Conceptual Basis of Probability and Quantum Information Theory

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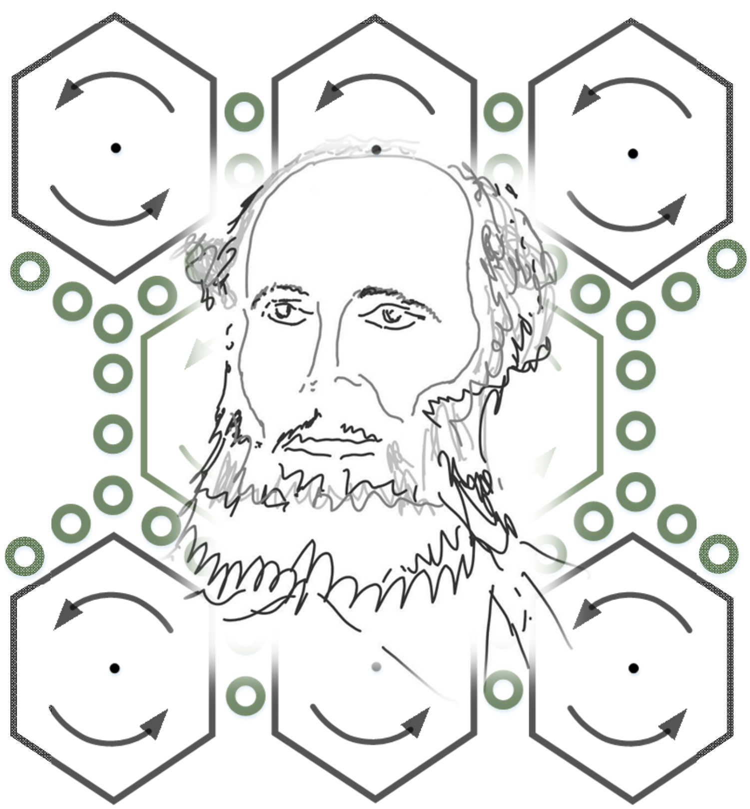

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Contents

Abstract

These notes present a probabilistic framework that enables a formulation of classical probability theory, thermodynamics, and quantum probability with a common set of four principles or axioms. It explains everything that usual quantum mechanics and classical probability theory does. We emphasize that this framework is not an interpretation of quantum mechanics, such as "many worlds", "Kopenhagen interpretation", or others. It is a probability algorithm that computes probabilities of future events and additionally enables a reconstruction of quantum theory, thermodynamics, diffusion, and Wiener processes.

We distinguish strictly between possibilities and outcomes. Moreover, we use a time concept based on the classification of future, present, and past. Well-known paradoxes are resolved. The superposition principle obtains a new meaning. The inclusion-exclusion principle, well-known in probability theory and number theory, is generalized to complex numbers.

Our probabilistic framework is not based on the Hilbert space formalism. It requires only simple set theory and complex numbers. Thus, this theory can be taught in schools.

Our framework may be viewed as an axiomatic approach to probability in the sense of Hilbert, who asked for an axiomatic probability theory in his sixth of the twenty-three open problems presented to the International Congress of Mathematicians in Paris in 1900.

We have applied our probabilistic algorithm to several problems, including classical problems, statistical mechanics and thermodynamics, diffraction at multiple slits, light reflection, interferometer, delayed-choice experiments, and Hardy's Paradox.

1 Introduction

The true logic of the world is in the calculus of probabilities.
James Clerk Maxwell

According to the Cambridge dictionary, a probability is a number that represents how likely it is that a particular outcome will happen. In 1900, Hilbert presented his famous fundamental problems, hereunder his sixth problem, which claimed to treat probability axiomatically, similar to geometry. Many responses reemerged[1]. However, more than 100 years later von Weizsäcker[2] wrote:

> *Probability is one of the outstanding examples of the epistemological paradox that we can successfully use our basic concepts without actually understanding them.* von Weizsäcker 2006

[1] Shafer, Vovk [2006]
[2] von Weizsäcker [2006, Page 59]

Even today, classical probability, its relationship to quantum probability, and the way of assigning probabilities to elementary events is a controversial philosophical discussion. However, there are various interpretations of probability, including one of the oldest, the frequency interpretation[3].

For quantum probability problems, the debates become strange and weird. Fuchs[4] writes about the annual conferences:

> *What is the cause of this year-after-year sacrifice to the "great mystery?" Whatever it is, it cannot be for want of a self-ordained solution: Go to any meeting, and it is like being in a holy city in great tumult. You will find all the religions with all their priests pitted in holy war - the Bohmians[3], the Consistent Historians[4], the Transactionalists[5], the Spontaneous Collapseans[6], the Einselectionists[7], the Contextual Objectivists[8], the outright Everettics[9, 10], and many more beyond that. They all declare to see the light, the ultimate light. Each tells us that if we will accept their solution as our savior, then we too will see the light.* Fuchs 2002

In the same sense, Weinberg[5] 2017 writes about quantum theory:

> *Even so, I'm not as sure as I once was about the future of quantum mechanics. It is a bad sign that those physicists today who are most comfortable with quantum mechanics do not agree with one another about what it all means. The dispute arises chiefly regarding the nature of measurement in quantum mechanics.* Weinberg 2017

In the following, we show that probability theory, thermodynamics, diffusion, Brownian motion, Wiener integrals, and quantum probability can be reconstructed with four principles. In particular, we present a predictive algorithm for computing probabilities about macroscopic future events or detector clicks. The principles distinguish strictly between internal possibilities and outcomes and are much more general than the axioms of quantum mechanics. Many well-known paradoxes can be explained, and interpretations like "many worlds" or "many minds" are avoided. This paper also summarizes some parts of two lecture notes[6], including some corrections. Both lecture notes contain many more issues.

The paper is organized as follows. Its main topic is a probabilistic framework consisting of four general principles which, in particular, describe and merge classical probability and quantum probability. These principles form the content of Section 2, and they may be viewed as an axiomatic approach to probability in the sense of Hilbert's sixth problem. Several examples are presented in Section 3. Three strange paradoxes, namely Renninger's Negative-Result Experiment, Delayed-Choice Experiments, and Interaction-Free Measurements are discussed in sections 4, 5, and 6, respectively. Then, in Section

[3] See the discussions in Ballentine [2014, Page 32], Drieschner [2020]

[4] Fuchs [2002]

[5] Weinberg [2017]

[6] Jansson [2017], Jansson [2019]

7, Hardy's Paradox is investigated in detail. This paradox is based on the idea of an experimental set-up where classic logic seems to prove that this set-up is not realizable, although it was realized later. In Section 8, we show that our probability theory is consistent and contains a $U(1)$ symmetry. The principle of inclusion and exclusion, a well-known counting technique belonging to combinatorics but also applied in measure theory, is generalized in Section 9. The reconstruction of statistical thermodynamics, described in Section 10, is a crucial touchstone when applying our probability theory. In the two following sections, we present the reconstruction of Feynman's formulation of quantum mechanics and its close relationships to Brownian motion, Wiener integrals, and diffusion. Finally, some conclusions are given.

Hamburg, Germany, September 2022
Christian Jansson

2 A Unified Probabilistic Framework

When introducing the axioms of quantum mechanics, Susskind and Friedman[7] write:

> *For a classical system, the space of states is a set (the set of possible states), and the logic of classical physics is Boolean. That seems obvious, and it isn't easy to imagine any other possibility. Nevertheless, the real world operates along different lines, at least whenever quantum mechanics is important. The space of states of a quantum system is not a mathematical set [6]; it is a vector space. Relations between the elements of a vector space are different from those between the elements of a set, and the logic of propositions is different as well.*

This section describes a probabilistic framework not based on the Hilbert space formalism. We understand probability as a prognostic prediction of decidable alternatives, which we call *outcomes*. These alternatives are described by sets consisting of *elementary possibilities*. This framework unifies classical mechanics, statistical thermodynamics, and quantum mechanics. It has a predictive power much more extensive than quantum mechanics. It supports partially the opinion of Fuchs and Peres[8]:

> *The thread common to all the nonstandard "interpretations" is the desire to create a new theory with features corresponding to some reality independent of our potential experiments. But, trying to fulfill a classical worldview by encumbering quantum mechanics with hidden variables, multiple worlds, consistency rules, or spontaneous collapse without any improvement in its predictive power only gives the illusion of a better understanding. Contrary to those*

[7] Susskind [2014], page 24
[8] Fuchs, Peres [2000]

> *desires, quantum theory does not describe physical reality. What it does is provide an algorithm for computing probabilities for the macroscopic events ("detector clicks") that are the consequences of our experimental interventions. This strict definition of the scope of quantum theory is the only interpretation ever needed, whether by experimenters or theorists.* Fuchs and Peres 2000

Shortly spoken, words say nothing but numbers describe everything.

What we observe, what we measure in experiments, are "detector clicks". In the following, we consider experiments in the broadest sense. We describe an experiment by three sets:

(i) The *possibility space* $\mathbf{P}$ is a set with elements $p \in \mathbf{P}$, which we call *elementary possibilities*.

(ii) The *possibility algebra* $\mathbf{F}$, sometimes called *field*, is defined as the collection of subsets of $\mathbf{P}$ that contains $\mathbf{P}$ itself, and is closed under complement and under countable unions. All subsets $F \in \mathbf{F}$ are called *possibilities*. If they don't coincide with the elementary possibilities $\{p\}$, then they are called *non-elementary*.

(iii) The *sample space* $\mathbf{O}$ is a set consisting of pairwise disjoint sets $F \in \mathbf{F}$ called *outcomes*. The outcomes form a partition of the possibility space, such that each elementary possibility $p \in \mathbf{P}$ is contained in exactly one outcome F. If an outcome contains more than one element, we speak of its *internal* elementary possibilities. The elementary possibilities contained in some outcome F are called *accessible* from F.

Moreover, we assume:

(iv) A *probability amplitude*[9] is given. It is defined as a mapping φ from the possibility algebra $\mathbf{F}$ into the set of complex numbers:

$$F \to \varphi_F = \varphi(F) \in \mathbb{C}, \quad F \in \mathbf{F}. \tag{1}$$

We request that two general principles or axioms are satisfied.

