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Hierarchical microcrack model for materials exemplified at enamel

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ABSTRACT

Objective. This article investigates the mechanical properties of a material with hierarchically arranged microcracks.

Methods. Hierarchically structured biomaterials such as enamel exhibit superior mechanical properties as being stiff and damage tolerant at the same time. The common mechanical explanation for this behavior is based on the hierarchically structured arrangement of hard minerals and soft organics and their cooperative deformation mechanisms. In situ mechanical experiments with mm-sized bovine enamel bending bars an scanning electron microscope reveal that enamel is able to withstand mechanical loading even if it contains microcracks on different lengths scales. To clarify this issue an analytical hierarchical microcrack model of non-interacting cracks is presented.

Results and Significance. The model predicts a decrease of the elastic modulus and the fracture strength with increasing levels of hierarchy. The fracture strain on the other hand may decrease or increase with the number of hierarchical levels, depending on the microcrack density. This simple hierarchical microcrack model is able to explain already published experiments with focused ion beam prepared μm-sized enamel cantilevers on different hierarchical levels. In addition it is shown that microcracking during loading in hierarchical materials may lead to substantial pseudoplastic behavior.

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1. Introduction

Many biomaterials such as enamel, bone or nacre consist of hard minerals surrounded by soft proteins, which are arranged in a hierarchical manner. Dental enamel, for instance, consists of nanometer-scaled hydroxyapatite (HAP) fibers arranged into rod-like structures. These rods are organized into groups with different orientations crossing each other to form the so-called decussation pattern. This kind of structure – from the HAP nanofiber to the decussation pattern level (as shown in Fig. 1) – is an example of a hierarchical structured biomaterial. In this example, one can define a single HAP nanofiber as the hierarchical level 0 and the bundled HAP nanofibers as the hierarchical level 1. Enamel rods can be defined as the hierarchical level 2, and the decussation pattern as the hierarchical level 3 [1,2].

To overcome the material availability problem with human teeth, bovine teeth – that are thicker and less curvy – are quite frequently used in dental studies, due to their similar morphologies. Both species exhibit three levels of hierarchy: nanofibers, rods and Hunter–Schreger Bands. Moreover, the studies of Yahyazadehfar et al. [3] and Bechtle et al. [4] on human and bovine enamel respectively, revealed that in both species similar toughening mechanisms prevail.

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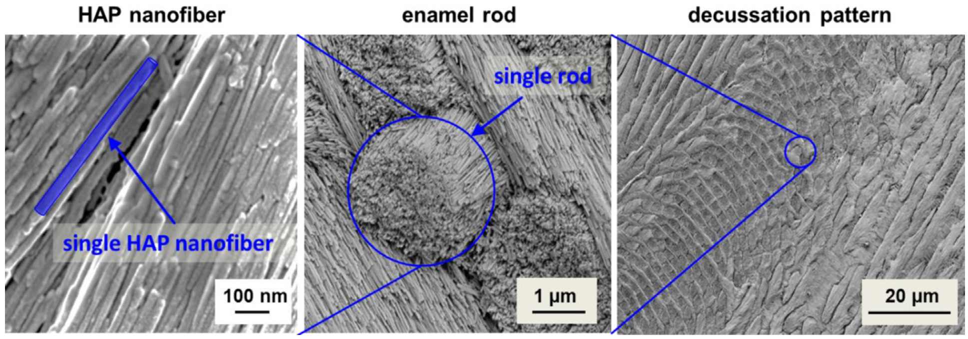

Fig. 1 – Hierarchical structure of bovine enamel: from a single HAP nanofiber to the decussation pattern.

Enamel is a highly mineralized tissue (up to ~85 vol%) and thus the hardest material in the human body. From a mechanical point of view, the most significant aspect is that the hierarchical structure enables enamel being stiff and damage tolerant at the same time. Toughening mechanisms such as crack bridging, microcracking or crack deflection operate during crack growth and are supposed to be even more efficient than in non-hierarchical materials [3–5]. Experiments with µm-sized cantilevers of different hierarchical levels have shown that bovine enamel shows a transition from a stiff and high-strength material to a damage-tolerant and low-strength material on the hierarchical level 3 (decussation pattern) [1]. This result matches very well with the hierarchical mechanics model developed by Gao and co-workers based on the suggestion from Jäger and Fratzl [6]. The model uses a tension-shear-chain model to explain the mechanical behavior of composite structures under tension. It bases on the assumption that the hard particles carry the tensile load, whereas the soft elements transfer the tensile load via shear to the hard particles. A comprehensive explanation of this model can be found elsewhere [7–10].

In this study, in situ bending experiments were performed with mm-sized bovine enamel specimens to get more insight into the role of hierarchy, the interaction of the hierarchical levels and the corresponding deformation mechanisms. While loading the enamel specimens, microcracks on different length scales could be observed on the sample surface in the scanning electron microscope (SEM). Despite of these microcracks, enamel could withstand mechanical loading and deformed macroscopically in a linearly elastic manner until catastrophic failure occurred. However, it is unclear if the microcracks were already in enamel before or if they were generated during the experimental procedure. It is possible that the microcracks are introduced via sample preparation, during evacuating the chamber of the SEM or while loading. Nevertheless, the experiments lead to the question: what is the influence of microcracks on the mechanical behavior of a hierarchically structured biomaterial. It is well known that enamel contains microcracks in different length scales [5] and that enamel continues its function through millions of chewing cycles even if it contains inherently crack-like defects [11].

