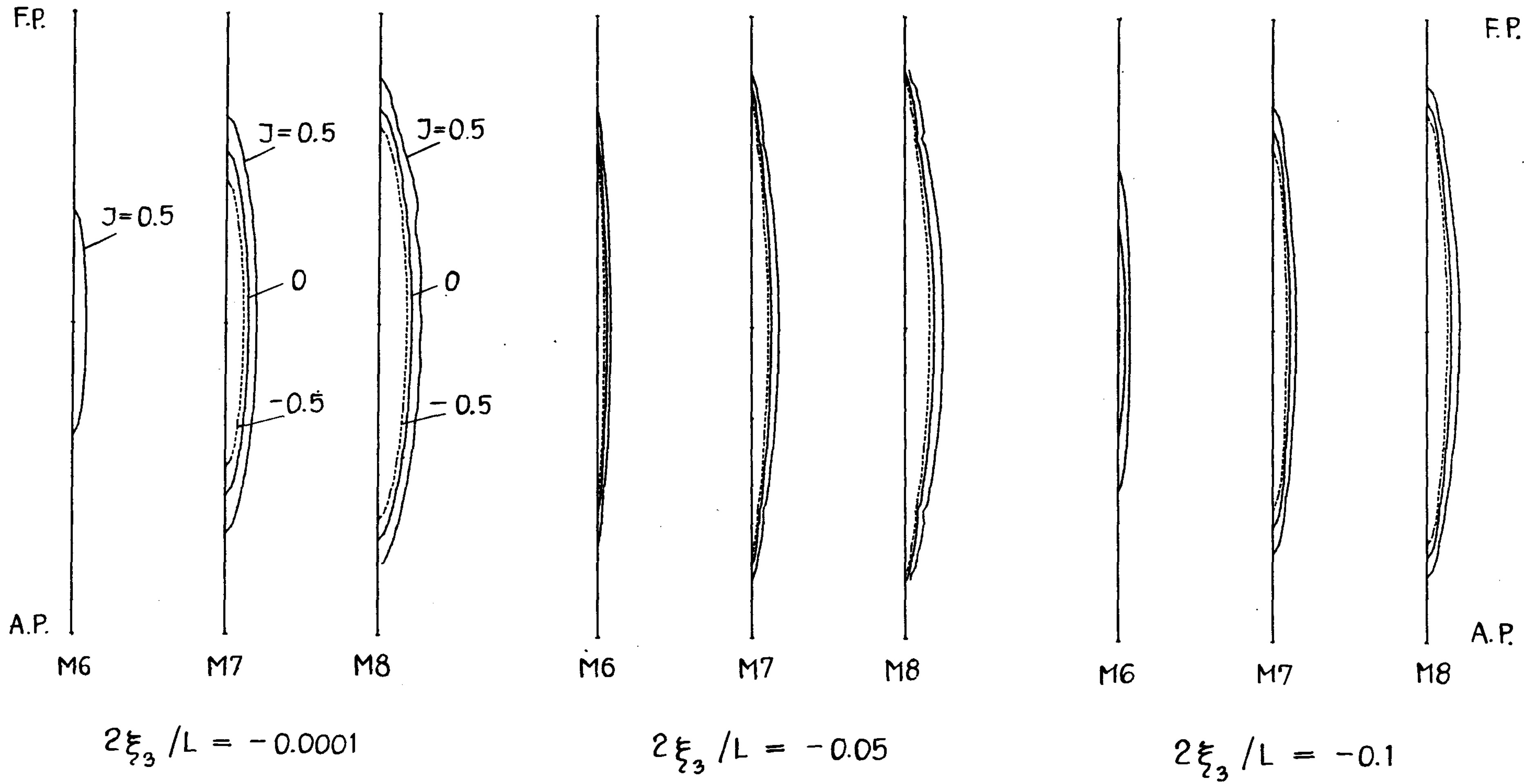



Fig. 22 Contour lines of Jacobian (2.2.2) for M6/M7/M8 at $\gamma_0 = 12$