First principle: For any countable set of pairwise disjoint possibilities $F_m \in \mathbf{F}$, such that $F = \cup_m F_m$, it is

$$\varphi_F = \varphi\left(\bigcup_m F_m\right) = \sum_m \varphi_{F_m}. \tag{2}$$

We speak of this principle as the principle of *superposition of probability amplitudes*. It is more general than Feynman's first principle: "When an event can occur in several alternative ways, the probability amplitude for the event is the sum of the probability amplitudes for each way considered separately"[10]. Feynman's quantum mechanics differs from our framework, since his theory

[9] We use the notation in Feynman [1948] p.4

[10] Feynman Lectures [1963] p.1-16

does not distinguish between outcomes, possibilities, and internal possibilities, and is not based on the three time concepts future, present, and past.

Second principle: This is *Born's rule* that transforms probability amplitudes of outcomes F to probabilities $\Pr(F)$:

$$\Pr(F) = |\varphi_F|^2 \text{ for all } F \in \mathbf{O}, \text{ and } \sum_{F \in \mathbf{O}} |\varphi_F|^2 = 1. \tag{3}$$

It says that computing the square of the magnitude of probability amplitudes for the outcomes yields classical probabilities. Summing up the probabilities of all outcomes gives one. In particular, Born's rule implies a real probability measure on the sample space $\mathbf{O}$. Thus, classical probability is incorporated.

We call the quadruplet $(\mathbf{P}, \mathbf{F}, \mathbf{O}, \varphi)$ together with these two principles a *possibility measure space*.

Notice that in the literature, frequently, a measure is a non-negative function in contrast to complex amplitudes. Complex numbers, however, are indispensable and fundamental for describing physical reality[11].

We'll examine some examples in more detail later. But for a first orientation only the coin toss and the double slit experiment are mentioned. For a coin toss, the elementary possibilities are Heads and Tails. They define the possibility space. The set of outcomes coincides with these two elementary possibilities. In the double-slit experiment, the paths from the source, via the wall of slits, to the detectors represent the elementary possibilities. Each outcome is described by the set of paths from the source to one of these detectors.

An experiment is called *deterministic* if the possibility space and the sample space consist of one element. In this case, the second principle implies that the probability of this unique outcome is one. An experiment is called *classical*, if the possibility space and the sample space coincide. The coin toss is classical, whereas the double-slit experiment is not classical, since each detector can be reached via several paths. The paths represent the internal possibilities.

The possibility space can be viewed as a $\mathbf{P}$-fold alternative, that is, the elementary possibilities are *mutually exclusive*, meaning that during the execution of an experiment exactly one possibility can happen. A $\mathbf{P}$-fold alternative generalizes an 2-fold alternative which is a *bit*. The sample space can be viewed as an $\mathbf{O}$-fold alternative of *mutually exclusive* outcomes, which are *empirically decidable* due to the detectors, in contrast to elementary possibilities.

Both principles provide mathematical conditions for probability amplitudes. The second principle implies that it is sufficient to calculate the amplitudes for all outcomes. The first principle shows that we need to compute the amplitudes for the elementary possibilities only. The content of the following two principles is about how to compute concrete probability amplitudes.

Third principle: The amplitudes φ_F contribute equally in magnitude for all accessible elementary possibilities. They are proportional to some constant times a complex number of magnitude one, namely

$$e^{\frac{i}{\hbar}S(F)}. \tag{4}$$

[11] Susskind [2014, page 44], Jansson [2017], Section 2.2, Wood [2021]

The function $S(F)$ is called the *action* of the elementary possibility F.

Feynman describes his formulation of quantum theory as follows:

> *The total amplitude can be written as the sum of amplitudes of each path - for each way of arrival. For every $x(t)$ that we could have - for every possible imaginary trajectory - we have to calculate an amplitude. Then we add them all together. What do we take for the amplitude for each path? Our action integral tells us what the amplitude for a single path ought to be. The amplitude is proportional to some constant times $exp(iS/\hbar)$, where S is the action for the path. If we represent the phase of the amplitude by a complex number, Planck's constant $\hbar$ has the same dimensions.*
> Feynman and Hibbs[12]

We make no further assumptions about the action, as is done in the case of space-time paths. Thus, we are very flexible in describing physical problems outside space-time.

This third principle can also be viewed as a generalization of Laplace's *principle of indifference* where all outcomes are equally likely assigned with unit one. We just have to replace the unit one by the set of complex numbers of magnitude one. Then we get back Laplace's theory, when we define the phase equal to zero.

Fourth principle: Two possibilities F and G are called *independent* if their intersection is non-empty, and if the occurrence of one possibility does not affect the other one. Mathematically, independence is expressed by the equation:

$$\varphi_{F \cap G} = \varphi_F \, \varphi_G, \tag{5}$$

that is, the joint amplitude is equal to the product of their amplitudes.

This principle is closely related to Feynman[13] : "When a particle goes by some particular route, the amplitude for that route can be written as the product of the amplitude to go partway with the amplitude to go the rest of the way.“ Basically, this property goes back to Laplace's theory. When an experiment that break down into a series of events happening independently, then the probability of the occurrence of all events is the product of the probability of each. Our first and fourth principle show that the *multiply-and-add rule* carries over to complex probability amplitudes for possibilities.

Let us compare our probability theory with the axiomatic system in geometry. Geometry consists of the following components:

(i) The primitives: points, lines, and planes. (ii) The axioms are statements about these primitives; for instance, two points are together incident with one line. (iii) The laws of logic. (iv) The theorems, that are the logical consequences of the axioms.

Hilbert remarked that primitive terms are empty shells or placeholders with no intrinsic properties. Instead of points, lines, and planes, we can also

[12]Feynman, Hibbs [1965], p.19

[13]Feynman Lectures [1963] p.3-4

talk about tables, chairs, and windows. When giving concrete meaning to the primitives of a geometrical system, we obtain a model of the system, and all theorems are true statements in this model.

Our possibility measure space consists of the following components:

(i) The primitives; elementary possibilities, outcomes, and amplitudes. (ii) The axioms are statements about these primitives; for instance, each elementary possibility is contained in exactly one outcome. (iii) The laws of classical logic. (iv) The theorems, like the inclusion-exclusion principle.

We shall discuss several concrete models, among them Feynman's formulation of quantum mechanics in spacetime, thermodynamics, or Wiener processes. Our possibility measure space has a wide range of applications and can be viewed as an axiomatic probabilistic system in the sense of Hilbert's sixth problem.

3 Examples

Some small examples are intended to understand better how to work with the principles of our *possibility measure space.*

Coin Toss

The first and most simple example is a fair coin toss. There are two elementary possibilities, Heads H and Tails T, yielding the possibility space $\mathbf{P} = \{H, T\}$ and the related possibility algebra $\mathbf{F}$, the power set of $\mathbf{P}$. Two outcomes coincide with the two elementary possibilities and form a partitioning of the possibility space. Hence, this is a classical experiment. We define the action as equal to zero such that the exponential interference term in (4) is equal to one. We set

$$\varphi_{\emptyset} = 0,\ \varphi_{\{H\}} = \frac{1}{\sqrt{2}},\ \varphi_{\{T\}} = \frac{1}{\sqrt{2}}. \tag{6}$$

Hence, the probabilities for both outcomes are $1/2$, according to the second principle. The first principle yields

$$\varphi_{\{H,T\}} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \sqrt{2}. \tag{7}$$

Obviously, $|\varphi_{\{H,T\}}|^2 > 1$. However, this is no contradiction since we use Born's rule only for the outcomes, not for arbitrary possibilities.

Atom in Two States

Smolin[14] discussed in his book a simple quantum model of an atom, which can be in two states, namely in an exited state E and in a ground state G with the lowest energy. In the unstable exited state E, the atom can decay into G by emitting a photon. We put a Geiger counter together with an exited atom

[14] Smolin [2019], Ch.4

into a box and close this box. The Geiger counter can also be in two states: in the yes-state Y when the counter has detected a photon and in the state N when no photon is detected. Initially, this system is in the state (E, N). When we open the box after some time, the system is **either** in state (E, N) **or** in state (G, Y). According to quantum mechanics, before opening the box, the system is assumed to be in a *superposition* of both states

$$(E, N) \textbf{ and } (N, Y), \tag{8}$$

which Smolin calls the "in between" state. But we never observe a superposition after opening the box. Smolin speaks of a seemingly weird situation and raises some questions. Why are there two dynamical rules, the *unitary evolution* before opening the box, and the *collapse* into one of the states (E, N) **or** (G, Y) when opening the box? Why do we treat measurements differently from other processes? Is a measurement device different from other ones, perhaps because of the size of complexity? When does the collapse really happen? Does it happen when the photon interacts with the counter or when we open the box and become conscious of the outcome? These are typical questions arising in quantum mechanics.

Our approach is a pure probability theory working on future events where such questions don't occur. It is not an interpretation of quantum mechanics, although it solves quantum problems. It is different from quantum theory and more comprehensive, as demonstrated below.

We define the possibility space

$$\mathbf{P} = \{(E, N), (E, Y), (G, N), (G, Y)\}. \tag{9}$$

The possibility algebra consists of all subsets of $\mathbf{P}$, and the outcomes coincide with the elementary possibilities. In other words, the sample space and the possibility space are identical, if we identify $p \in \mathbf{P}$ with $\{p\} \in \mathbf{F}$. There are no internal possibilities, hence yielding a simple classical statistical system.

Assuming that the system works correctly, we set

$$\varphi(E, Y) = \varphi(G, N) = 0, \ \varphi(E, N) \neq 0, \ \varphi(G, Y) \neq 0. \tag{10}$$

All quantities belong to the prognostic future, describing what might happen but not what happens in the present. The non-zero amplitudes depend on the set-up, the atoms, and how long the box is closed[15]. Born's rule implies that

$$|\varphi(E, N)|^2 + |\varphi(G, Y)|^2 = 1. \tag{11}$$

In the present, the system tends to move to states of higher probability, although rarely it choose an outcome of low probability. Nothing is strange; we have no "in between" superpositions.

