## **Torque Vectoring**

Linear Parameter-Varying Control for an Electric Vehicle

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This external PhD thesis was carried out between December 2009 and December 2014 as co-operation between Intedis GmbH & Co. KG and Hamburg University of Technology (TUHH). The aim of this work is to improve vehicle dynamics for electric vehicles with two electric motors, using linear parameter-varying control.

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I dedicate this thesis to my family and to dear ones who did not see this day.

# Summary

In this thesis, advanced control theory is applied to the problem of controlling an electric vehicle with independent propulsion actuators. Here, a linear parameter-varying (LPV) vehicle dynamics controller is designed and implemented to control two independent electric machines driving the front wheels of a prototype vehicle developed as part of the European project eFuture. The control concept is implemented on a standard automotive microcontroller and deals with safety, performance and efficiency limitations of the vehicle. The thesis is divided into three main parts:

- The first part deals with the vehicle dynamics and the torque vectoring application. To begin, the dynamics of the vehicle are analysed in Chapter 2, and several vehicle models and tyre models are presented. The vehicle drivetrain is briefly discussed, and limitations and constraints on vehicle movement are pointed out. The validation of simulation models with experimental data is also shown. In Chapter 3, several torque vectoring applications are analysed.
- The second part is concerned with control theory and controller design for torque vectoring. Chapter 4 reviews the modelling and control of LPV systems. Different approaches to LPV control are compared, and it is shown how non-linear vehicle dynamics can be represented as LPV models. Chapter 5 investigates the influence of actuator limitations on vehicle behaviour and explains how an anti-windup design is implemented to deal with saturations of the electric drivetrain. This anti-windup concept is extended to cope with spinning or locking wheels.
- The third part of the thesis presents the implementation of the control design and experimental results using the prototype vehicle of the eFuture project. Chapter 6 discusses the general driveline software of the eFuture prototype and the interaction of different software functions with torque vectoring. A discrete-time controller design is proposed and the fixed-point representation of the controller is discussed. Chapter 7 discusses real test drives, which demonstrate the performance improvements achieved with torque vectoring, as compared to an equal torque distribution, as typically used in conventional vehicles.

## Abstract

In this thesis, a torque vectoring control strategy is proposed for the propulsion of an electric vehicle with two independent electric machines at the front wheels. The proposed control scheme comprises a linear parameter-varying (LPV) controller and a motor torque and wheel slip limiter which deals with drivetrain saturations and wheel slip limitations. This control strategy was implemented on a microcontroller in a test car. As part of the European project eFuture, test drives were carried out and measurements were performed in several test manoeuvres, which demonstrate the benefits of the proposed method as compared with equal torque distribution.

**Key words:** LPV systems, Torque vectoring, active yaw control,  $H_{\infty}$  control, eFuture, single track model, vehicle dynamics, tyre model, anti-windup, chassis control, drive-train, fixed-point representation

# Kurzzusammenfassung

Ziel der Arbeit war die Entwicklung einer Antriebsmomenten-Verteilungsstrategie für ein elektrisches Fahrzeug. Ein Prototyp wird mit zwei elektrischen Maschinen an der Vorderachse angetrieben. Die Antriebsmomenten-Verteilung besteht aus einem linearen, parameterveränderlichen (LPV) Fahrdynamik-Regler und einem Motormoment- und Radschlupf-Begrenzer. Das Regelkonzept wurde auf einem Mikrocontroller integriert, welcher für den automobilen Einsatz qualifiziert ist. Im Rahmen des europäischen Projekts eFuture wurden mehrere Testfahrten durchgeführt, die die Vorteile der vorgeschlagenen Regelstrategie gegenüber einer Gleichverteilung des Antriebsmoments zeigen.

**Schlüsselwörter:** LPV-Systeme, LMI, Torque vectoring, aktive Gierraten-Regelung,  $H_{\infty}$ -Regelung, eFuture, Einspurmodell, Fahrdynamik, Reifenmodell, Anti-Windup, Fahrwerkregelung, Antriebsstrang, Festkomma-Arithmetik

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- Design of a robust plausibility check for an adaptive vehicle observer in an electric vehicle (M. Korte, G. Kaiser, V. Scheuch, F. Holzmann, H. Roth) In Proceedings of the 16<sup>th</sup> Advanced Microsystems for Automotive Applications(AMAA), Berlin, Germany, May 2012.
- A New Functional Architecture for the Improvement of eCar Efficiency and Safety (V. Scheuch, G. Kaiser, R. Straschill, F. Holzmann), In Proceedings of the 21<sup>st</sup> Aachener Colloquium Automobile and Engine Technology, Aachen, Germany, October 2012.
- Torque Vectoring for an Electric Vehicle Using an LPV Drive Controller and a Torque and Slip Limiter (G. Kaiser, Q. Liu, C. Hoffmann, M. Korte and H. Werner), In Proceedings of the 51<sup>st</sup> IEEE Conference on Decision and Control(CDC), Maui, Hawaii, USA, December 2012.
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- LPV Torque Vectoring for an Electric Vehicle with Experimental Validation (G. Kaiser, M. Korte, Q. Liu, C. Hoffmann and H. Werner), In Proceedings of the 19<sup>th</sup> World Congress of the International Federation of Automatic Control(IFAC), Cape Town, South Africa, August 2014.