In this work, we propose a hierarchical microcrack model for hierarchically structured materials. The analytical model shows the influence of microcracks on different hierarchical levels on the elastic modulus, the fracture strength and the fracture strain.

2. Experiments

2.1. *Sample preparation*

Bovine permanent mandibular incisors were used in this study due to their larger size compared to human enamel. They were extracted at a local slaughterhouse (Lippeck & Richter GmbH, Hamburg). Roots were cut off, the pulp interior was removed and the teeth were disinfected within a 0.1 wt% thymol solution for 24 h. Teeth were then rinsed and further stored in Hank's Balanced Salt Solution (HBSS, Invitrogen, USA) to avoid any dehydration and correlated changes in mechanical properties in the sample preparation process. Slices of around 2 mm thickness, as shown in Fig. 2a, were cut out of the middle of bovine incisors using a Buehler Isomet 4000 precision saw. These slices were further sectioned from the labial side into bar-like shape by the saw. The gross bars are grinded down using the profile forms with differently sized gaps for the tooth bar getting smaller with each profile form (shown in Fig. 2b) so that the final required rectangular shape ($\sim 1 \times 1 \times 10\,mm^3$) can be obtained. The last grinding step is done with 1200 grit SiC paper and only the side, which is monitored in the SEM, was further polished with 1 µm and 0.25 µm diamond suspension. The polished surface layer was then subsequently etched for 1 s using 36% hydrochloric acid to make the structural features visible and was sputter-coated with a thin gold layer (some nanometers).

2.2. *Observation of microcracks during in situ bending experiments in the SEM*

The bovine enamel bars were loaded with a 3-point bending device (Microtest 200, Gatan GmbH, Germany) with a span distance of 7 mm, where the sample surface was observed simultaneously by means of the Zeiss Supra 55VP scanning electron microscope (3 kV, 10^{-6} mbar). Before performing the test, the specimen was firstly pre-loaded with a force of ~1N to ensure that the specimen did not move while evacuating the chamber of the SEM. The experiments were performed

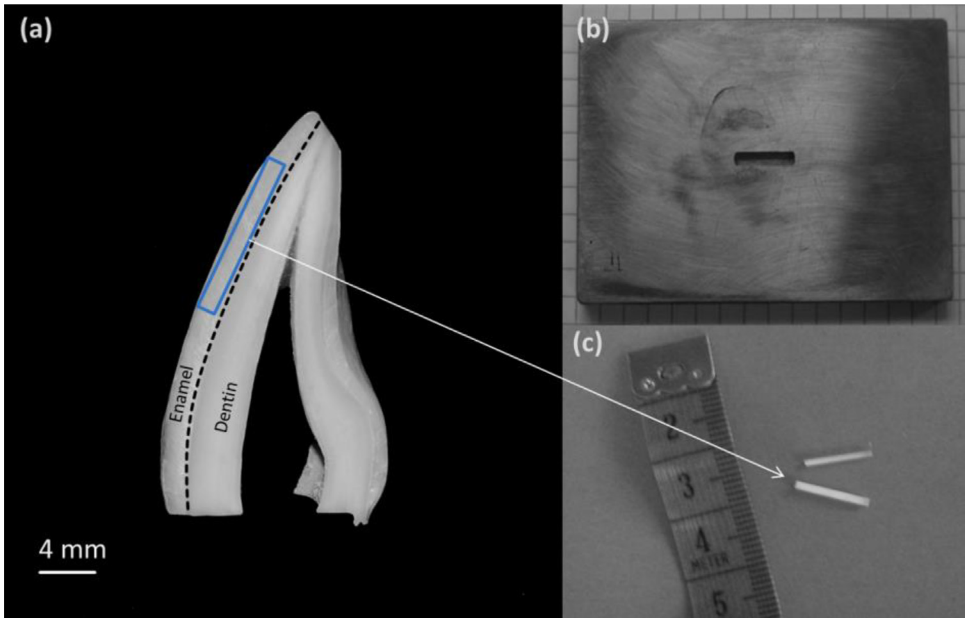

Fig. 2 – (a) The longitudinal view of the bovine incisor cross-section (b) a profile form used for the bending bar preparation (c) mm-sized enamel bending bars.

with a displacement rate of 0.1 mm/min. A representative force-displacement curve of a bending experiment with enamel specimen is shown in Fig. 3. The specimen failed catastrophically at a force of 16N. The sample surface was scanned at several load levels (indicated by the dashed rectangles). For this, the loading was stopped and the displacement was kept constant. This explains the data points with constant displacement marked with dashed rectangles. The scanning time between each loading step was approximately 10 min. Due to technical problems the data points between 0 N and 4 N could not be recorded. The solid line was fitted with all displayed data points by linear regression, where the curve is extrapolated to the origin of the coordinate system. It should be noted that the measured displacement includes the compliance of the device as well. The overall slope of the measured curve does not change significantly until catastrophic failure.

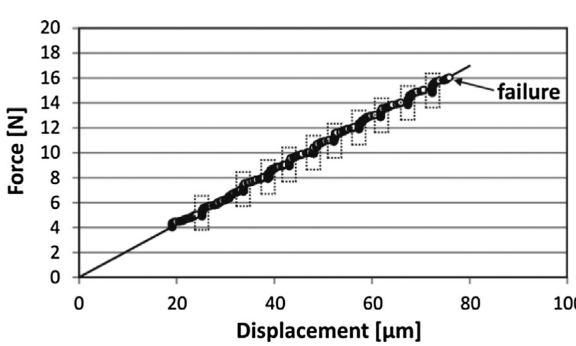

Fig. 3 – Force-displacement curve of a bending experiment with enamel specimen. The sample fails catastrophically at a force of 16N. The solid line is fitted by linear regression using all displayed data points. The experiment was performed in the SEM in order to be able to scan the sample surface at several loads, where the displacement was kept constant. These scanning periods are indicated by the dashed rectangles.