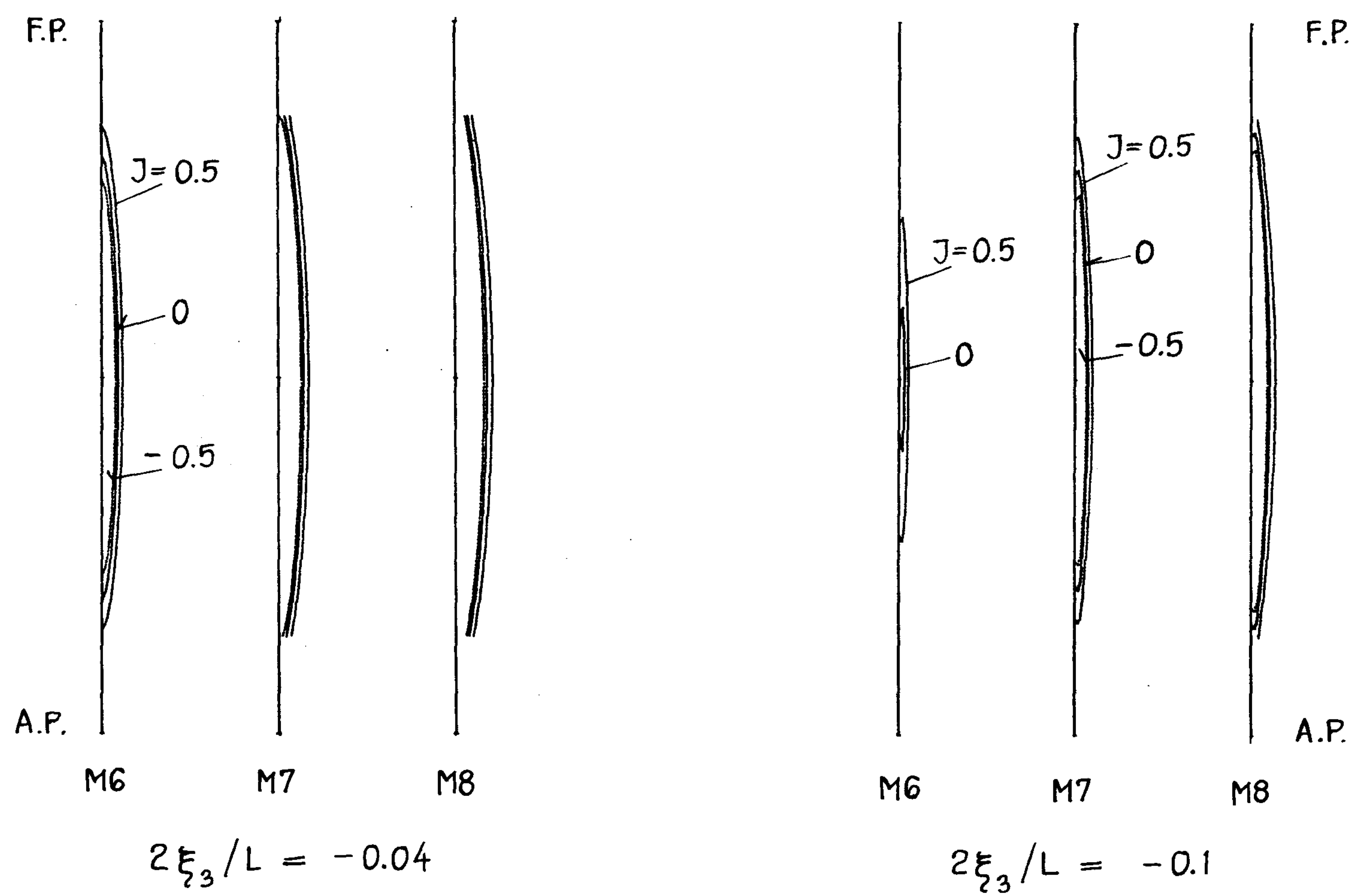



Fig. 23 Contour lines of Jacobian (2.2.2) for M6/M7/M8 at $\gamma_0 = 0$