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# **1** Introduction

Today, most vehicles are powered by internal combustion engines (ICEs) and most ICEs run on products that are extracted from fossil fuels. These derivatives are mainly "Petrol", "Diesel" and sometimes natural gas. However, it is well known that fossil fuels are finite. Another source of energy is necessary. One possibility is to use ethanol, produced from biological crops. However, these plants create competition with food generation plants, and are not desired if not everybody has access to sufficient food supply. Another problem of ICEs is their generation of local emissions, which are unwanted in areas of high population density. Novel combustion engines produce fewer toxic substances and consume less fuel than previous ICEs, but they still produce exhaust gases and carbon dioxide (CO<sub>2</sub>). For example, Beijing had major smog problems, especially in the winter of 2013-2014 [1]. Also, Paris partly banned the use of purely internal combustion engine (ICE)-based vehicles in the beginning of 2014 [2] because of smog problems. Air pollution is a global problem that is not purely related to personal transportation, but automotive vehicles are one part of the problem.

Driven by their "green conscience", customers are starting to request vehicles that consume less fuel and pollute the environment less than in the past decades. Additionally, different sources of propulsion energy are being requested. The development of hydrogen fuel cell electric vehicles (FCEVs) and battery electric vehicles (BEVs) has increased sharply over the last two decades. A combination of ICEs and BEVs, known as hybrid electric vehicles (HEVs), have become popular. For example, Japan had a market share of over 20% for HEVs in 2013 [3] and in California, USA the actual market share of HEVs is around 7.2% [4]. Also, the sales of electric driven vehicles (vehicles which are propelled only by an electric motor) has risen during recent years. Technical and economic limitations of FCEVs and BEVs limit their production. However, it seems that the problems with BEVs have nearly been solved. At the beginning of 2014, the number of electric driven vehicles rose to 400 000 worldwide [5]. Additionally, the battery electric vehicle (BEV) "Tesla Model S" was the most sold vehicle in Norway for September and December of the year 2013. The market share of electric vehicles was 6.1% in Norway at the end of 2013 [6]. Besides the environmental and health considerations of the customer, economic considerations play a major role in these changes, and the economic environment is influenced by politics. In Norway, for example, subsides for an electric vehicles (EV) range from "free parking", "free travel on ferries" and "usage of bus lane" to "value added tax exemption" and "register fee exemptions" [7]. With all these reliefs and with new electric vehicless (EVs) entering the market, Carranza et al. [7] claim in 2013 that "the market penetration of electric vehicles in Norway could exceed 10% by the end of 2014".

New design possibilities for electric vehicles arise from new drivetrain structures. The

basic architectural changes related to electric vehicles are discussed in Section 1.1. The scope of this work is explained in Section 1.2. In Section 1.3, the main objective of the thesis is explained. The scientific contribution of this work is described in Section 1.4. An overview of this thesis is provided in Section 1.5.

## **1.1 Problem Description**

The motivation for this thesis is associated with the drivetrain of the vehicle. The drivetrain of a purely ICE-based vehicle architecture has requirements which do not apply to EV architectures. The category EV is used because here it does not matter if the electric energy is provided through a battery, a capacitor, a hydrogen fuel cell or even an ICE within a serial hybrid electric vehicle (HEV). The important fact is that the vehicle is equipped with electric machines for propulsion. The following review clarifies the differences among drivetrain architectures.

#### Drivetrain - internal combustion engine

In an ICE-based architecture, the drivetrain starts with a fuel tank. The fuel is transported to the ICE, where it is burned. During this process, chemical energy is converted into mechanical propulsion energy and dissipative heat energy. From the ICE, the mechanical energy is routed through the clutch to the gearbox. The clutch is installed to operate the ICE in its physical operation range. In the gearbox, the torque and angular velocity of the mechanical energy are modulated. From the gearbox, the mechanical energy is routed to the differential. The differential splits the energy to the left and right wheels. The order of this sequence is fixed, and only individual components may differ. Today, most automotive vehicles have an internal combustion engine (ICE), clutch (C) and gearbox (GB) located in the front, and actuate the front wheels with the differential (D), as shown in Figure 1.1. The tank (T) is located in the back of the vehicle, some-



Figure 1.1: drivetrain - internal combustion engine

where below the rear seats. To keep the diagram simple, Figure 1.1 represents these drivetrain components as spread through the length of the vehicle, although in fact they are located in the front of the vehicle.