Fig. 4 shows the middle section of the specimen (near the middle support). This image was scanned in situ at a load level of 15 N. Several cracks with different sizes (up to several hundred µm) – in the inner enamel layer exposed to tensile stresses during bending – can be seen. These cracks are denoted in the following as microcracks. Some of them are indicated by arrows. It should be noted that these cracks were not visible in the SEM before loading. Nevertheless, it is unclear if these microcracks were generated while loading, or if they were already in the sample before loading. In Fig. 5, the SEM-image from another specimen is shown at higher magnification. The SEM-images in Fig. 5a and b were scanned at 1 N and 9 N, respectively.

In further experiments, enamel specimens were loaded and unloaded several times without catastrophic failure. The slope of the loading and unloading force-displacement curve did not change significantly from one cycle to the other cycle (less than 1%). The specimen surface was scanned at several load levels and microcracks were observed as well. Furthermore, it could be observed that some of these microcracks grow in a stable manner without becoming unstable. The specimens were loaded up to stresses of ~200 to 300 MPa and did not fail.

3. Hierarchical microcrack model

The in situ observation of the bending experiments shows that enamel can withstand mechanical loading even if it contains microcracks of different sizes. Furthermore, it can be concluded that while loading – or from one loading cycling to the next one – the slope of the force-displacement curve is

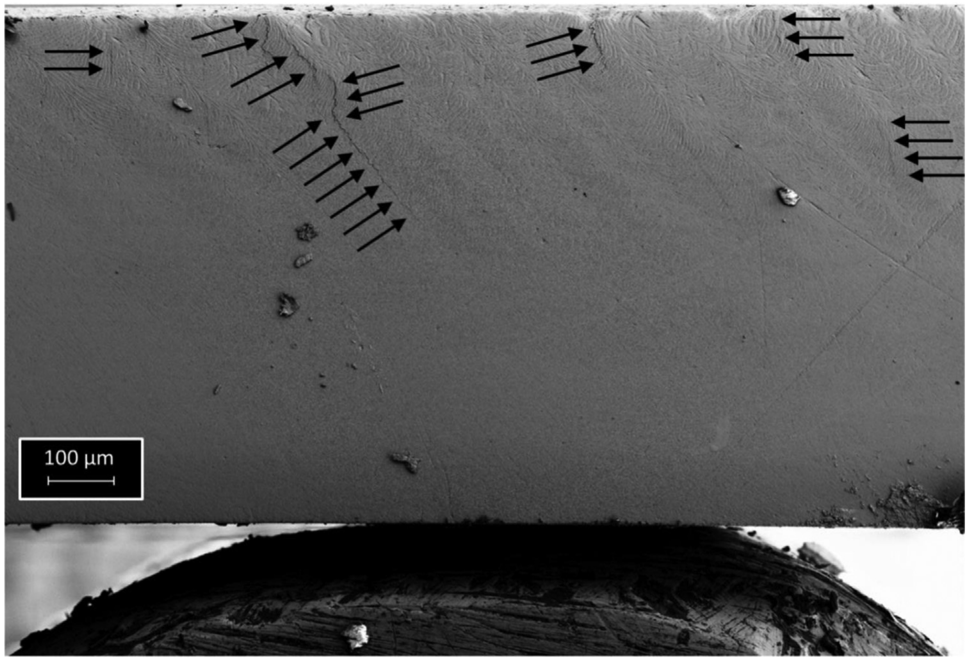

Fig. 4 – SEM image (for the experiment from Fig. 1) scanned at a load level of 15N. The black arrows indicate some of the cracks, which become visible when loading the specimen.

linear until catastrophic failure occurs. It is well known that the slope of the force-displacement curve decreases due to crack growth, microcracking or plastic deformation. In the performed experiments, none of these phenomena have caused a significant slope decrease until catastrophic failure occurred. However several microcracks could be observed. Hence, the question arises whether the microcracks have an influence on the mechanical behavior. Microcracks have already been investigated intensively for materials such as ceramics, concrete or rocks as well on different scales [12–17]. But to the authors knowledge, up to now there is no microcrack model for hierarchically structured materials, which describes the influence of the microcracks on the elastic modulus, fracture strength and fracture strain in dependence of the hierarchical level. In the following, we propose an analytical microcrack model for a material, which consists of n self-similar hierarchical levels. We assume that the material exhibits microcracks on all hierarchical levels and that the microcracks are already in the specimen before loading. All other effects like plastic deformation, stable crack growth or toughening mechanisms are neglected in the following. This study investigates the influence of a hierarchical microcrack-pattern on the elastic modulus, fracture strength and fracture strain of a isotropic linear elastic material.