The Double-Slit Experiment

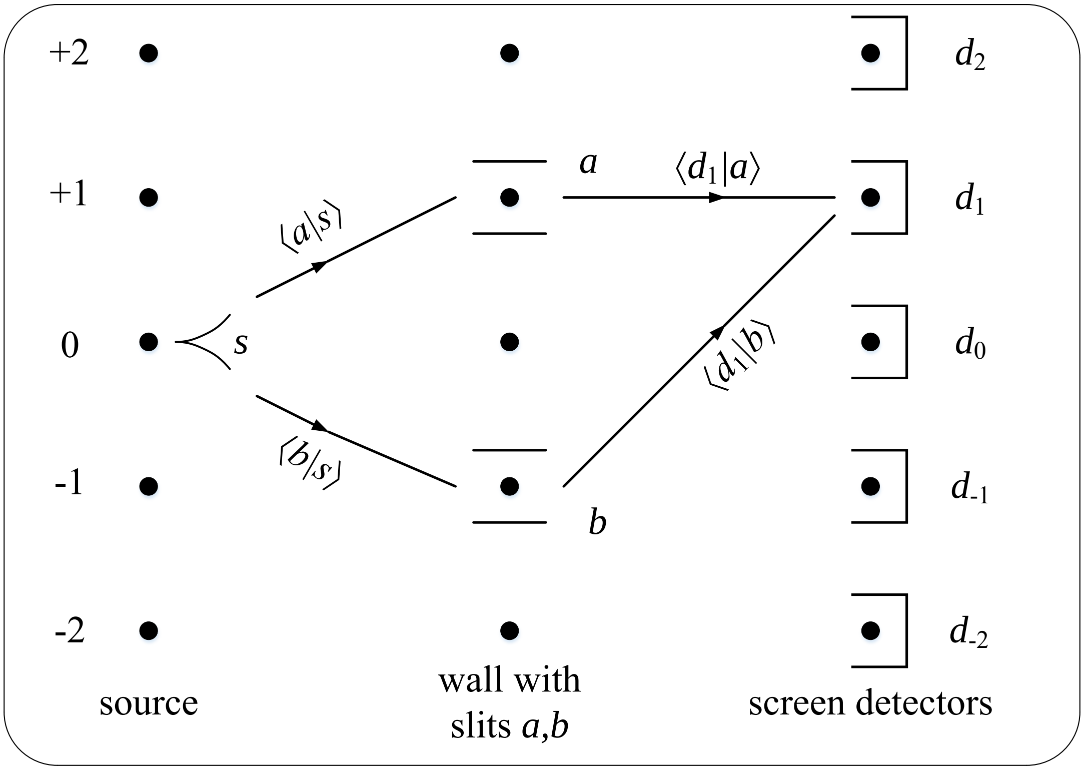

Figure 1: The double-slit experiment described for a discrete spacetime. The particle leaves source s, passes one of the two slits a or b, and is finally detected in d_1.

Now let us discuss the double-slit experiment, see Figure 1. In this experiment, the elementary possibilities are piecewise straight paths sad_m, sbd_m from the source s, via the wall W with two slits a and b, to the detectors $d_m, m = -l, ..., l$ positioned on the screen D.

We show in detail how changes in the experimental set-up change the statistics. At first, we look at the experiment where slit b is closed. Only paths through slit a are relevant. Therefore, the possibility space is

$$\mathbf{P} = \{sad_m : \ d_m \in D\} . \tag{12}$$

There are no internal possibilities. The sample space of outcomes

$$\mathbf{O} = \{O_{d_m} : \ d_m \in D\} , \ O_{d_m} = \{sad_m\} \in \mathbf{F}. \tag{13}$$

coincides with the set of elementary possibilities. Thus, we have a classical experiment. Our third principle gives us the amplitude

$$\varphi(O_{d_m}) = \varphi_{sad_m} \tag{14}$$

via the action on the path sad_m. The squared magnitudes of the amplitudes are, according to the second principle, the probabilities:

$$\Pr(O_{d_m}) = |\varphi_{sad_m}|^2 \tag{15}$$

[15]We do not assume a time parameter. Clocks can be built outside spacetime; see Jansson [2017] Ch. 4.14

Similarly, if slit a is closed, we obtain the amplitudes

$$\varphi(O_{d_m}) = \varphi_{sbd_m}, \tag{16}$$

and

$$\Pr(O_{d_m}) = |\varphi_{sbd_m}|^2. \tag{17}$$

Now, we assume that both slits are open. Then the possibility space is given as

$$\mathbf{P} = \{sad_m, sbd_m : \ a, b \in W, d_m \in D\}. \tag{18}$$

We have internal possibilities due to the slits a and b, since without detectors at the slits, it cannot be observed through which slit the particle goes in the present. The outcomes are defined via the detector clicks. Hence, the sample space of outcomes is

$$\mathbf{O} = \{O_{d_m} : \ d_m \in D\}, \text{ where } O_{d_m} = \{sad_m, sbd_m\}. \tag{19}$$

In agreement with the third principle, we set

$$\varphi_{sad_m} = \frac{1}{\sqrt{2}} e^{\frac{i}{\hbar}S(sadm)}, \ \varphi_{sbd_m} = \frac{1}{\sqrt{2}} e^{\frac{i}{\hbar}S(sbdm)}. \tag{20}$$

These are disjoint elementary possibilities, and the first principle yields the amplitudes of the outcomes

$$\varphi(\{O_{d_m}\}) = \varphi_{sad_m} + \varphi_{sbd_m} \text{ for all } d_m \in D. \tag{21}$$

Using the third principle, we can calculate the amplitudes for the outcomes by inserting the concrete amplitudes for the elementary possibilities. Born's rule provides the probabilities of the outcomes:

$$\begin{aligned} \Pr(O_{d_m}) &= |\tfrac{1}{\sqrt{2}} e^{\frac{i}{\hbar}S(sadm)} + \tfrac{1}{\sqrt{2}} e^{\frac{i}{\hbar}S(sbdm)}|^2 \\ &= \tfrac{1}{2}\left(|e^{\frac{i}{\hbar}S(sadm)}|^2 + |e^{\frac{i}{\hbar}S(sbdm)}|^2\right) + \\ &= \tfrac{1}{2}(e^{\frac{i}{\hbar}S(sadm)})^* e^{\frac{i}{\hbar}S(sbdm)} + (e^{\frac{i}{\hbar}S(sbdm)})^* e^{\frac{i}{\hbar}S(sadm)}). \end{aligned} \tag{22}$$

The first term in this sum corresponds to the classical probability, and the second term describes interference.

If $e^{\frac{i}{\hbar}S(sadm)} = e^{\frac{i}{\hbar}S(sbdm)}$, from (22) it follows that

$$\Pr(O_{d_m}) = 2|e^{\frac{i}{\hbar}S(sadm)}|^2. \tag{23}$$

This doubles the probability when only one slit is open. Hence, we have *constructive interference*. If $e^{\frac{i}{\hbar}S(sadm)} = -e^{\frac{i}{\hbar}S(sbdm)}$, the probability that detector d_m clicks is

$$\Pr(O_{d_m}) = 0, \tag{24}$$

yielding *destructive interference*. So far, nothing has happened. We have computed quantities belonging to the future, yielding a pattern of constructive and destructive interference.

Finally, let us assume that we have information about through which slit the particle passes, given by two additional detectors d_a and d_b, which click when a particle passes slit a or b, respectively. The detectors should work correctly, that is, it cannot happen that a particle arrives at detector d_m via slit b and detector d_a clicks, or both detectors d_a and d_b don't click.

In this case, the possibility space is defined in the form

$$\mathbf{P} = \{sad_a d_m, sbd_b d_m : \quad a, b \in W, d_m \in D\} . \tag{25}$$

The outcomes are defined via the detector clicks. But now we have two additional detectors d_a and d_b. Hence, the sample space of outcomes is

$$\mathbf{O} = \{O_{d_a d_m}, O_{d_b d_m} : \ d_m \in D\} , \tag{26}$$

where

$$O_{d_a d_m} = \{sad_a d_m\} , O_{d_b d_m} = \{sbd_b d_m\} . \tag{27}$$

Thus, we get classical probabilities:

$$\Pr(O_{d_a d_m}) = |\varphi_{sad_m}|^2, \ \Pr(O_{d_b d_m}) = |\varphi_{sbd_m}|^2. \tag{28}$$

This is a natural explanation of the double-slit experiment. Probabilities, part of the future, are calculated. In the present, a particle has the tendency to move on exactly one path of higher probability. The particle doesn't know anything about the set-up. This experiment is not a classical model because of the internal possibilities. In particular, we see that it is essential to distinguish clearly between elementary possibilities and outcomes.

We have investigated three different experimental setups that lead to different statistics. In the quantum mechanical literature, there are discussed several questions, among them:

- Non-locality: How does the particle know which slits are open?
- Wave-particle duality: Is every particle also a wave? Is the complete information about a particle encoded in its wave function?
- Superposition: Is a massive object at various places simultaneously?

The answers depend on the interpretation, frequently accompanied by the comment that quantum mechanics is weird and magic. What makes quantum mechanics seemingly hard to understand? The first reason is the mental fixation on the particle, much less than on the experimental set-up. Secondly, it is the fixation on the (3+1)-spacetime. In our probability theory, time is replaced by the trinity of time, namely future, present, and past. Future is timeless, and probability is a measure of future events. The experimental set-up plays the primary role. The particle is limited to the interaction with the experimental set-up in the present, with the tendency to move to outcomes of larger probability. The particles obey the probabilities calculated from the experimental set-up. Whether only one slit or both slits are open is a property of the set-up, not of the particle.

For photons, electrons, and several atoms, slit experiments were performed in agreement with the statistics above. But they become more difficult for increasing size of the particles. However, even huge molecules, combined of 810 atoms, show interference. At the University of Vienna, in 2012, a double-slit experiment was performed using large molecules called phthalocyanine. Only one molecule is in the set-up at the same time. In a video camera, they exhibit their macroscopic nature. They arrive localized at small places on the screen of detectors. The molecules, one after the other, build up an interference pattern. There is no collapsing wave but a simple probability distribution.

Large molecules and other large macroscopic objects can be viewed as a cloud of elementary particles, the constituents. If the binding force between the constituents is weak, then the particles in this cloud can pass independently of one another both slits yielding interference. If the binding force between the constituents is large, then all must pass the same slit yielding a stochastic pattern as in the case where one slit is closed.

The double-slit experiment can be generalized to finitely many slits and finitely many subsequent walls. Then, the elementary possibilities consist of all possible paths from the source via the walls to the detectors. Going to infinitely many slits and walls leads to Feynman's path integral.

For some other aspects of slit experiments, see Jansson[16].

Light Reflection

The seemingly simple problem of how a mirror reflects light is slightly more complicated. Here, we give a very short presentation. The figures below, corresponding to the reflection of light, are slight modifications of some figures in the beautifully written book of Feynman[17] about QED.

Usually, the reflection of light is solved using the *ray model of light* which holds in many practical situations. The mirror reflects light such that the angle of incidence is equal to the angle of reflection. The length of the mirror, as well as the ends of the mirror, do not influence the light that reaches the detector. In the experimental set-up, the light of one color is emitted, and at some other point, there is a photomultiplier. During the execution, only one photon is present in the experiment.