### Drivetrain - generation one electric vehicle

There are two classes of EV-based drivetrains. In the first generation of EVs, the ICE is replaced with one electric machine. The electric machine does not need a clutch and gearbox anymore. However, many electric machines have a single gear to modulate the torque and angular velocity of the electric machine. Because of this single gear, the machine can be built with a smaller diameter and operates at a higher velocity. This is just a construction constraint, and from now on, the single gear is regarded as part of the electric machine. Additionally, the electric machine consists of a stator, a rotor and an inverter. The first-generation EV drivetrain is shown in Figure 1.2a.



(a) EV drivetrain, first-generation

(b) EV drivetrain, second generation

Figure 1.2: Drivetrain - electric vehicle

storage system (ESS) normally consists of a battery but may also consist of a fuel cell, a capacitor, a fly wheel, or some other source of electric energy storage. The ESS is connected to the electric machine (EM). The mechanical output of the EM is connected to the differential (D) which routes the energy to the wheels. These electric vehicles are available as serial production vehicles. Popular vehicles include the Mitsubishi i-Miev [8], Nissan Leaf [9], Renault Fluence [10], Renault ZOE [10], Smart ED [10], VW e-up! [10], Volvo C30 Electric [11], Tesla Model S [10], BMW i3 [12], Ford Focus Electric [10], Toyota RAV4 EV [13], Chevrolet Spark EV [14], Honda Fit EV [15] and many more.

### Drivetrain - generation two electric vehicle

For the drivetrain of the second generation EVs, the differential is removed with the integration of two (or four) EMs. Figure 1.2b gives an idea of this concept. The advantage of such a concept is the control of individual wheels and the possibility of different vehicle packaging designs. Drivetrain reliability is improved because it is possible to drive the vehicle even if one motor fails.

At present, only the Mercedes AMG SLS Electric Drive [16] is available as a serial product of second generation EVs. However, it is somewhat misleading to speak of a serial vehicle, given that the price of this vehicle is above  $\notin$ 400 000. Other prototypes, such as the Mitsubishi MIEV concept model [17], the Audi R8 e-tron [18], the Rimac Concept One [19] and so on, are being developed, showing a trend toward this technology. The location of the electric machines is not yet fixed. Some prototypes are equipped

with hub motors; others are equipped with in-chassis electric machines. Some vehicles have two motors at the front, some two at the rear, and some even have four motors for every wheel.

## 1.2 Scope of Work

This study aims to improve vehicle behaviour by developing a distributed propulsion system, driven by two independent electric motors. The safety and performance of the vehicle will be enhanced with a proper controller design. The non-linear, parameterdependent vehicle dynamics result in a ambitious control problem; in this work the challenge is addressed within the framework of linear parameter-varying systems, by developing, implementing and testing an LPV controller that is designed to guarantee stability and performance. Additionally, the controller will be implemented on an automotive microcontroller and validated with real test drives.

## 1.3 Main Objective

The developed controller should be integrated into the prototype-vehicle of the European project eFuture [20]. The eFuture project develops a new safe and efficient vehicle architecture. The project focuses on electric vehicles and on necessary considerations for producing such vehicles in serial production. Standard electric machines are used for this prototype, and all controllers are implemented on standard microcontrollers. Standards like AUTOSAR [21] for code generation or ISO 26262 [22] for functional safety are followed as closely as possible for a research project. Defined tests show the proper operation of all controllers which improve the vehicle dynamics and safety.

Following a series of computer simulations, experiments are performed using a carrier vehicle, shown in Figure 1.3. These tests validate the proper operation of the developed torque vectoring function. In this prototype, the electric drivetrain can be fully controlled and all necessary safety requirements for operating such a vehicle must be satisfied within the prototype. For this vehicle, the basic task of torque vectoring is generating proper torque commands for the front left and front right electric machines such that the vehicle operates safely and has an optimal performance, given the constraints of the hardware. This is achieved by designing and implementing an LPV controller which copes with non-linear vehicle dynamics.

## **1.4 Scientific Contribution**

The control design is implemented in a prototype-vehicle. The main contributions of this thesis are the following:

• An affine, linear parameter-varying vehicle model is defined which includes longitudinal and lateral vehicle movement. Existing linear fractional transformation



Figure 1.3: Prototype of the eFuture project

and polytopic linear parameter-varying design methods are applied to find linear parameter-varying controllers.

- An existing anti-windup controller design method is applied here to deal with motor limitations and is extended to meet different vehicle constraints in various operating conditions.
- To solve the problem of an underactuated system, the requirements of wheel slip limitation are integrated into the anti-windup design to achieve a "functionally controllable model" [23]. The extension of the anti-windup design to the motor torque and wheel slip limiter is developed.
- A polytopic linear parameter-varying controller for the longitudinal and lateral vehicle dynamics is implemented on an automotive microcontroller.