3.1. *The hierarchical geometry*

Let us assume a material, which consists of n self-similar hierarchical levels. It is assumed that the material is built of cubic volume elements, where the smallest volume element has a side length of l_0 and is defined as the hierarchical level 0. Each following hierarchical level itself has a cubic shape as well. In

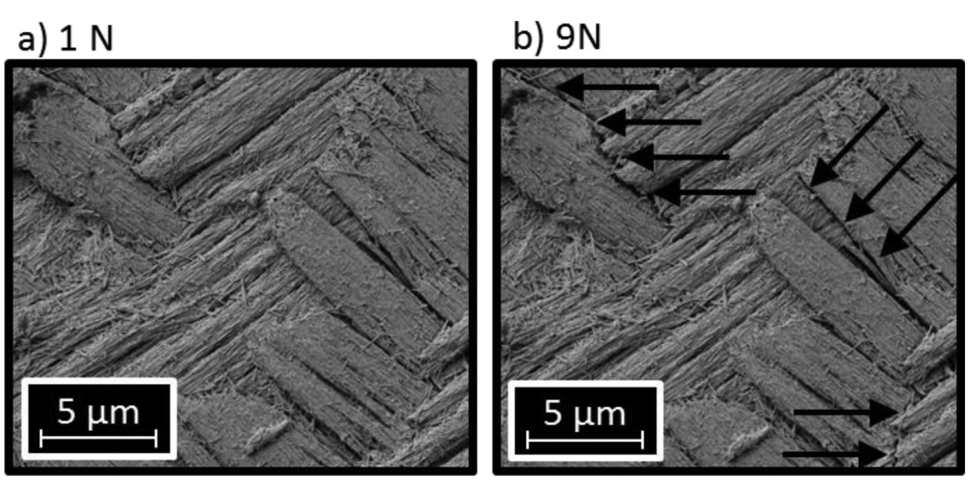

Fig. 5 – SEM-image of an enamel specimen a) after pre-loading b) at a load level of 9 N.

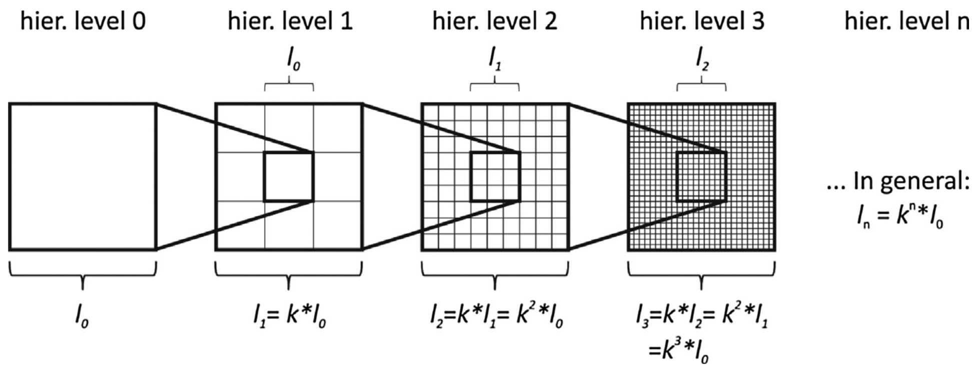

Fig. 6 – Schematic of the self-similar hierarchical geometry of a material with n hierarchical levels. In this figure as an example, k = 3 is assumed for all hierarchical levels.

Fig. 6 a two-dimensional schematic of the hierarchical geometry is shown. Assuming that the hierarchical level 1 consists of k^3 cubes from the hierarchical level 0, the total side length of the hierarchical level 1 is equal to $l_1 = k_1 \times l_0$. Here k_1 is a scaling factor between the hierarchical level 0 and the hierarchical level 1. For the sake of simplicity, it is assumed that the scaling factor is constant for all hierarchical levels ($k_n = k$). Accordingly, the hierarchical level 2 consists of k^3 cubes from the hierarchical level 1. Thus, the total side length of the hierarchical level 2 is $l_2 = k \times l_1 = k^2 \times l_0$. From this hierarchical geometry it follows that the total side length of a material with *n* hierarchical levels is $l_n = k^n \times l_0$.

3.2. *Microcracks*

The material consists of penny-shaped microcracks on each hierarchical level. The microcrack diameter – for microcracks on the hierarchical level *n* – is assumed to be l_{n-1}. A two-dimensional schematic is shown in Fig. 7. For the sake of simplicity it is assumed that the material is under a uniaxial tensile load in x-direction. Therefore, only microcracks in y–z-plane with a surface area of $\pi l^2{}_{n-1}/4$ (as shown in Fig. 7) are assumed to influence the mechanical behavior of the material.

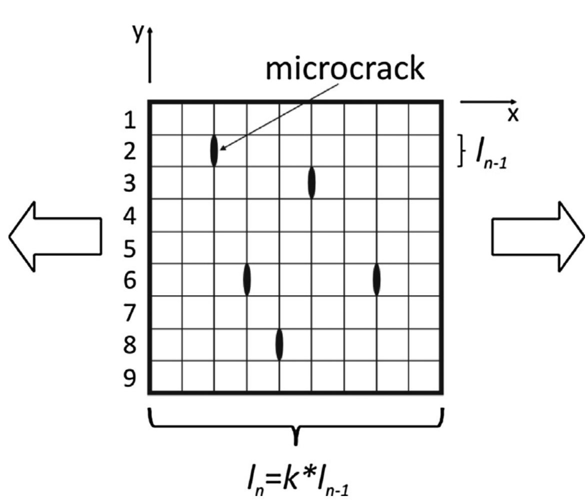

Fig. 7 – Schematic of microcracks for a structure with hierarchical level n. In this figure k = 9, N = 5.