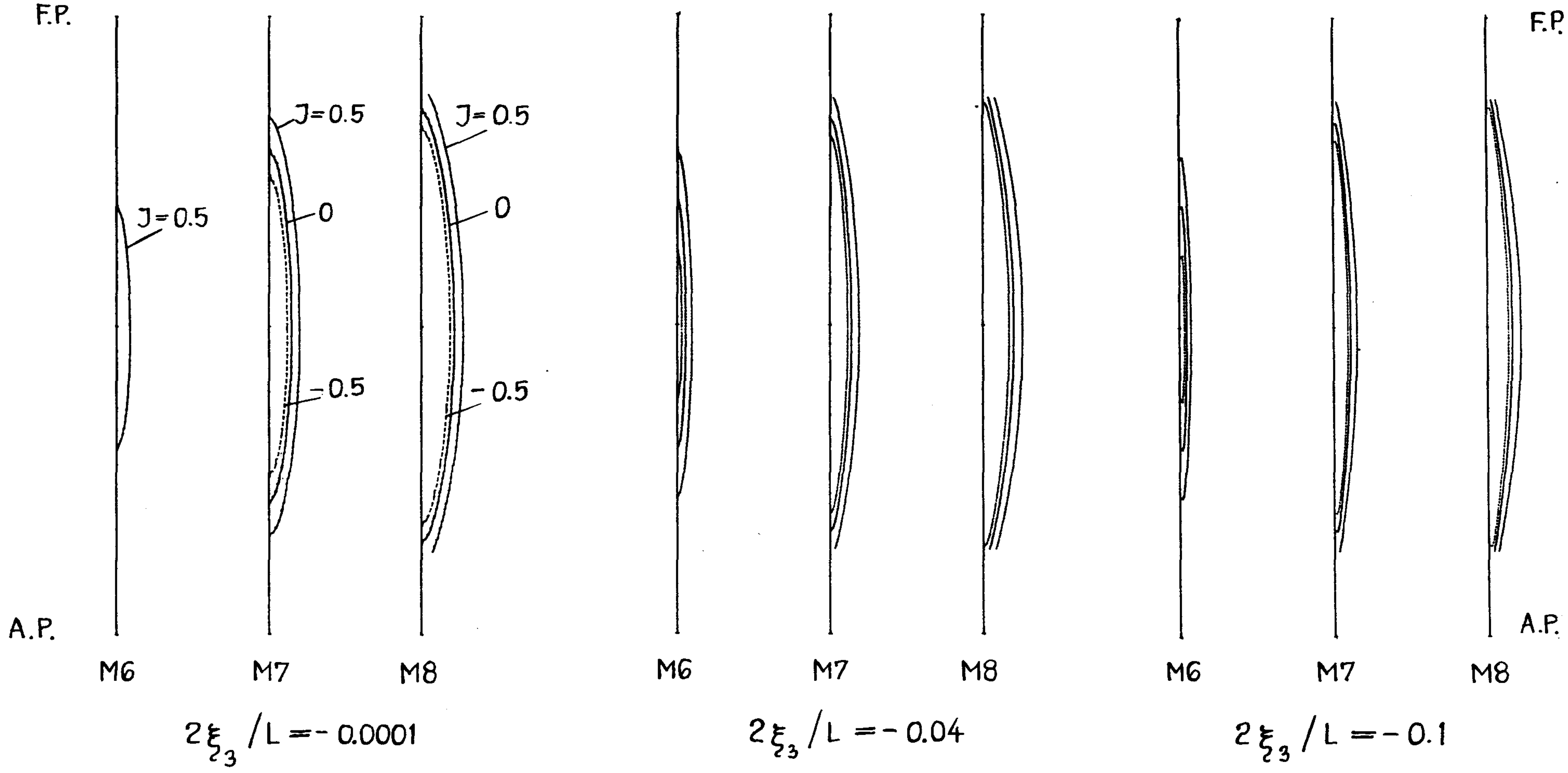



Fig. 24 Contour lines of Jacobian (2.2.2) for M6/M7/M8 at $\gamma_0 = \infty$