## 1.5 Thesis Overview

The rest of the thesis is organised as follows. In Chapter 2, the basic physical relations and equations for vehicle movement are discussed, especially the planar dynamics that are relevant for torque vectoring. The general idea of torque vectoring is explained in detail in Chapter 3. A review of different controller designs and implementations is provided. In Chapter 4, linear parameter-varying control is briefly explained and applied to the problem of torque vectoring. Chapter 5 develops an anti-windup concept to deal with the limitations of the electric drivetrain. This concept is extended to the motor torque and wheel slip limiter, which also suppresses spinning or locking of the driven wheels. Chapter 6 gives an overview of the steps needed to implement the torque vectoring controller on an automotive microcontroller. Results of test drives are discussed in Chapter 7. Conclusions and an outlook for future work are given in Chapter 8.

# 2 Automotive Vehicles

To discuss the design of a new torque vectoring controller, more information about vehicle dynamics and vehicle components is necessary. A brief account of these topics is presented in this chapter. Section 2.1 offers an overview of vehicle dynamics and equations to model dynamic vehicle behaviour. Section 2.2 summarises important components of dynamic vehicle behaviour. Different tyre models are described because tyres have a major influence on the movement of the vehicle. Additionally, information about the electric drivetrain is provided. Section 2.3 validates different simulation models with measurement data obtained during the eFuture project [20].

## 2.1 Vehicle Model

A vehicle model predicts the behaviour of the vehicle for given changes, inside or outside the vehicle. Computer simulations are used for defining and comparing such scenarios under different conditions. The field of vehicle simulations is used for various investigations. For example, crash simulations help to predict the deformation of the vehicle under certain test scenarios taken from real accidents. Injuries to driver and passengers are made visible and devices to prevent these injuries can be developed. Thermal stress simulations help to improve the durability of electric components.

In the present study, vehicle simulations are related to the movement of the vehicle with given inputs and disturbances. Inputs to the vehicle are the change of the steering wheel angle and torques acting on the wheels of the vehicle. Disturbances or external inputs are the aerodynamic drag forces, the incline of the road, varying road conditions and so on. A general vehicle model for movement in space is described. Afterwards, reduced models are derived from the general vehicle model, and are used in controller design and controller tuning.

## 2.1.1 Global vehicle model

For simulating vehicle dynamics, the vehicle is simplified to a single point in space, with a given mass m at the centre of gravity (CoG) and a moment of inertia I. The CoG moves along three dimensions in space which are described using a coordinate system. As an automotive standard [24] x is defined as the forward direction of the vehicle. The positive y direction is to the left side of the vehicle (looking from the top). The positive z direction is to the top side of the vehicle. Besides the three transversal movements, the vehicle rotates along the three axes. Rotation around the x-axis is referred to rolling and is determined by the angle  $\phi$ . Rotation around the y-axis is known as pitch angle  $\theta$ . Rotation around the vertical z-axis is defined as yaw angle  $\psi$ . The combination and orientation of the vehicle coordinate system is called "vehicle frame." If forces, moments or states are described in the vehicle frame, no superscript is used. Besides the CoG, the four wheel location points, i.e. front left (FL), front right (FR), rear left (RL) and rear right (RR), are important. These points are defined by the intersection of the wheels with the road. Besides the vehicle frame, coordinate frames for the wheels are defined. The wheel coordinate frames are indicated with a superscript <sup>w</sup>. The orientations of these frames are different from the vehicle frame if the wheels are steered or the position of the wheel frames changes with the bouncing of the vehicle. A diagram of the angles, movements and coordinate frames is given in Figure 2.1. For reviewing purposes, a global coordinate system is introduced with the



Figure 2.1: Three dimensional vehicle coordinate system and front right wheel frame

superscript g. This frame is important for describing the position of and the trajectory travelled by the vehicle.

### Vehicle motion

The movement of the vehicle is calculated using the equations of motion from Newton

$$\mathbf{m} \left( \mathbf{a} - \mathbf{v} \times \boldsymbol{\omega} \right) = \mathbf{F}_{\text{ext}} + \sum_{i=1}^{4} \left( \mathbf{F}_{\text{wheel},i} + \mathbf{F}_{\text{susp},i} \right)$$
(2.1)

and Euler

$$\mathbf{I}(\boldsymbol{\alpha} - \boldsymbol{\omega} \times \boldsymbol{\omega}) = \mathbf{M}_{\text{ext}} + \sum_{i=1}^{4} \left( \mathbf{M}_{\text{wheel},i} + \mathbf{M}_{\text{susp},i} \right)$$
(2.2)

as defined in [25]. The transversal acceleration **a** is defined by accelerations in directions x, y and z with  $\mathbf{a} = [a_x; a_y; a_z]$ . The angular acceleration  $\boldsymbol{\alpha}$  is defined by the angular acceleration around the three coordinate axes with  $\boldsymbol{\alpha} = [\alpha_x, \alpha_y, \alpha_z]^T$ . The velocity **v** is described by the velocities along the three axes  $\mathbf{v} = [v_x, v_y, v_z]^T$ . The angular velocity  $\boldsymbol{\omega}$  is described by the angular velocities around the three axes with  $\boldsymbol{\omega} = [\omega_x, \omega_y, \omega_z]^T$ .