With the absolute number of microcracks on level n, N_n in the volume $l_n{}^3$ the microcrack number density is $c_n = N_n/l_n{}^3$. The density ω_n of penny-shaped microcracks of diameter l_{n-1}, on the hierarchical level n is defined as:

$$\omega_n = c_n \left(\frac{l_{n-1}}{2}\right)^3 = \frac{N_n}{8k^3}, \tag{1}$$

where $\omega_n < 1$, meaning that the microcracks should not interact. For the following consideration we assume ω_n as a material constant, which does not change while loading and is constant for all hierarchical levels ($\omega_n = \omega$; $N_n = N$). In other words, the hierarchical microcrack distribution exists before loading and there is no crack propagation until the material fails catastrophically.

3.3. *Elastic modulus*

The influence of microcracks on the stiffness of a linearly elastic material has been studied in detail. As it is intuitively expected the elastic modulus decreases as a function of the number of microcracks and their size. Both the size and the number of microcracks is cast into the dimensionless parameter? which can be interpreted as a measure of the volume density of cracks. Here we use the result for the effective modulus E_{eff} for non-interacting penny-shaped microcracks oriented perpendicular to the loading direction in an isotropic medium with elastic modulus E_0 and Poisson ratio ν_0 [17]:

$$E_{eff} = \frac{E_0}{(1 + A\omega)} \tag{2a}$$

The parameter A depends on the shape and orientation of voids (or more general inclusions) used in the model. As we are interested in cracks the parameter A is identified as [18]:

$$A = \frac{16}{3}\left(1 - \nu_0^2\right) \tag{2b}$$

In the following a typical Poisson's ratio of 0.25 for brittle materials is assumed, which gives exactly $A = 5$.

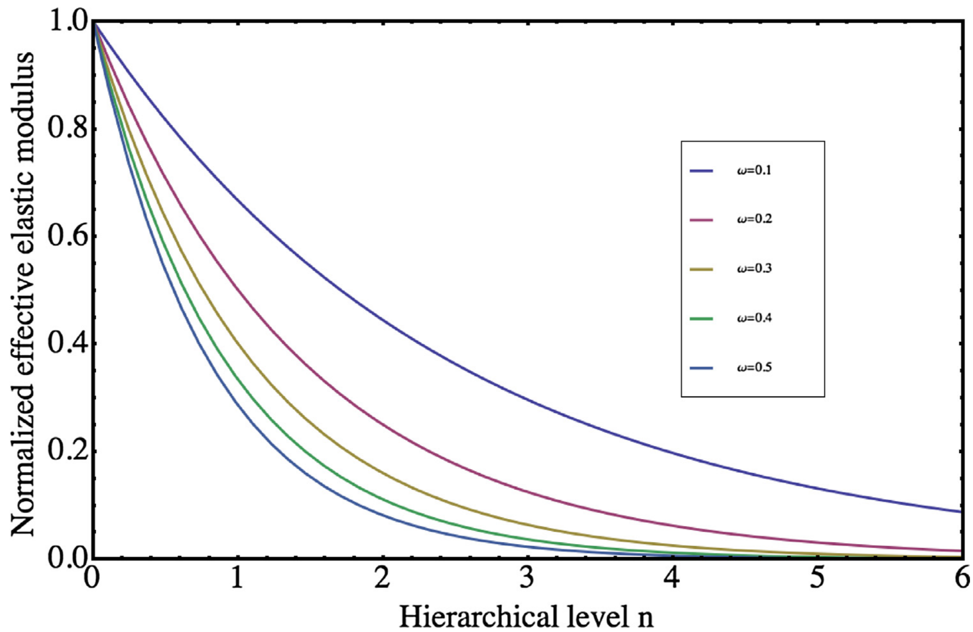

Fig. 8 – Normalized effective elastic modulus (E_n/E_0) as a function of the hierarchical level n according to Eq. (3) with different microcrackdensities ω (A = 5).

Applying Eq. (2a) on our model the effective modulus of hierarchical level 1 can be written as:

$$E_1 = \frac{E_0}{(1 + A\omega)} \tag{2c}$$

E_0 is equal to the elastic modulus of the smallest length scale (hierarchical level 0).

The generalization of this argument gives the elastic modulus for the hierarchical level n:

$$E_n = \frac{E_0}{(1 + A\omega)^n} \tag{3}$$

Accordingly, the elastic modulus decreases with increasing hierarchical level n as well as with increasing ω, as shown in Fig. 8.

3.4. *Fracture strength*

We assume that the fracture toughness of the material is constant (no R-curve behaviour) and the same on each hierarchical level, $K_{Ic,n} = K_{Ic}$. Moreover we assume that for $n \geq 1$, only the longest crack on the highest hierarchical level with crack length l_{n-1} leads to fracture. It does not matter whether all microcracks on the highest hierarchical level propagate in parallel or whether only on crack is going to propagate, as the material will fail catastrophically in both cases. Hence the scaling law for the hierarchical strength is as follows:

$$\sigma_{c,n} = \frac{K_{Ic}}{\sqrt{2\pi l_{n-1}}} = \frac{K_{Ic}}{\sqrt{2\pi k^{n-1} l_0}}, \textit{for } n \geq 1 \tag{4}$$

The factor $\sqrt{2}$ is due to the penny-shaped crack geometry, which differs from the 2-dimensional situation of a Griffith crack. The normalized strength is defined as:

$$\sigma_{c,n}^{normalized} = \frac{\sigma_{c,n}}{\sigma_{c,1}} = \frac{1}{\sqrt{k^{n-1}}} \tag{5}$$

and plotted in Fig. 9. The length l_0 is chosen that it corresponds to the size where flaw tolerance is achieved [9].