Unfortunately, the ray model does not match several experimental results. We observe reflection, that is, the detector clicks even if we cut off essential parts of the mirror, including large areas of the middle part of the mirror. This observation suggests that the photon may move from the source to the detector on each possible path.

The third principle in our probabilistic framework says that each possible path is furnished with a complex amplitude, as shown in Figure 2. In agreement with the first principle, we sum up all vectors corresponding to the paths

[16] Jansson [2021], Ch. 4

[17] Feynman [1985]

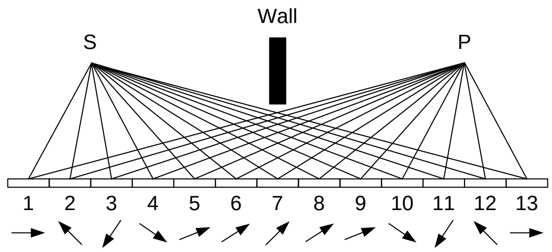

Figure 2: The third principle says that a photon has an amplitude equal in magnitude for each possible path from the source to the detector. Hence, it can be reflected from every part of the mirror.

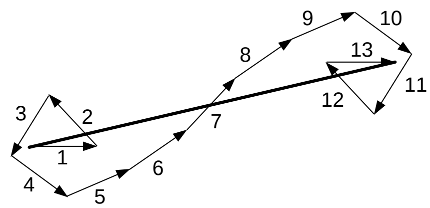

Figure 3: The amplitudes for the paths (the elementary possibilities) are added together. The major contribution of the final amplitude is due to the paths of minimal action.

ending at the photomultiplier. Then we obtain the amplitude of the outcome that the multiplier clicks, see Figure 3.

In many textbooks, it is argued that a photon has to move on all possible paths from the source to the detector, simultaneously[18]. In our probabilistic framework, the amplitudes are related to elementary possibilities, the paths. The probabilities correspond to future events. Each particle chooses exactly one path in the present.

In Figures 2 and 3, we have divided the mirror into little squares. Each path corresponds to one square. The final amplitude length evolves mainly from the amplitudes of the middle part of the mirror; see Figure 3. The contributions from the left and right parts almost cancel each other out.

All amplitudes, corresponding to paths distant from the classical path of least action, interfere destructively. Only the paths in the neighborhood of the classical path interfere constructively, causing the ray model of light approximately.

But this is not the whole story. The question is, how the photon finds the path of extremal action? Does the photon move on all possible paths to find the right path? Is this imagination only a mathematical description far away from any reality? However, we should show in an experimental set-up that a photon also sometimes chooses other paths.

Let us cut off a large part of the mirror such that only three segments on

[18]For example, this strange interpretation can be found in the pleasant talk of Girvin in the KITP Public Lectures, see online `kitp.edu/online/plecture/girvin`.

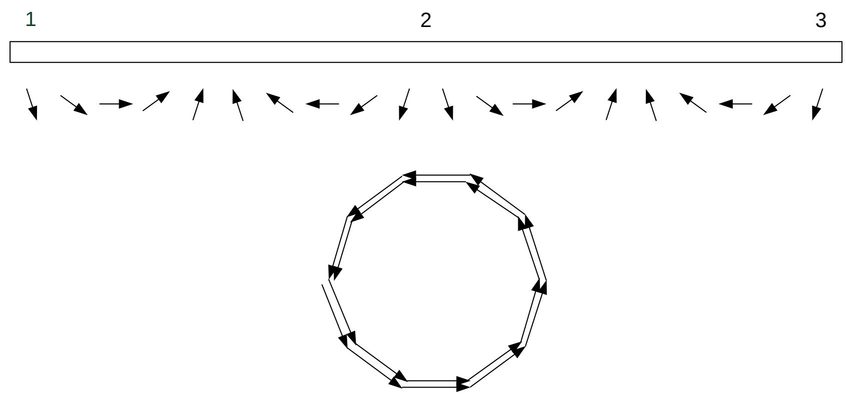

Figure 4: If the set-up contains only three segments on the left part of the mirror, then the detector does not click since the amplitudes add up to zero.

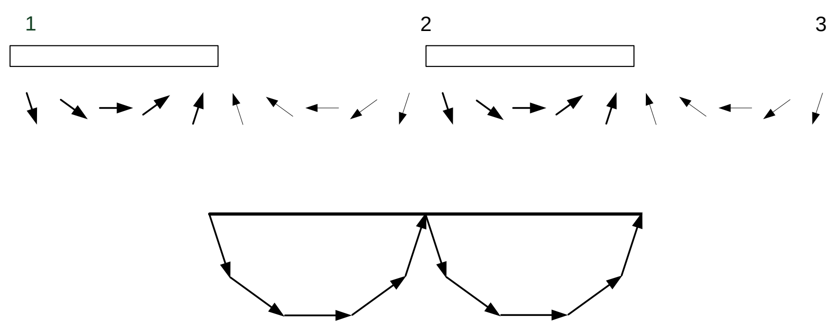

Figure 5: A striped mirror reflects a substantial amount of light and is called a diffraction grating.

the left side are left over; see Figure 4. The amplitudes are displayed in greater detail than before. When adding all arrows, they cancel out approximately. Hence, the probability of being detected in the photomultiplier is almost zero.

We now cut off further small segments, as shown in Figure 5. Most of the vectors point to the right. Adding up the amplitudes, the sum predicts possible reflection, in agreement with the fact that the photomultiplier sometimes clicks.

It sounds crazy that if you cut off the essential middle of the mirror, then sometimes we observe reflection. This weird view vanishes in our probabilistic framework: The paths obtain prognostic amplitudes, but the dynamics happen in the present. There, the photon moves only on one path with the tendency that this path belongs to an outcome with high probability.

Dice Unlike Any Dice

Finally, we mention Anthony Zee[19]. He used the title "Dice Unlike Any Dice" in his book "Fearful Symmetry" and writes:

> *Welcome to the strange world of the quantum, where one cannot determine how a particle gets from here to here. [...] When a*

[19] Zee [2015], p.141

> *die is thrown, the probability of getting a 1 is 1/6. The probability of getting a 2 is, of course, also 1/6. Now, consider the following question: What is the probability of getting a 1 or a 2 in one throw? The answer is evident to gamblers and non-gamblers alike: The probability is 1/6 + 1/6 =1/3. In everyday life, to obtain the probability of either A or B occurring, we simply add the probability of A occurring and the probability of B occurring.*
>
> *The quantum die is astonishingly different. Suppose we are told that for the quantum die the probability of throwing a 1 is 1/6, and the probability of throwing a 2 is also 1/6. In contrast to what our experience with ordinary dice might suggest, we cannot conclude that the probability of getting either a 1 or a 2 in one throw is 1/3! It turns out that the probability of throwing a 1 or a 2 can range between 1/3 and 0!*

It seems that quantum theory, and its various interpretations, yield other results than classical probability theory. Apparently, Zee views both theories as incompatible. Our unified probabilistic framework, however, allows us to treat classical and quantum probabilistic problems. Since we clearly distinguish between internal possibilities, possibilities, and outcomes, we have a theory that is not an interpretation of quantum mechanics but is much more extensive. This will become more clear in the following applications.

4 Renninger's Negative-Result Experiment

Already in 1960, Renninger[20] invented an experiment based on interaction-free measurements, which is well-known under the name *Renninger's negative-result experiment*[21]. He was the first to point to the surprising effect that we can receive information by observing that **"nothing happens"**.

He considered a simple experimental set-up where two spherical scintillation screens with radii $r_1 < r_2$ are placed around a light source. The radius of the outer screen is very large compared to the radius of the inner one.

In large time intervals, a photon is emitted from source S that can move in all directions. Quantum mechanics describes the state of the photon in terms of a radially symmetrical wave function ψ depending on both screens. Suppose, one photon is emitted but not detected at the inner screen. Then we have the information that the photon is moving outside the inner radius. But this state does not depend on the inner screen; hence, must correspond to another wave function ψ' which reflects the possibility to interact only with the outer sphere. Without any interaction of the photon with the screens, we have a reduction of the photon's wave function $\psi \to \psi'$.

[20] In 2012, in the hospital of St. Georg in Hamburg, I was cured from an acute Leukemia. I am indebted to two doctors, Prof. Dr. M. Zeis and Dr. H. Hauspurg, who helped me to recover via several chemotherapeutics and a bone marrow transplantation. Dr. H. Hauspurg is a grandson of Mauritius Renninger.

[21] Renninger [1960]

Renninger's experiment shows that knowledge is gained, although nothing is measured. The sheer possibility that the photon could be detected, although it does not click, yields a collapse of the wavefunction $\psi \to \psi'$. Some physicists think that this is a link between the mystery of matter and the observer's mind; see the discussions about the many mind interpretations of quantum mechanics in several books. This interaction-free collapse, however, seems to falsify the well-known *wave particle duality*.

In our probability theory, we have a simple two-state system with two elementary possibilities, namely that the photon interacts with the inner screen r_1 or the outer screen r_2. These two possibilities also represent the two outcomes. The amplitudes depend on the geometry of the experiment. They deliver two probabilities. In the present, the photon chooses exactly one possibility, with the tendency to the one with the higher probability. No strange arguments, no collapse of a wave function.

5 Delayed Choice Experiments

For a better understanding of wave-particle dualism, Wheeler[22] proposed 1983 so-called *delayed choice experiments*. These are related to questions about whether a delayed choice of several ingredients in the experimental set-up for measuring point-like or wave-like properties change the past?

Let us look at a *Mach-Zehnder interferometer*.

In this experiment, a photon is generated in a source s_0. Then it meets a beam splitter b_1. It can pass b_1 and follow the lower path, or it can be reflected and follow the upper one. On the lower and the upper path, the photon will be reflected by a mirror, say m_1 and m_2, respectively. Then both paths meet at a point where a second half-silvered mirror b_2 is positioned. Finally, behind b_2 two detectors d_1 and d_2 are placed, which detect the photon after it has passed the second beam splitter.

It is a simple task to describe this experiment within our probabilistic framework. At first, we define the *possibility space* $\mathbf{P}$ which consists of four *elementary possibilities*:

$$\begin{aligned}\mathbf{P} = \quad \{ \quad & p_1 = s_0 b_1 m_1 b_2 d_1, \\ & p_2 = s_0 b_1 m_1 b_2 d_2, \\ & p_3 = s_0 b_1 m_2 b_2 d_1, \\ & p_4 = s_0 b_1 m_2 b_2 d_2\}.\end{aligned} \tag{29}$$

The element p_1 denotes the elementary possibility that a photon would travel on the lower path from source s_0, via the half-silvered mirror b_1, towards detector d_1. The possibility p_2 is the same as p_1, except that the photon moves towards detector d_2. The two remaining elementary possibilities are on the upper path.