The CoG changes its movement depending on the forces  $\mathbf{F} = [F_x, F_y, F_z]^T$  and moments  $\mathbf{M} = [M_x, M_y, M_z]^T$  which are generated by the wheel forces  $\mathbf{F}_{wheel}$  and the suspension forces  $\mathbf{F}_{susp}$ . Sidewind, gravitational forces and so on act as external forces  $\mathbf{F}_{ext}$  on the vehicle. Knowing the forces and the geometric properties of the vehicle, the wheel moment  $\mathbf{M}_{wheel}$ , the suspension moment  $\mathbf{M}_{susp}$  and the external moment  $\mathbf{M}_{ext}$  are calculated. The index *i* is defined as i = 1 for FL, i = 2 for FR, i = 3 for RL and i = 4 for RR.

The velocity **v** and angular velocity  $\boldsymbol{\omega}$  are defined as

$$\mathbf{v} = \int \mathbf{a} \, dt + \mathbf{v}_0 \tag{2.3}$$

$$\boldsymbol{\omega} = \int \boldsymbol{\alpha} \, dt + \boldsymbol{\omega}_0 \tag{2.4}$$

as the integrals of the acceleration and angular acceleration, where  $\mathbf{v}_0$  represents the initial velocity and  $\boldsymbol{\omega}_0$  the initial angular velocity.

For the torque vectoring development, it is sufficient to calculate (2.1 - 2.4). These equations describe the vehicle forces and their effects on the vehicle velocity. For visualisation, or other vehicle controllers like active cruise control, it is advantageous to calculate the position  $\mathbf{p}$  of the vehicle in the global coordinate frame  $\mathbf{p}^g = [p_x^g, p_y^g, p_z^g]$ . To calculate the global vehicle position  $\mathbf{p}^g$ , the velocity of the vehicle  $\mathbf{v}$  is described in the global coordinate frame as  $\mathbf{v}^g$  with the transformation matrix  $\mathbf{T}$ . Similarly, the global vehicle angle  $\mathbf{\Phi}^g = [\phi^g, \theta^g, \psi^g]$  is calculated from the angular velocity  $\boldsymbol{\omega}$  of the vehicle which is represented in the global coordinate system as  $\boldsymbol{\omega}^g$ . The transformation matrix  $\mathbf{T}^g$  from the vehicle to the global frame is defined as

$$\mathbf{T}^{g} = \begin{bmatrix} \cos\psi^{g} & \sin\psi^{g} & 0\\ -\sin\psi^{g} & \cos\psi^{g} & 0\\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta^{g} & 0 & -\sin\theta^{g}\\ 0 & 1 & 0\\ \sin\theta^{g} & 0 & \cos\theta^{g} \end{bmatrix} \begin{bmatrix} 1 & 0 & 0\\ 0 & \cos\phi^{g} & \sin\phi^{g}\\ 0 & -\sin\phi^{g} & \cos\phi^{g} \end{bmatrix}.$$
(2.5)

In the global frame the velocity and angular velocity are defined as

$$\mathbf{v}^g = \mathbf{T}^g \mathbf{v}$$
  
$$\boldsymbol{\omega}^g = \mathbf{T}^g \boldsymbol{\omega}.$$
 (2.6)

For the transformation matrices  $\mathbf{T}$ , the superscript indicates the new coordinate system where the subscript defines the actual coordinate system.  $\mathbf{T}^{g}$  defines the transformation from the vehicle coordinate system to the global coordinate system.

Integrating the velocity  $\mathbf{v}^g$  and angular velocity  $\boldsymbol{\omega}^g$  over the time t defines the global position  $\mathbf{p}^g$  and angle  $\Phi^g$  as

$$\mathbf{p}^g = \int \mathbf{v}^g \, dt + \mathbf{p}_0^g \tag{2.7}$$

$$\mathbf{\Phi}^g = \int \boldsymbol{\omega}^g \, dt + \mathbf{\Phi}^g_0, \tag{2.8}$$

where  $\mathbf{p}_0^g$  defines the initial vehicle position and  $\mathbf{\Phi}_0^g$  describes the initial vehicle angle in the global frame.