$$l_0 = \frac{2\pi \Gamma E_0}{\sigma_{c,0}^2} \tag{6}$$

with $\Gamma = \frac{K_{Ic}^2}{E}$. Hence, $\sigma_{c,0}$ corresponds to the theoretical strength of the material. Solving Eq. (6) for $\sigma_{c,0}$ gives:

$$\sigma_{c,0} = \frac{K_{Ic}}{\sqrt{2\pi l_0}} = \sigma_{c,1} \tag{7}$$

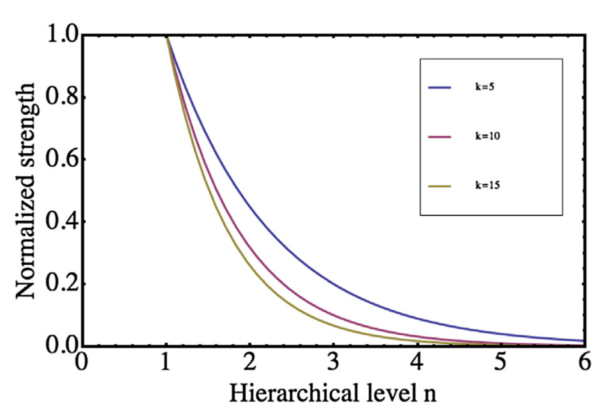

Fig. 9 – Normalized fracture strength ($\sigma_{c,n}/\sigma_0$) as a function of hierarchical level n according to Eq. (5).

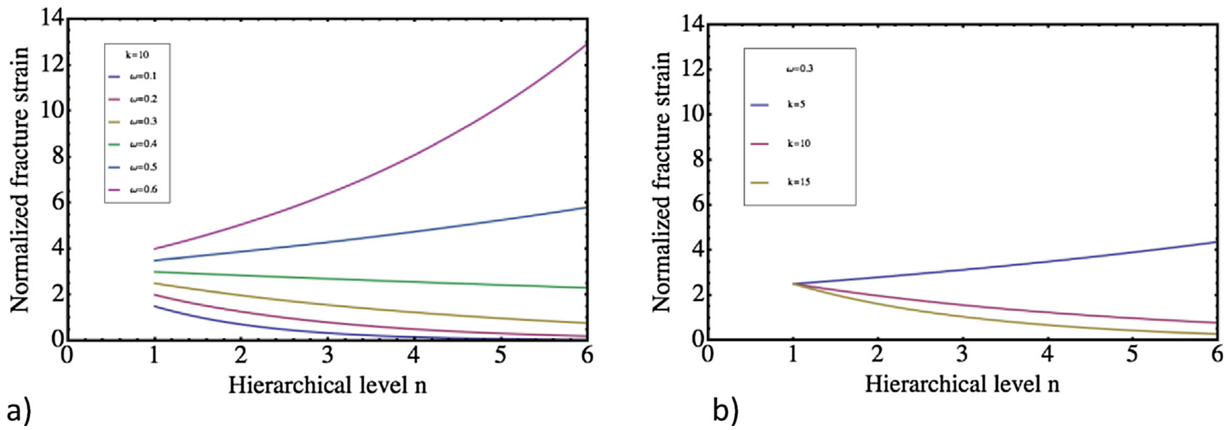

Fig. 10 – Normalized fracture strain as a function of the hierarchical level according to (10). In a) k = 10 and ω is varied whereas in b) ω = 0.3 is kept constant and k is varied.

This means that the fracture strength on the hierarchical level 1 and 0 is identical. This result is not astonishing because flaw tolerance means that for crack sizes smaller or equal l_0 the material does not fail because of the – typically – nanosized crack but rather the atomic bonds break and determine the strength.

It has to be mentioned that Eq. (6) is not exactly identical with the result given in Ref. [8], because here a 3-dimensional geometry with a penny-shaped crack is used whereas in Ref. [8] a Griffith-crack in a 2-dimensional geometry was treated.

3.5. *Fracture strain*

The hierarchical material is also with microcracks linear elastic, hence:

$$\sigma_n = E_n \varepsilon_n \tag{8}$$

Together with Eqs. (3) and (4) it follows for the fracture strain $\varepsilon_{c,n}$:

$$\varepsilon_{c,n} = \frac{\sigma_{c,n}}{E_n} = \frac{\frac{K_{Ic}}{\sqrt{2\pi k^{n-1} l_0}}}{\frac{E_0}{(1+A\omega)^n}} = \frac{K_{Ic}(1+A\omega)^n}{E_0\sqrt{2\pi k^{n-1} l_0}}. \tag{9}$$

The corresponding normalized fracture strain is:

$$\varepsilon_{c,n}^{normalized} = \frac{\varepsilon_{c,n}}{\varepsilon_{c,0}} = \frac{(1+A\omega)^n}{\sqrt{k^{n-1}}} \tag{10}$$

with $\varepsilon_{c,0} = \frac{\sigma_{c,0}}{E_0}$. There exists a critical microcrack density ωc,n which depends on k:

$$\omega_c(k) = \frac{1}{A}\left(\sqrt{k} - 1\right) \tag{11}$$

For $\omega > \omega_c(k)$ the fracture strain increases with hierarchical level and vice versa.

This can be seen in Fig. 10 where the normalized fracture strain is plotted as a function of the hierarchical levels. In Fig. 10 k = 10 is kept constant and the microcrack density is varied. The critical microcrack density is $\omega_c(k) = 0.43$ and for ω > 0.43 the fracture strain increases with n. Accordingly, the fracture strain increases with increasing hierarchical level n as shown in Fig. 10b when, for a fixed ω, the factor k exceeds a critical value $k_c > (1 + A\omega)^2$, which is kc > 6.25.