[22]Wheeler [1978, 1983]

The outcomes are defined as the possibilities

$$\mathbf{O} = \{O_1, O_2\},\ O_1 = \{p_1, p_3\}, O_2 = \{p_2, p_4\}\ \in \mathbf{F}. \tag{30}$$

They describe the two events that the photon would be detected in d_1 or would be detected in d_2, respectively. Hence, we have a non-classical experiment with internal possibilities.

The action for each elementary possibility consists of the interaction with the mirrors and detectors in succession. The third principle shows how to compute the amplitudes of the elementary possibilities. When the action is assumed to be additive, the exponential term is a product of complex numbers.

We define the action such that the number $i = e^{i\pi/2}$ is the probability amplitude when the photon is reflected, and $\frac{1}{\sqrt{2}}$ is the amplitude when the photon passes a half-silvered mirror straightforward. In the first case, the path is rotated by a right angle. In the second case, passing straightforwardly has a 50:50 chance.

It follows immediately that the amplitudes of the elementary possibilities are

$$\begin{aligned}
\varphi(\{p_1\}) &= \tfrac{1}{\sqrt{2}} \cdot i \cdot \tfrac{i}{\sqrt{2}} = -\tfrac{1}{2}, \\
\varphi(\{p_2\}) &= \tfrac{1}{\sqrt{2}} \cdot i \cdot \tfrac{1}{\sqrt{2}} = \tfrac{i}{2}, \\
\varphi(\{p_3\}) &= \tfrac{i}{\sqrt{2}} \cdot i \cdot \tfrac{1}{\sqrt{2}} = -\tfrac{1}{2}, \\
\varphi(\{p_4\}) &= \tfrac{i}{\sqrt{2}} \cdot i \cdot \tfrac{i}{\sqrt{2}} = -\tfrac{i}{2} \\
&.
\end{aligned} \tag{31}$$

From the first principle, it follows that

$$\varphi(O_1) = \varphi(\{p_1\}) + \varphi(\{p_3\}) = -1 \tag{32}$$

yielding the probability

$$\Pr(O_1) = |-1|^2 = 1, \tag{33}$$

that the photon is detected in d_1.

In the same way, we obtain

$$\varphi(O_2) = \varphi(\{p_2\}) + \varphi(\{p_4\}) = 0. \tag{34}$$

yielding the probability zero. Hence, the photon is never detected in d_2.

Now we change our experiment by removing the second beam-splitter b_2. Then, we obtain the possibility space.

$$\mathbf{P} = \{p_1 = s_0 b_1 m_1 d_2,\ p_2 = s_0 b_1 m_2 d_1\}. \tag{35}$$

and the outcomes

$$\mathbf{O} = \{O_1, O_2\},\ O_1 = \{p_1\}, O_2 = \{p_2\}\ \in \mathbf{F}. \tag{36}$$

This is a classical experiment without internal possibilities. The amplitudes of the elementary possibilities are

$$\begin{aligned} \varphi(\{p_1\}) &= \tfrac{1}{\sqrt{2}} \cdot i = \tfrac{i}{2}, \\ \varphi(\{p_2\}) &= \tfrac{i}{\sqrt{2}} \cdot i = -\tfrac{1}{2}. \end{aligned} \tag{37}$$

Hence, the probabilities are 1/2 for each outcome.

Imagine realizing this experiment on a cosmic scale: Let a star emit photons some billions of light-years ago. Each photon must pass a galaxy before arriving at our experiment. Gravitation makes the light bend around the galaxy. Billions of years later, when the photons are not far away, we can change our experimental arrangements. If we put in a beam-splitter with a photographic plate behind it, we would measure an interference pattern, as calculated above. Alternatively, we can put in detectors before the beam splitter. Then, we obtain a clump of photons at one detector and a clump of photons at the other one. In our experiment, we decide whether the beam-splitter is put in when the particles have already passed the galaxy.

All this is not strange when using our probabilistic framework based on future events or future possibilities provided by the experimental set-up. But believing in wave-particle duality, the experiment implies the well-known paradox that our late choice influences the statistical patterns.

More precisely, if the second beam-splitter is in front, only detector d_1 clicks. Wheeler argues as follows:

> *[this] is evidence . . . that each arriving light quantum has arrived by both routes.*

Hence, he believes that a particle can move on different routes simultaneously. If the second beam-splitter is turned out, each photon follows one path or the other. With probability 1/2, the photon reaches one of both detectors. Wheeler writes in the same paper:

> *[either] one counter goes off, or the other. Thus the photon has traveled only one route.*

This seems to be in accordance with the *wave-particle duality* and the Copenhagen interpretation. Either we can measure wave-like properties or particle-like properties. Since the experimenter could randomly switch the second beam splitter in the interferometer on and off shortly before the photon arrives, Wheeler writes:

> *Thus, one decides the photon shall have come by one route or by both routes after it has already done its travel.*

Since no signal traveling at a velocity less than that of light can connect the two events described above, he writes:

> *We have a strange inversion of the normal order of time. We, now, by moving the mirror in or out, have an unavoidable effect on what we have a right to say about the already past history of that photon.*

Such experiments have been carried out on a small scale, see Jacques et al.[23]. In actual experiments, activation and deactivation of the beam splitter is done when the photon is already in the experiment. This is decided by a random number generator, not by a person.

The wave-particle duality is doubtful or even falsified if we don't accept the inversion of the normal order of time.

6 Interaction-Free Measurement

An interaction-free measurement is a type of measurement, where an object is located without any interaction or touch between it and the measuring device.

Elitzur and Vaidman[24] proposed a modified *Mach-Zehnder interferometer*, where an object might be placed on one of the routes.

Let us look at the interferometer as described in Section 5. An object might be placed on the lower path between beam splitter b_1 and mirror m_1. Elitzur and Vaidman describe the object as a bomb that explodes when touched by a photon. The interferometer is mantled such that it is not possible to decide whether the bomb is placed in the interferometer or not.

We describe this situation in terms of two experimental setups. The first one is the interferometer without the object. Then we have interference, such that all photons are detected in d_1.

The second one is the interferometer with the object. Then, we obtain the possibility space

$$\mathbf{P} \;= \{p_0 = s_0 b_1 A,\ p_1 = s_0 b_1 m_2 d_1,\ p_2 = s_0 b_1 m_2 d_2\}, \tag{38}$$

where p_0 denotes the possibility that the photon is absorbed by the object. The set of outcomes is

$$\mathbf{O} = \{O_0, O_1, O_2\},\ O_0 = \{p_0\},\ O_1 = \{p_1\},\ O_2 = \{p_2\}. \tag{39}$$

Thus, we have a classical experiment without internal possibilities. The amplitudes of the elementary possibilities are

$$\begin{aligned} \varphi(\{p_0\}) &= \tfrac{1}{\sqrt{2}} \\ \varphi(\{p_1\}) &= \tfrac{i}{\sqrt{2}} \cdot i \cdot \tfrac{1}{\sqrt{2}} = -\tfrac{1}{2}, \\ \varphi(\{p_2\}) &= \tfrac{i}{\sqrt{2}} \cdot i \cdot \tfrac{i}{\sqrt{2}} = -\tfrac{i}{2}. \end{aligned} \tag{40}$$

Hence, the probability is 1/2 that the photon is absorbed and is 1/4 that the photon is detected in d_1 or d_2, respectively. It follows that the object is put into the interferometer if the photon is detected in d_2. In this case, we have proved that the object is put into the interferometer without any interaction between the photon and the object.

[23]Jacques et al. [2006]

[24]Elitzur, Vaidman [1993]

It is not strange that the photon is detected in d_2. We have a classical stochastic problem without interference if the object is put in. The photon chooses in the present one of the three possibilities p_0, p_1, or p_2 in agreement with the calculated probabilities. Otherwise, we have interference if the object is outside, as described above. We can say, either the first or the second experimental set-up applies. Nothing seems to be strange in our probability theory describing future events.

Elitzur and Vaidman state that obtaining information about an object without any contact is a paradox. The authors argue that this paradox vanishes when using the *many-world interpretation*, where each possible state of a quantum superposition is real, thus, yielding a unitary dynamics without collapse. Actually, the photon interacts with the bomb, causing an explosion, but not in our world. This explanation represents an unnecessary, tremendous effort.

7 Hardy's Paradox

In 1992, Hardy published the idea of an experimental set-up where simple classical logical arguments show that this set-up is not realizable, although it was later realized with photons. This experiment is well-known under the name *Hardy's paradox*[25]. For a nice presentation see also Laloë[26]. Hardy's paradox is a challenge for each quantum interpretation, of course, and also for our probability theory.

We discuss Hardy's paradox in terms of a three-slit experiment. It consists of a source that produces two particles simultaneously. When produced, these two particles arrive at a wall with three slits. Finally, they interact with position detectors placed behind the wall of slits. The space of possibilities is described as follows:

(a) The value $a = +$ represents the possibility that, in the present, the first particle passes the topmost slit. The value $a = -$ denotes the possibility that it passes the middle slit.

(b) Analogously, the value $b = +$ represents the possibility that the second particle passes the lowest slit, and the value $b = -$ denotes the possibility that the second particle passes the middle slit.

(c) There is a partition in the middle of the wall of slits such that the first particle cannot pass the lowest slit, and the second particle cannot pass the topmost slit. Moreover, it is not allowed that both particles pass the middle slit simultaneously; perhaps, they annihilate each other, or the slit is too small. If the first particle passes the topmost slit, the middle slit is open for the second particle, yielding the same interference patterns as one particle in a double-slit experiment. Similarly, interference occurs if the second particle passes the lowest slit.

[25] Hardy [1992]
[26] Laloë [2001]

Outcomes $a', b' = \pm$	Related elementary possibilities $p = aba'b' \in \mathbf{P}$
$a' = +, b' = +:$	$p_1 = + + ++, p_2 = + - ++, p_3 = - + ++, p_4 = - - ++$
$a' = +, b' = -:$	$p_5 = + + +-, p_6 = + - +-, p_7 = - + +-, p_8 = - - +-$
$a' = -, b' = +:$	$p_9 = + + -+, p_{10} = + - -+, p_{11} = - + -+, p_{12} = - - -+$
$a' = -, b' = -:$	$p_{13} = + + --, p_{14} = + - --, p_{15} = - + --, p_{16} = - - --$

Table 1: Four outcomes and sixteen elementary possibilities of Hardy's paradox.