## 2.1.2 Dual-track model

The dual-track model (DTM) is a simplification of the global vehicle model into a twodimensional model which moves in the horizontal x-y plane. There is no standard DTM but most models [26], [27], [28] cover the longitudinal, lateral and yaw movements of the CoG. The DTM neglects transversal movement in the vertical direction and rotation about the x and y axes. Wheel dynamics in longitudinal and lateral directions are covered by static, normal forces. More advanced models use the longitudinal and lateral acceleration of the CoG to estimate load transfer to the vehicle wheels [29]. Figure 2.2a shows an average two-track model. A DTM simulates the horizontal movement of the



Figure 2.2: Dual-track model and single-track model

CoG and the angular velocity of the wheels. Normally, seven states are defined as

$$\dot{v}_x = v_y r + \frac{1}{m} \left( F_{ext,x} + \sum_{i=1}^4 F_{\text{wheel},x,i} \right)$$
(2.9)

$$\dot{v}_y = -v_x r + \frac{1}{m} \left( F_{ext,y} + \sum_{i=1}^4 F_{\text{wheel},y,i} \right)$$
 (2.10)

$$\dot{r} = \frac{1}{I_z} \left( M_{ext,z} + \sum_{i=1}^4 M_{\text{wheel},z,i} \right)$$
(2.11)

where the states are represented with the longitudinal velocity  $v_x$ , the lateral velocity  $v_{y}$ , the yaw rate r and the angular velocities of the four wheels  $\omega_{i}$ . The longitudinal tyre forces  $F_{x,i}$ , the lateral type forces  $F_{y,i}$  and the restoring moment  $M_{z,i}$  of the types act on the vehicle. The external forces  $F_{x,ext}$  in the longitudinal direction,  $F_{y,ext}$  in lateral direction and the external moment  $M_{z,ext}$  are additional disturbances to the vehicle's movement. External forces are related to air-drag, tyre-friction, trailer operation and so on. The mass m represents the weight of the vehicle. The vehicle moment of inertia around the vertical axis is described by  $I_z$  and the wheel moment of inertia around the spinning wheel axis is labelled  $I_w$ . The effective roll radius of the tyre R is defined as the distance from the road contact point to the centre point of the wheel. The tyre model is not fixed for the two-track model. The longitudinal type force  $F_{x,\text{wheel}}$ , the lateral type force  $F_{y,\text{wheel}}$  and tyre yaw moment  $M_{z,\text{wheel}}$  depend mainly on the longitudinal velocity of the vehicle  $v_x$ , the wheel slip  $\lambda$ , the type slip angle  $\alpha$ , the road surface conditions  $\mu$  and the vertical type load  $F_{z,\text{wheel}}$ . Different type models have been developed, and the accuracy of the calculation of type forces has a major effect on the quality of the two-track model. A detailed explanation of the type models is given in Section 2.2.1.

### 2.1.3 Single-track model

The single-track model (STM) is the most common model in the literature [25], [30], [31], [32] for lateral vehicle control. The basic idea of the STM is to merge both wheels of an axle into a single wheel. This idea is shown in Figure 2.2b. The model assumes that the left and right wheels generate the same lateral forces. The lateral force generation is linear to the combined tyre slip angle  $\alpha$ . The longitudinal tyre force generation is combined to a general, longitudinal input force  $F_x$ . The STM is non-linear but can be linearised for a certain longitudinal velocity  $v_{x_0}$ . Here, the STM expects that the tyre slip  $\lambda$  and tyre slip angle  $\alpha$  are limited and in the range of  $|\lambda| < 0.15$  and  $|\alpha| < 0.1$  rad. Furthermore, the vehicle must drive forward with  $v_x > 1$  kph to achieve numerically stable results. For reverse driving the equations (2.13 - 2.15) or (2.17 - 2.18) must be modified; see [25] for more details. The linear and non-linear vehicle models are regarded as front steering vehicles with additional devices, required to apply a yaw moment  $M_z$ .

#### Non-linear single-track model

The non-linear model is defined as

$$\dot{v}_x = v_y r + \frac{1}{m} F_x \tag{2.13}$$

$$\dot{v}_y = -\frac{C_{y,F} + C_{y,R}}{mv_x}v_y + \left(\frac{-l_F C_{y,F} + l_R C_{y,R}}{mv_x} - v_x\right)r + \frac{C_{y,F}}{m}\delta$$
(2.14)

$$\dot{r} = \frac{-l_F C_{y,F} + l_R C_{y,R}}{I_z v_x} v_y - \frac{l_F^2 C_{y,F} + l_R^2 C_{y,R}}{I_z v_x} r + \frac{l_F C_{y,F}}{I_z} \delta + \frac{1}{I_z} M_z.$$
(2.15)

Symbol	Value	Comment
$l_F$	1.240	Distance from front axle to centre of gravity[m]
$l_R$	1.228	Distance from rear axle to centre of gravity [m]
$C_{y,F}$	70,000	Cornering stiffness of the front axle [N]
$C_{y,R}$	84,000	Cornering stiffness of the rear axle [N]
$w_F$	1.445	Width of the front axle [m]
m	1624	Mass of the vehicle [kg]
$I_z$	1800	Moment of inertia around vertical axis $[\text{kg m}^2]$