3.6. *Stress–strain curve*

The general stress–strain behaviour of a brittle material with a hierarchical crack system can be described by a normalized stress–strain curve as follows.

$$\sigma_n^{normalized} = \frac{\sigma_n}{\sigma_{c,0}} = \frac{1}{(1+A\omega)^n}\varepsilon_n^{normalized} \tag{12}$$

with $\varepsilon_n^{normalized} = \frac{\varepsilon_n}{\varepsilon_{c,0}}$.

Together with the fracture strain $\varepsilon_{c,n}$ from Eq. (9) the influence of the hierarchical levels and crack densities can be investigated as shown in Fig. 11. If the crack density is below the critical microcrack density ω_c the stress strain curve has a decreasing elastic modulus and fracture strain with hierarchical level (Fig. 11a). This behaviour was identified in micro-beam tests on enamel [1]. Interestingly it is also possible that the facture strain increases with increasing hierarchical level for $\omega > \omega_c(k)$ even though the elastic modulus decreases (Fig. 11b). Finally Fig. 11c shows that the fracture stress is independent of the microcrack density because only the absolute crack length determines the strength. This feature of the model is due to the assumption that the cracks do not interact and propagate.

4. Discussion

The in situ experiments have shown that enamel is able to withstand mechanical loading despite of the occurrence of microcracks on different lengths scales. It should be noted that in some in situ experiments stable crack growth could be observed, i.e., some cracks at the specimen surface grew in a stable manner. However, the enamel specimens behaved linear elastic until catastrophic failure occurred. Furthermore, it has been observed that the microcracks are inhomogeneously distributed. It can be assumed that the generation of microcracks reduces local stress concentrations, which avoids an immediate catastrophic failure. Since the slope of the force-displacement curve is linear until catastrophic failure, it can be assumed that the majority of the microcracks are already in the specimen before mechanical loading. Otherwise the slope of the force-displacement curve would have been decreased significantly. Moreover, the linear shape of the force-displacement curve implies that there is no significant crack growth, toughening mechanisms or nonlinear response of the protein during the in situ experiments.

All these experimental findings lead to the question, how microcracks influence the mechanical behavior in a hierarchically structured material. The proposed hierarchical microcrack model is simplistic in the sense that it is neglects the complex anisotropic and even inhomogeneous structure of natural enamel. This reduction in complexity is due to a analytical solution of the mechanical problem, which enables the determination of the elastic modulus, the fracture strength and the fracture strain as a function of the hierarchical level and microcrack density; as displayed in Fig. 11.

Already published experiments with μm-sized focused ion beam (FIB) prepared cantilevers of different hierarchical levels [1] show the same trend as in Fig. 11a. In addition, the FIB experiments have revealed that enamel shows a transition from a brittle to a damage-tolerant behavior for the highest investigated increasing level of hierarchy $n = 3$.

In addition the proposed hierarchical microcrack model is able to explain a pseudoductile mechanical behavior under the following assumptions.

1. It is assumed that the hierarchical microstructure determines the typical microcrack length as given in chapt. 3.2. That means that the microcrack diameter in level n is the length corresponding to the length of the hierarchical level l_{n-1}.
2. Due to the complex interwoven microstructure like for example in enamel microcrack growth is not allowed.
3. In order release local stresses the number density N_n of microcracks can be increased during loading.

A qualitative stress strain curve for such a situation is shown in Fig. 12 for a material with 3 hierarchical microcrack levels. The elastic modulus of the material at the beginning is determined by the initial crack densities ω_1, ω_2 and ω_3, which correspond to the number densities c_1, c_2, and c_3 (stage 1 in Fig. 12) for a given l_0, l_1, l_2. If we assume that at a certain stress level the microcrack density increases on level 1 because the internal stresses are higher on this length scale than on level 2 the number density c_1 increases and the elastic modulus decreases at almost constant or slightly increasing stress – as some microcracks on level 1 may need higher stresses to be initiated (stage 2 in Fig. 12). The result is a pseudoplastic behavior. If c_1 reaches a saturation density c_1^{sat} the stress strain curve increases with the slope corresponding to (c_1^{sat}, c_2) (stage 3 in Fig. 12). If now at a higher stress the same szenario as in level 1 happens on level 2 until c_2^{sat} is reached (stage 4 in Fig. 12). This model assumes that the either the internal stresses on the