(d) The primed value $a' = +$ describes the possibility that, in the present, the first particle ends in the area of destructive interference. This is possible only if the second particle passes the middle slit and therefore destroys the area of destructive interference for the first particle. The primed value $a' = -$ is the negation of $a' = +$, thus being detected in the area of constructive interference. Analogously, the primed values $b' = +$ and $b' = -$ are defined.

When performing this experiment several times, from the description above, it follows immediately:

(i) The not-primed result, $a = -$ and $b = -$, never occurs.

(ii) The not-primed result, $a = +$ and $b = +$, sometimes occurs.

(iii) Both mixed-primed results, $a' = +$ and $b = +$, or $a = +$ and $b' = +$, never occur.

(iv) The doubly-primed result, $a' = -$ and $b' = -$, never occurs.

Classical logic implies that this experiment cannot be realized: The second condition (ii) guarantees that sometimes the result $a = +$ and $b = +$ is obtained when performing the experiment several times. But then condition (iii) implies that $b' = -$, since we have $a = +$, and $a' = -$, since $b = +$. This violates condition (iv). In other words, these conditions cannot be fulfilled simultaneously. The experiment seems to be not realizable.

A surprise is a realizable experimental set-up with photons based on a pair of Mach-Zehnder interferometers that interact through a beam splitter[27]. Why is this possible, although classical logic tells us that there cannot be a realization? In the following, we want to explain this paradox with our probability theory.

The first step is the definition of the possibility space. We have four binary quantities a, b, a', b' with values $\pm$. Hence, there are sixteen elementary possibilities $p = aba'b'$ forming the possibility space $\mathbf{P}$. The possibility algebra $\mathbf{F}$ is the power set of $\mathbf{P}$. The sample space $\mathbf{O}$ of outcomes consists of four subsets of $\mathbf{P}$ that are characterized by the values $a' = \pm, b' = \pm$. All outcomes and related elementary possibilities are displayed in Table 1.

Next, the probability amplitude must be defined. It depends on the concrete experimental set-up as well as on the conditions (i), (ii), (iii), and (iv).

[27]Irvine, et al. [2005]

Conditions	Related possibilities
$a = -, b = -:$	$F_{\{a=-,b=-\}} = \{p_4, p_8, p_{12}, p_{16}\}$
$a = +, b = +:$	$F_{\{a=+,b=+\}} = \{p_1, p_5, p_9, p_{13}\}$
$a' = +, b = +:$	$F_{\{a'=+,b=+\}} = \{p_1, p_3, p_5, p_7\}$
$a = +, b' = +:$	$F_{\{a=+,b'=+\}} = \{p_1, p_2, p_9, p_{10}\}$
$a' = -, b' = -:$	$F_{\{a'=-,b'=-\}} = \{p_{13}, p_{14}, p_{15}, p_{16}\}$

Table 2: The possibilities corresponding to the conditions.

Amplitudes of the conditions
$\varphi(F_{\{a=+,b=+\}}) = \varphi_{p_1} + \varphi_{p_5} + \varphi_{p_9} + \varphi_{p_{13}} = \alpha \neq 0$
$\varphi(F_{\{a'=+,b=+\}}) = \varphi_{p_1} + \varphi_{p_3} + \varphi_{p_5} + \varphi_{p_7} = 0$
$\varphi(F_{\{a=+,b'=+\}}) = \varphi_{p_1} + \varphi_{p_2} + \varphi_{p_9} + \varphi_{p_{10}} = 0$
$\varphi(F_{\{a'=-,b'=-\}}) = \varphi_{p_{13}} + \varphi_{p_{14}} + \varphi_{p_{15}} + \varphi_{p_{16}} = 0$

Table 3: The linear equations representing the conditions (i), (ii), (iii), and (iv).

These conditions can be expressed simply in terms of possibilities contained in $\mathbf{F}$. The first one, $a = -$ and $b = -$, is $F_{\{a=-,b=-\}}$ which contain the four elementary possibilities $\{p_4, p_8, p_{12}, p_{16}\}$. These are forbidden. A possibility that does not happen has the amplitude zero. Hence, we obtain four linear equations:

$$\varphi_{p_4} = \varphi_{p_8} = \varphi_{p_{12}} = \varphi_{p_{16}} = 0. \tag{41}$$

All further conditions are given in Table 2.

Principle 1 implies that the probability amplitude of any possibility $F \in \mathbf{F}$ is the sum of the probability amplitudes of the elementary possibilities $p \in F$. Thus, we obtain four linear equations which represent the conditions (i), (ii), (iii), and (iv). They are displayed in Table 3.

These eight linear equations, displayed in (41) and Table 3, form an inhomogeneous system of full rank with sixteen variables. The space of solutions is an eight-dimensional linear manifold. This proves that Hardy's experiment is realizable. The concrete amplitudes depend on the experimental set-up. The amplitudes of the outcomes must be normalized such that their sum of the squared magnitudes is one, yielding probabilities. Given the probabilities, the two particles choose in the present an elementary possibility p that is not forbidden but with the tendency to move to high probability outcomes.

8 Consistency and Symmetry

In this section, we show that our probability theory is consistent, that is, it does not lead to any contradiction. Moreover, we prove that this theory contains a $U(1)$ symmetry: All probabilistic statements are invariant if one transforms the amplitudes of all elementary possibilities with one element of $U(1)$.

Firstly, we prove that the probability amplitude is well-defined. The amplitude φ_F should not depend on the partitioning of F. If F contains only one element, nothing is to prove. Let two disjoint elements be given such that $F = \cup\{F_1, F_2\}$. Then $\varphi_F = \varphi_{F_1} + \varphi_{F_2} = \varphi_{F_2} + \varphi_{F_1}$ is well-defined. For three pairwise disjoint possibilities F_1, F_2, F_3, we can partition $F = \cup\{F_1, F_2, F_3\}$ as follows:

$$F_1, F_2, F_3;\ \cup\{F_1, F_2\}, F_3;\ \cup\{F_1, F_3\}, F_2;\ \cup\{F_2, F_3\}, F_1. \tag{42}$$

Since complex addition is associative and commutative, in each case the first principle yields

$$\varphi_F = \varphi_{F_1} + \varphi_{F_2} + \varphi_{F_3}. \tag{43}$$

Thus, φ_F is well-defined. The same holds true when we partition F into more than three elements:

$$\varphi_F = \sum_m \varphi_{F_m}. \tag{44}$$

The second principle says that the sum of the square of the magnitudes of probability amplitudes corresponding to the outcomes is one. This simple normalization condition can always be achieved.

Secondly, the multiplication of all probability amplitudes with the same element $e^{i\phi} \in U(1)$ does, due to Born's rule, not change the probabilities. Thus, our framework contains $U(1)$ symmetry. The fundamental symmetry group $U(1)$ leaves the inner product of two complex numbers and thus their norm constant. It is well-known that this group is locally isomorphic to the symmetry group $SO(2)$, the group of rotations in the two-dimensional real space. $U(1)$ gauge symmetry is well-known in quantum electrodynamics. One cannot measure the absolute phase of the wave functions of electrons, photons, or other particles.

9 The Inclusion-Exclusion Principle

The principle of inclusion and exclusion is known to find its most general formulation in the area of measure theory[28]. Moreover, it is a basic counting tool for calculating the number of elements contained in a union of sets. Frequently this principle is used in combinatorics[29]. Rota[30] put this principle in the form:

> *One of the most useful principles of enumeration in discrete probability and combinatorial theory is the celebrated principle of inclusion-exclusion. When skillfully applied, this principle has yielded the solution to many a combinatorial problem.* Rota

Now, it has a wide area of applications: Graph coloring, matching theory, Euler's ϕ function, Stirling numbers, occupancy problems, chromatic polynomials, and many more.

[28] Cerasoli, Fedullo [2002]
[29] Roberts, Tessman [2009]
[30] Rota [1964]

We generalize this principle to complex amplitudes, which satisfy our first principle. For two finite sets F and G, it is

$$\varphi(F \cup G) = \varphi(F) + \varphi(G) - \varphi(F \cap G). \tag{45}$$

The simple proof is as follows. Since

$$F = (F - F \cap G) \cup (F \cap G),\ G = (G - F \cap G) \cup (F \cap G), \tag{46}$$

and

$$F \cup G = (F - F \cap G) \cup (G - F \cap G) \cup (F \cap G), \tag{47}$$

we have pairwise disjoint partitions of F,G, and $F \cup G$, respectively. Thus, our first principle implies

$$\varphi(F) = \varphi(F - F \cap G)) + \varphi(F \cap G),\ \varphi(G) = \varphi(G - F \cap G)) + \varphi(F \cap G). \tag{48}$$

Moreover, it follows that

$$\varphi(F \cup G) = \varphi(F - F \cap G)) + \varphi(G - F \cap G) + \varphi(F \cap G). \tag{49}$$

Inserting formula (48) finishes the proof.

With induction, the general principle of *inclusion and exclusion* for n sets $F_1, ..., F_n$ can be proved:

$$\begin{aligned} \varphi\left(\bigcup_{i=1}^{n} F_i\right) &= \sum_{i=1}^{n} \varphi(F_i) - \sum_{i<j} \varphi(F_i \cap F_j) + \sum_{i<j<k} \varphi(F_i \cap F_j \cap F_k) \\ &+ \cdots + (-1)^{n-1} \sum_{i<\ldots<n} \varphi\left(\bigcap_{i=1}^{n} F_i\right). \end{aligned} \tag{50}$$

This inclusion and exclusion formula is fulfilled for all complex-valued functions φ that obey our first principle. For example, the function

$$\varphi(F) = \text{const} \sum_{p \in F} \varphi(\{p\}), \tag{51}$$

where const is any constant, satisfies the first principle. A special case is the cardinality of F:

$$\varphi(F) = \sum_{p \in F} 1 = |F|, \tag{52}$$

which is widely used in combinatorics. This principle remains valid when the classical probability replaces the cardinality. Then our function takes the form

$$\varphi(F) = \sum_{p \in F} \Pr(\{p\}). \tag{53}$$

It follows immediately that inclusion and exclusion can be used in measure theory and statistical thermodynamics. Finally, we mention the case

$$\varphi(F) = \text{const} \sum_{p \in F} e^{\frac{i}{\hbar} S(\{p\})}, \tag{54}$$

which is related to Feynman's path integral. More precisely, if F is an outcome in a slit experiment, p denotes the path from a source to a detector corresponding to F, then the formula (53) is the path integral.