Table 2.1: Parameters of the simulation model

The first state is the longitudinal velocity  $v_x$  of the vehicle. The second state is the lateral velocity  $v_y$  or the sideslip angle  $\beta$ . These two variables are interchangeable with

$$\beta = \arctan\left(\frac{v_y}{v_x}\right). \tag{2.16}$$

The third state is the yaw rate r of the vehicle. The physical parameters of the eFuture prototype are defined in Table 2.1. The mass m and the vertical moment of inertia  $I_z$ are physical properties. The distance from the front axle to the CoG  $l_F$  and the distance from CoG to the rear axle  $l_R$  are geometrical vehicle properties. The cornering stiffness of the front axis  $C_{y,F}$  and the rear axis cornering stiffness  $C_{y,R}$  are related to the wheel and suspension characteristics. The steering angle of the front wheels is defined as  $\delta$ . The input variable  $M_z$  in equation 2.15 is often not used within STMs because there is normally no additional yaw moment  $M_z$ . The additional yaw moment  $M_z$  can be generated by different wheel forces on the left and right side, as in [33], [34]. The same effect is also possible with varying surface or wheel conditions. Here, the yaw moment  $M_z$  is included because the basic idea of torque vectoring is to apply a yaw moment  $M_z$ to control the lateral movement of the vehicle. The single-track model is appropriate for describing the vehicle movement if the wheel forces are in their linear force generation regime.

### Linear single-track model

To achieve a linear single-track model, the non-linear model from 2.1.3 is linearised around a fixed, longitudinal velocity  $v_{x_0}$ . The state equations simplify to a linear model with two states, the lateral velocity  $v_y$  (or sideslip angle  $\beta$ ) and the yaw rate r. These states are defined as

$$\dot{v_y} = -\frac{C_{y,F} + C_{y,R}}{mv_{x_0}}v_y + \left(\frac{-l_F C_{y,F} + l_R C_{y,R}}{mv_{x_0}} - v_{x_0}\right)r + \frac{C_{y,F}}{m}\delta$$
(2.17)

$$\dot{r} = \frac{-l_F C_{y,F} + l_R C_{y,R}}{I_z v_{x_0}} v_y - \frac{l_F^2 C_{y,F} + l_R^2 C_{y,R}}{I_z v_{x_0}} r + \frac{l_F C_{y,F}}{I_z} \delta + \frac{1}{I_z} M_z.$$
(2.18)

The inputs to the linear STM are the steering angle of the front wheels  $\delta$  and the yaw moment  $M_z$ . The parameters of the model are defined in Table 2.1.

## 2.2 Vehicle Components

As mentioned before, the simulation of the vehicle dynamics relies on the physical laws of Newton and Euler. Therefore, the generated wheel forces acting on the chassis must be calculated. The resulting wheel forces are influenced by the propulsion system, the wheel steering and external forces. These components will be briefly discussed in the next section.

### 2.2.1 Wheels

The wheel tyres are one of the most important components for vehicle dynamics because the wheels are the vehicle's connection to the ground. The wheels have to fulfil various tasks. Firstly, wheels act as springs and dampers for the vehicle. Secondly, wheels generate longitudinal and lateral forces to manoeuvre the vehicle. To accelerate or brake the vehicle, a torque T is applied to the wheel through the electric motor or the hydraulic brake. The torque acting on the wheel changes the angular acceleration  $\dot{\omega}$  of the wheel and hence the movement of the wheel. This relationship is defined as

$$\dot{\omega} = \frac{1}{I_w} \left( T - F_x^w R \right), \qquad (2.19)$$

where the angular acceleration  $\dot{\omega}$  of the wheel depends on the wheel's moment of inertia  $I_w$ , the effective wheel radius R and the traction force  $F_x^w$ . The traction force  $F_x^w$  also acts on the chassis and moves the vehicle. The free body diagram of a quarter car model illustrates these connections, as shown in Figure 2.3. As well as the longitudinal tyre



Figure 2.3: Wheel dynamics - side and top view

force  $F_x^w$ , the lateral type force  $F_y^w$  is also important for vehicle movement. Research on types [35], [36], [37] began simultaneously with the development of the first vehicles and

continues today. The results for type force generation are mostly as shown as in Figure 2.4a. The longitudinal force  $F_x^w$  of the type is displayed over the longitudinal slip  $\lambda$  and



Figure 2.4: Wheel states and type forces

different wheel slip angles  $\alpha$ . The wheel slip is defined as

$$\lambda = \frac{\omega R - v_x^w}{(|\omega R|) \cup (|v_x^w|)},\tag{2.20}$$

where the tyre slip  $\lambda$  is calculated using the longitudinal velocity of the tyre centre  $v_x^w$ , the angular velocity of the tyre  $\omega$  and the effective tyre radius R. In (2.20), the tyre slip normally<sup>1</sup> ranges from [-1,1] and can be used for traction, braking and reverse driving conditions. Numerical problems arise for low velocities, so (2.20) is applied for  $(|v_x| \cap |\omega R|) > 1 \frac{\mathrm{m}}{\mathrm{s}}$ .