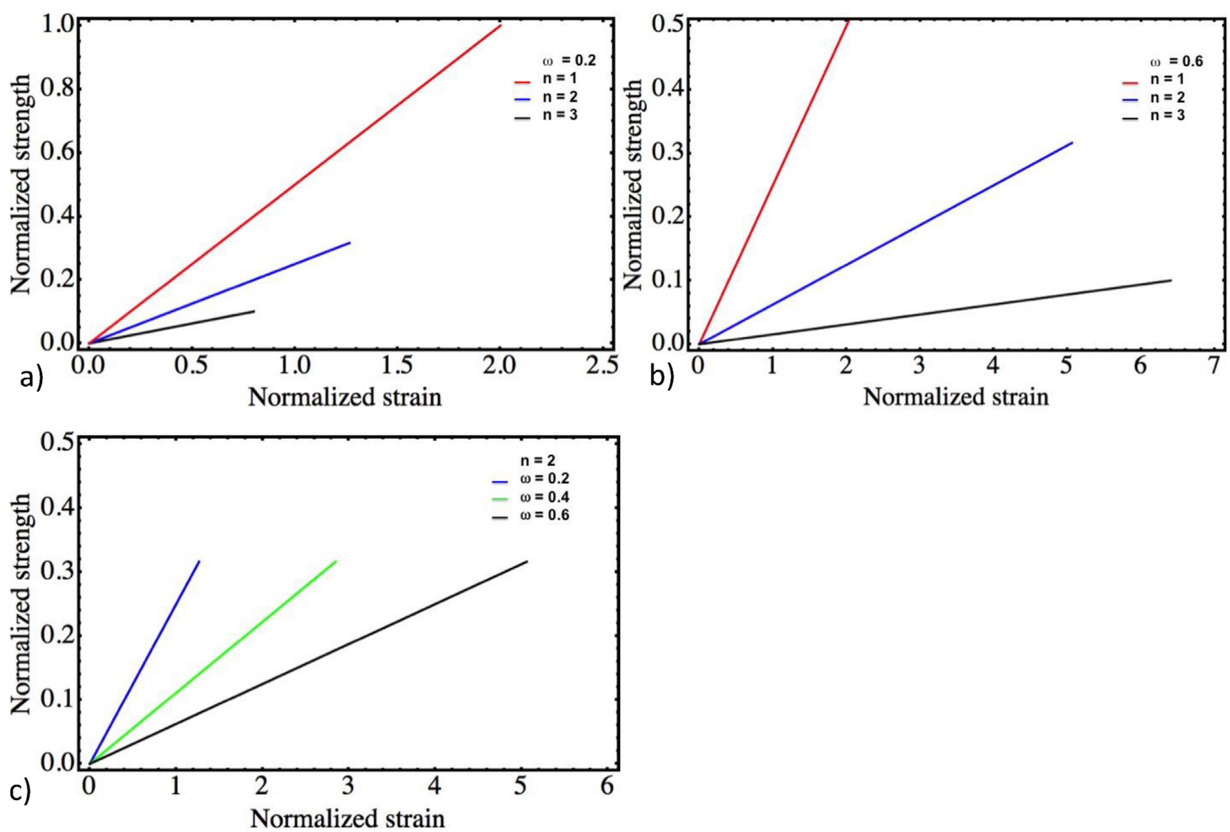

Fig. 11 – Normalized stress–strain curves of materials with up to 3 hierarchical levels and different microcrack densities, a) $\omega = 0.2$ and b) $\omega = 0.6$. It can be seen that above a critical microcrack density ω_c the fracture strain increases with increasing hierarchical level. Below ω_c the opposite is true. For a given hierarchical level the fracture stress is independent of the crack density ω but the fracture strain increases with increasing crack density (c).

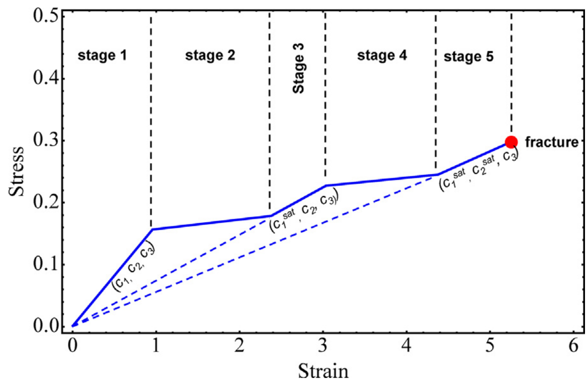

Fig. 12 – Stress–strain curve in arbitrary units for a hierarchical material with 3 hierarchical levels. In stage 2 the microcrack density is allowed to increase due to local inhomogeneities, which produce internal stresses, which are high enough to trigger additional microcracks on level 1 at the stress level in stage 2. When the microcrack density on level 1 is saturated the material behaves linear elastic and the stress level increases until the microcrack density on level 2 increases due to internal stresses on that length scale. After this process is saturated the overall microcrack density is assumed to stay constant until failure occurs.

length scale of hierarchical level 1 are higher than the internal stresses on the length scale on the level 2. The modulus at this point corresponds to (c_1^{sat}, c_2^{sat}) and the stress increases until fracture (stage 5 in Fig. 12).

The anisotropy and inhomogeneity of natural enamel may be approached by the model as follows. Anistropy on different hierarchical levels can be introduced by changing microcrack densities ω_n on the different hierarchical levels n. In addition the model can be refined by adjusting the shape and orientation dependent parameter A on each hierarchical level leading to hierarchy dependent A_n. The inhomogeneity of natural enamel results in internal stresses, which will lead to a local change in microcrack density. All this can be put into a complex hierarchical finite element model, where our model is part of the constitutive laws. This should be the next step to understand in more depth the significance of microcracks in hierarchical materials. However, the intention of this work is to convince the scientific community that microcracks play an important role in the mechanical behavior of hard hierarchical materials. Even more they offer the possibility to explain experimental findings, which are mostly attributed to the "brick and mortar" model.

5. Conclusion

In situ bending experiments reveal that enamel is able to withstand mechanical loading and behaves linearly elastic until catastrophic failure occurs despite the occurrence of microcracks on different length scales. A microcrack model for hierarchical structured materials is proposed, which contains microcracks on all hierarchical levels. The model shows that – solely due to the occurrence of microcracks on different hierarchical levels – the elastic modulus, the fracture strength and the fracture strain can be tailored.

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