Although the important inclusion-exclusion principle is fulfilled for probability amplitudes, several fundamental inequalities well-known in probability theory are not satisfied. Examples are

$$F \subseteq G \Rightarrow \varphi(F) \leq \varphi(G), \tag{55}$$

and

$$\varphi\left(\bigcup_{i=1}^{n} F_i\right) \leq \sum_{i=1}^{n} \varphi(F_i). \tag{56}$$

Both inequalities are satisfied if φ denotes the cardinality or a non-negative measure. Otherwise, they are not satisfied.

For example, let $\mathbf{P} = \{1,2,3\}$, let $\varphi(\{1\}) = \varphi(\{2\}) = 1$, $\varphi(\{3\}) = -1$, and let $F = \{1,2\}$, $G = \{1,2,3\}$. Then $F \subseteq G$, but $\varphi(F) = 2 > \varphi(G) = 1$, thus violating inequality (55).

If we define $F_1 = \{1,3\}$, $F_2 = \{2,3\}$, then $\varphi(F_1 \cup F_2) = 1 > \varphi(F_1) + \varphi(F_2) = 0$, thus violating inequality (56).

Such violations are natural in physics. For instance, let us look at the experiment of light reflection. Let G denote all paths reflected at the segments $1,2,3$ as displayed in Figure 4, and let F be the set of all paths at the segments displayed in Figure 5. Then $F \subseteq G$, but $\Pr(F) > \Pr(G) = 0$, thus violating inequality (55).

10 Thermodynamics

> *A theory is the more impressive the greater the simplicity of its premises is, the more different kinds of things it relates, and the more extended is its area of applicability. Therefore the deep impression which classical thermodynamics made upon me. It is the only physical theory of universal content concerning which I am convinced that within the framework of the applicability of its basic concepts, it will never be overthrown.* Albert Einstein, Autobiographical Notes (1946)

Hence, it is an important touchstone to reconstruct thermodynamics using our probability theory. More details about this reconstruction can be found in Jansson[31]. For a nice introduction to the theory of thermodynamics, we mention Penrose[32], and moreover four textbooks[33] which are well-suited for

[31] Jansson [2021], Chapter 5
[32] Penrose [2005], Chapter 27
[33] Schroeder [1999], Gould, Tobochnik [2010], Schwarz [2017], Swendsen [2020]

engineers.

In thermodynamics, huge numbers of constituents are considered. For example, 1 mole of molecules corresponds to Avogadro's number $\approx 10^{23}$. Therefore, thermodynamics is basically a statistical theory.

A thermodynamic system consists of a large set of constituents. These are described in terms of *microstates*; each constituent may have position and momentum or a quantum of energy. A microstate is a specific configuration of a system such that all possible microscopic variables are fixed. The microstates form distinguishable alternatives. They either happen or do not happen in the present, but two or more microstates cannot occur simultaneously.

Macrostates refer to the thermodynamic system as a whole. A system has only a few macroscopic variables, like the total energy E, pressure P, volume V, temperature T, or the total number N of gas molecules. In the following, we write shortly M for a macrostate and μ for a microstate.

The number of microstates, the configurations with all exact values, may be huge. In contrast, a macrostate emerges by fixing the value of a few macroscopic variables. Each macrostate consists of many microstates, the so-called *accessible* ones. The *multiplicity* of a *macrostate* M is defined as the number of its microstates, and is denoted by $\Omega(M)$. The *total multiplicity* Ω_{tot} is defined as the sum over all multiplicities $\Omega(M)$.

The macrostates can be measured in contrast to the microstates. Macrostates form a partitioning of the set of all microstates of the system.

The fundamental principle in statistical thermodynamics states that **all microstates of a system are equally probable**. It follows that the probability of a macrostate M is the multiplicity of this macrostate divided by the total multiplicity:

$$\Pr(M) = \frac{\Omega(M)}{\Omega_{tot}}. \tag{57}$$

The obvious way to connect the concepts of thermodynamics with our probability theory is to identify the microstates μ as the elementary possibilities $p \in \mathbf{P}$. The macrostates M, as measurable states, correspond to the outcome $F \in \mathbf{O}$.

Now, we reconstruct the probabilities for macrostates (57) with our probability theory. Our **third principle** says that all elementary possibilities contribute equally in magnitude, that is, the microstates μ have the amplitudes

$$\varphi_\mu = \text{const } e^{\frac{i}{\hbar}S(\mu)}. \tag{58}$$

We have no further knowledge about the actions of the constituents. Therefore, it seems natural to set the action $S(\mu) = 0$ for all microstates. Then the exponential term is one, implying that there is no interaction and interference. Moreover, we set

$$\text{const} = \frac{1}{\sqrt{\Omega_{tot}}\sqrt{\Omega(M)}}. \tag{59}$$

Then

$$\varphi_\mu = \frac{1}{\sqrt{\Omega_{tot}}\sqrt{\Omega(M)}} \cdot 1. \tag{60}$$

Since the microstates are pairwise disjoint, we can apply the first principle. Then the probability amplitude of a macrostate M takes the form:

$$\varphi_M = \sum_{\mu \in M} \varphi_\mu = \Omega(M) \frac{1}{\sqrt{\Omega_{tot}}\sqrt{\Omega(M)}} = \sqrt{\frac{\Omega(M)}{\Omega_{tot}}}. \tag{61}$$

Computing the square of the magnitude of probability amplitudes according to Born's rule yields the classical probabilities (57) for the outcomes.

Obviously, in our derivation, we did not apply the thermodynamic principle of indifference. Instead, we define the action of all elementary possibilities (microstates) as equal to zero, which is a statement about the experimental set-up, not about probabilities.

11 Reconstruction of Quantum Mechanics

We reconstruct Feynman's quantum mechanics[34], which is based on path integrals. His theory is known to be mathematically equivalent to Schrödinger's and Heisenberg's formulations of quantum mechanics. Our probability theory differs from quantum mechanics: At first, our language is set theory in contrast to the usual quantum axioms formulated with Hilbert spaces. It uses classical logic only. Our theory significantly distinguishes between possibilities and outcomes. It works with complex numbers in contrast to the usual measure theory. Classical probability and thermodynamics can be reconstructed. A massive object is not at several places simultaneously, as frequently remarked in the literature; for example, see Penrose[35] who writes:

> *As we have seen, particularly in the previous chapter, the world actually does conspire to behave in a most fantastical way when examined at a tiny level at which quantum phenomena hold sway. A single material object can occupy several locations at the same time and like some vampire of fiction (able, at will, to transform between a bat and a man) can behave as a wave or as a particle seemingly as it chooses, its behavior being governed by mysterious numbers involving the "imaginary" square root of -1.* Penrose 2016

Parallel universes are not required; our theory is formulated as a single-world theory. It is not an interpretation of quantum mechanics. It is a probability theory, much more general than classical probability and quantum mechanics.

In the following, we derive *Feynman's path integral* by using zigzag paths $x(t)$: For a particle moving from position x_a at time t_a to x_b at time t_b in

[34] Feynman [1948]
[35] Penrose [2016, p.216]

spacetime, the time is divided up into n smaller segments $t_a = t_0 < t_1 < \cdots < t_{n-1} < t_n = t_b$ all of length $\varepsilon = (t_b - t_a)/n$.

The *possibility space* $\mathbf{P}$ consists of all paths from $a = (x_a, t_a)$ to $b = (x_b, t_b)$ where b varies in a spacetime subset B. This set may consist of finitely many points where detectors are positioned. In the following, we consider only finitely many zigzag paths. Thus the *possibility algebra* $\mathbf{F}$ is the power set of $\mathbf{P}$. For fixed $b \in B$, the non-elementary possibility

$$F(b,a) = \{x(t) \in \mathbf{P} : x(t_a) = x_a, x(t_b) = x_b\} \in \mathbf{F}. \tag{62}$$

The set of outcomes $\mathbf{O}$ consists of all sets $F(b,a)$ where b varies in B. Hence, they form a partitioning of $\mathbf{P}$.

Let $c = (x_c, t_c) \in C$ be a point such that $t_a < t_c < t_b$, then we define the non-elementary possibility

$$F(b,c,a) = \{x(t) : x(t_a) = x_a, x(t_c) = x_c, x(t_b) = x_b\} \in \mathbf{F}. \tag{63}$$

It follows that

$$F(b,c,a) = F(b,c) \cap F(c,a), \tag{64}$$

where the sets on the right-hand side are defined as above. Moreover,

$$F(b,a) = \bigcup_{c \in C} F(b,c,a). \tag{65}$$

The paths $x(t) \in \mathbf{P}$ are pairwise disjoint. Thus, the first principle yields

$$\varphi(F(b,a)) = \sum_{x(t) \in F(b,a)} \varphi(\{x(t)\}). \tag{66}$$

In the literature, the amplitude $\varphi(F(b,a))$ is called *Green's kernel of motion* and is denoted by $K(b,a)$. The second principle implies the probability $\Pr(b,a) = |K(b,a)|^2$ to move from a to b.

With the third principle, we obtain the amplitudes for the elementary possibilities

$$\varphi(\{x(t)\}) = \text{const} \ \exp\left(\frac{i}{\hbar} S(\{x(t)\})\right), \tag{67}$$

where

$$S(\{x(t)\}) = \int_{t_a}^{t_b} L(\dot{x}, x, t) dt. \tag{68}$$

is the action, defined as the integral over the Lagrangian L. Since we consider only zigzag paths, the action takes the form

$$S(\{x(t)\}) = \sum_{j=1}^{n} L\left(\frac{x_j - x_{j-1}}{\varepsilon}, \frac{x_j + x_{j-1}}{2}, \frac{t_j + t_{j-1}}{2}\right). \tag{69}$$

Formula (66) is the fundamental point in Feynman's theory. Hence, we ask how to perform the sum over all paths. The idea is to proceed as in the definition of the Riemann integral. There, the integral of a function is approximated in the form

$$\int_{x_a}^{x_b} f(x)dx \propto \sum_{j=0}^{n} f(x_j), \tag{70}$$

where the points x_j are equally spaced. This sum depends on n, and a limit would not exist. But with the normalization factor $\delta = (x_b - x_a)/n$ we obtain

$$\int_{x_a}^{x_b} f(x)dx = \lim_{\delta\to 0}(\delta \sum_{j=0}^{n} f(x_j)). \tag{71}$$

A similar normalization factor must be introduced for the path integral. But this turns out to be not trivial.

Putting all together and taking the limit $\varepsilon = (t_b - t_a)/n \to 0$, formula (66) yields *Feynman's path integral*

$$K(b,a) = \lim_{\varepsilon\to 0} \frac{1}{A} \int \cdots \int \exp\left(\frac{i}{\hbar} S(\{x(t)\})\right) \frac{dx_1}{A} \cdots \frac{dx_{n-1}}{A}, \tag{72}$$

where A is a normalization constant depending on the Lagrangian.

From (65), the first and fourth principle, and the fact that the classical action is additive, that is,

$$S(b,a) = S(b,c) + S(c,a), \tag{73}$$

it follows that

$$K(b,a) = \int_{x_c} K(b,c)K(c,a)dx_c. \tag{74}$$

More general for $(n+1)$ points we obtain

$$K(b,a) = \int_{x_1}\int_{x_2}\cdots\int_{x_{n-1}} K(b,n{-}1)K(n{-}1,n{-}2)\cdots K(1,a)\; dx_1 dx_2 \cdots dx_{n-1} \tag{75}$$

where

$$K(j,j-1) = \frac{1}{A}\exp\left(\frac{i}{\hbar}\varepsilon L\left(\frac{x_j - x_{j-1}}{\varepsilon}, \frac{x_j + x_{j-1}}{2}, \frac{t_j + t_{j-1}}{2}\right)\right). \tag{76}$$

Changing slightly the notation $x_b = x, t_b = t, x_a = y, t_a = s$ and using (74), we obtain the *wave function*

$$\varphi(x,t) = \int K(x,t;y,s)\varphi(y,s)dy. \tag{77}$$

Hence, the probability amplitude for the outcome of arriving at the point (x, t) is equal to the sum over all amplitudes to arrive at (y, s) multiplied by the amplitude to move from (y, s) to (x, t).

In the most used formulation of quantum mechanics, the Schrödinger equation is postulated. This equation can be derived from (77). We approximate the wave function in the first order of the time interval ε such that

$$\varphi(x, t+\varepsilon) = \frac{1}{A} \int \exp\left(\varepsilon \frac{i}{\hbar} L\left(\frac{x-y}{\varepsilon}, \frac{x+y}{2}, t\right)\right) \varphi(y, t) dy. \tag{78}$$

Let us look at the special Lagrangian $L = m\dot{x}^2 + V(x)$. If we substitute $y = x + \mu$, integrate and expand the resulting equation to first order in ε and second order in μ, then we obtain the *Schrödinger equation*

$$i\hbar \frac{\partial \varphi}{\partial t} = \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} \varphi + V\varphi. \tag{79}$$

Moreover, the normalization constant turns out to be[36]

$$A = \sqrt{\frac{2\pi\hbar\varepsilon i}{m}}. \tag{80}$$

Thus, with our probability theory, we have reconstructed Feynman's formulation in terms of path integrals, and have derived the Schrödinger equation. Quantization is an immediate consequence of this equation and hence a consequence of our probabilistic framework. However, quantization can be derived directly from the path integral, see Kleinert[37]. In classical probability theory, quantization cannot be derived.

It can be shown that the construction of the paths leads to the result that the paths may be continuous in the limit case, but they are nowhere differentiable, that is, the velocity is discontinuous at all points in spacetime.

The phase space path integral is a more general expression than the spacetime path integral presented here. The momentum is an essential parameter, connecting quantum mechanics with Hamiltonian formalism. We will not derive this path integral formulation. Almost all details about path integrals can be found in the monograph written by Kleinert[38] and the literature therein. We refer the reader also to Feynman[39].

12 Diffusion and Wiener Integral

Readers with knowledge of statistical mechanics might notice the similarity between Feynman's formulation and Brownian motion, where discretization corresponds to discrete time random walks. The path integral formulation is almost indistinguishable from that of Brownian motion. In this final section,

[36] Feynman [1948], Section 6
[37] Kleinert [2009], Sections 2.6 and 9.2
[38] Kleinert [2009]
[39] Feynman [1948], Feynman Lectures [1963], Feynman, Hibbs [1965]

we sketch some relationships between quantum path integrals, Brownian motion, Diffusion, and the Wiener Integral. The reader is referred to the book of Zeidler[40] and the literature therein.

The *heat equation* is a partial differential equation defined as the initial-value problem with initial time s:

$$\frac{\partial\varphi(x,t)}{\partial t} = -\kappa\frac{\partial^2}{\partial x^2}\varphi(x,t) - V(x)\varphi(x,t),\ t \geq s,\ \varphi(x,s) = \varphi_0(x). \tag{81}$$

Besides applications in many scientific fields like probability theory, financial mathematics, and image analysis, this equation describes the flow of heat in an isotropic and homogeneous medium, where $\varphi(x,t)$ denotes the temperature at point x and time t. Moreover, this equation can be regarded as a diffusion equation for a mass density $\varphi(x,t)$. Microscopically, diffusion is related to *Brownian motion*, that is, the random movement of microscopic particles in a gas or a liquid.

It can be proved[41] that its solution is

$$\varphi(x,t) = \int K(x,t;y,s)\varphi_0(y)dy. \tag{82}$$

where the heat kernel reads as

$$K(x,t;y,s) = \lim_{\varepsilon\to 0}\frac{1}{A}\int\cdots\int \exp\left(-S(\{x(t)\})\right)\,\frac{dx_1}{A}\,\cdots\,\frac{dx_{n-1}}{A}. \tag{83}$$

The value S denotes the discrete action for a linear zigzag path $x(t) = (x(t_i))$. For a Lagrangian, defined as the difference between the kinetic energy and the potential energy V, it is

$$S(\{x(t)\}) = \sum_{j=1}^{n}\frac{1}{4\kappa}\left(\frac{x_j - x_{j-1}}{\varepsilon}\right)^2 + V(x_j)\varepsilon. \tag{84}$$

Here, we use the same discretization as in Section 11. The normalization constant for points in the three-dimensional position space is

$$A = (4\pi\kappa\varepsilon)^{3/2}. \tag{85}$$

As in Feynman's path integral, the heat kernel $K(x,t;y,s)$ represents a summation over all paths connecting the starting point y with the final point x.

The path integral (83), also called a *Wiener integral*, has a rigorous meaning in the sense of a classical measure on a space of continuous functions[42].

13 Time

Time is, perhaps, the most commonly discussed concept in physics and philosophy[43]? Various questions emerge. Why does time appear in so many

[40] Zeidler [2006], Ch. 11
[41] Zeidler [2006], Section 11.8
[42] Reed, Simon [1972], Vol. II, Section X.11
[43] Taschner [2007], Ben-Naim [2016] ,Smolin [2013], Rovelli [2018]

equations? Is time a derived variable, or is it a fundamental variable? How many fundamental variables exist in physics? In other words, what can we say about the dimensions of space[44]?

The time parameter t appears in almost all physical equations. The fundamental variables such as the position $x(t)$, the velocity $v(t)$, the momentum $p(t)$, the energy $E(t)$, and so on, are time-dependent. The harmonic oscillator, for instance, is described by the well-known Euler-Lagrangian differential equation.

Equations without time dependence seem questionable at first or even very strange. However, the variable time appears unnecessary for describing physical systems. For example, the harmonic oscillator is implicitly defined by its total conserved energy, namely the sum of kinetic and potential energy. It represents an ellipse in the phase space. Another example is the famous *Wheeler-de Witt equation* which contains no time parameter. This equation is a candidate for the solution of the well-known quantum-gravitation problem. See the book of Hamber[45], which covers the theory of Quantum Gravitation with particular attention to Feynman path integrals. The fundamental theory of statistical thermodynamics, an almost universal physical theory, is timeless[46]. The second law of thermodynamics and entropy has nothing to do with time. Some physicists put forward the idea of "physics without time ", among them Rovelli[47].

We have introduced a probability theory describing future events. The future is timeless. However, our third principle requires calculating classical action, defined as Lagrangian's integral over time. Thus, a variable time comes into play through the back door.

Our approach is to replace the (3+1)-dimensional spacetime with a timeless Euclidean (3+3)-position-velocity space as the basis of physics. This space has a $(+ + + + + +)$ Euclidean signature, see my lecture notes[48]. In this position-velocity space, we can define clocks as machines that produce a derived quantity, which we call time. Then with some simple arguments, we reconstruct the key of relativity theory, namely the *Lorentz transform* without any assumption about "propagation of light". The rest of the mathematical framework of special relativity follows immediately. We don't use Einstein's two postulates, namely that (i) the laws of physics are the same in all inertial frames of reference, and that (ii) the speed of light in free space has the same value c in all inertial frames of reference.

There are two other physical models with six-dimensional world coordinates.

Firstly, the twister theory of Penrose[49] based on a non-Euclidean signature (+ + - - - -). He did not pursue this theory because he could not interpret the six variables appropriately.

[44] Mirman [2006]
[45] Hamber [2009]
[46] Ben-Naim [2018]
[47] Rovelli [2018]
[48] Jansson [2017, Sections 4.13 and 4.14]
[49] Penrose [2005] Ch. 33

Secondly, the theory of Heim[50], which is based on a non-Euclidean signature (+ + + - - -). His deep and fundamental theory opens a new worldview with various predictions: We live in a 6-dimensional world, all elementary particles and their dynamics are 6-dimensional, and human beings are too. Moreover, he states that there was never a big bang. He is not alone, see also Penrose[51], Ben-Naim[52] and Rovelli[53].

A surprising and important result of Heim's theory was to predict the masses and lifetimes of all known elementary particles to high precision. His formulas are used in CERN and DESY. Therefore, Heim's theory goes far beyond all known theories about quantum mechanics.

14 Conclusion

John Wheeler[54] argued that real insight into quantum theory requires that we would be able to describe this theory in one simple statement that anyone could understand. Our simple statement is:

Quantum theory can be reconstructed from a probability theory describing future events in terms of possibilities and outcomes, using classical logic, simple set theory for listing possibilities and outcomes, and complex numbers.

In more detail, a complex number is assigned to each possibility or outcome so that the absolute square of an outcome provides a probability. The described theory can even be taught in schools, similar to Kolmogorov's theory of probability.

[50] Auerbach, von Ludwiger [1992], von Ludwiger [2013], Eckardt, [2020]
[51] Penrose [2010]
[52] Ben-Naim [2016]
[53] Rovelli [2018]
[54] Ball, [2017]

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