The lateral force  $F_y^w$  is shown in Figure 2.4b over the type slip angle  $\alpha$ , for different longitudinal slip values. The angle  $\alpha$  is defined as the angle between the direction of motion and the orientation of the wheel with

$$\alpha = \arctan\left(\frac{v_y^w}{v_x^w}\right). \tag{2.21}$$

The wheel slip angle is calculated using the longitudinal velocity of the wheel  $v_x^w$  and the lateral velocity  $v_y^w$ . If the longitudinal and lateral velocities of the wheel are not

<sup>&</sup>lt;sup>1</sup>an range from [-2, 2] is possible if the vehicle moves forward and the tyre rotates backwards (or vice versa). However, this scenario is very unusual and will be neglected here.

available, the tyre sideslip angle is calculated using

$$\alpha = \delta - \arctan\left(\frac{v_y + d_{x,i}r}{v_x - d_{y,i}r}\right),\tag{2.22}$$

where  $v_x$  is the longitudinal velocity,  $v_y$  is the lateral velocity and r the yaw rate of the vehicle. The signed, longitudinal distance from position i to the CoG is described using  $d_{x,i}$  and the signed, lateral distance from the CoG to point i is described using  $d_{y,i}$ . The sign is determined within the coordinate system from [24], which is displayed in Figure 2.2a, e.g. the rear, right wheel has negative sign values for  $d_{x,RR}$  and  $d_{y,RR}$ .

Force generation between the tyre and the road surface is highly non-linear and depends on many different factors. Various models have been developed in order to approximate the behaviour of the tyres. As well as longitudinal and lateral forces, the tyre's yaw moment is also significant, detailed information on this topic is given in [36]. Several models have been used and the most important models are described in the following pages.

#### Cogwheel tyre model

The "cogwheel" model is the simplest model because no wheel slip  $\lambda$  is possible between the road surface and the tyre. The velocity over ground  $v_x^w$  is directly linked to the angular velocity  $\omega$  of the wheel by

$$v_r^w = \omega R, \tag{2.23}$$

where R is the effective type radius. The applied moment T is related to the accelerating force  $F_x$  as

$$F_x^w = \frac{T}{R}.$$
(2.24)

This model is not valid for most driving conditions because it assumes a fixed interconnection of the surface and the tyre. However, this model is used for one special case. At low velocities  $((|v_x| \cap |R\omega|) < 1\frac{\mathrm{m}}{\mathrm{s}})$ , wheel slip can not be calculated accurately. In this condition, the cogwheel model is a numerically stable and is used instead of tyre slip-based models. For low velocities, the lateral wheel force  $F_y^w$  is estimated as

$$F_y^w = -C_l v_y^w, \tag{2.25}$$

where  $v_u^w$  is the lateral velocity of the wheel and  $C_l$  is a friction constant.

### Linear tyre model

In the linear tyre model, the wheel forces

$$F_x^w = C_x \lambda \tag{2.26}$$

$$F_y^w = C_y \alpha \tag{2.27}$$

generated are proportional to the wheel slip  $\lambda$  and the wheel slip angle  $\alpha$ . Longitudinal tyre stiffness  $C_x$  and cornering stiffness  $C_y$  are constant values. This model is accurate if wheel slip is limited to  $|\lambda| < 0.15$  and the wheel slip angle to  $|\alpha| < 0.1$  rad. For higher wheel slip or wheel slip angles, the linear tyre model calculates forces that are stronger than the real tyre forces.

### Dugoff tyre model

One of the earliest non-linear tyre models was developed by Howard Dugoff in 1970 and is referred to as the HSRI<sup>2</sup> tyre model. The wheel forces  $F_x^w$  and  $F_y^w$  are calculated from longitudinal slip  $\lambda$ , the wheel slip angle  $\alpha$ , the vertical load  $F_z$  and the road surface adhesion coefficient  $\mu$ . The wheel properties are combined in the parameters longitudinal wheel stiffness  $C_x$  and cornering stiffness  $C_y$  [37]. A modified version to correct high slip characteristics is provided in [38]. It calculates the tyre forces using

$$F_x^w = -\frac{C_x \lambda}{1 - \lambda} f(\kappa) \tag{2.28}$$

$$F_y^w = -\frac{C_y \tan^2 \alpha}{1 - \lambda} f(\kappa) \tag{2.29}$$

$$\kappa = \frac{\mu F_z^w (1 - \epsilon v \sqrt{\lambda^2 + \tan^2 \alpha})(1 - \lambda)}{2\sqrt{C_x^2 \lambda^2 + C_y^2 \tan^2 \alpha}}$$
(2.30)

$$f(\kappa) = \begin{cases} \kappa(2-\kappa) & \text{for } \kappa < 1\\ 1 & \text{for } \kappa \ge 1, \end{cases}$$
(2.31)

where  $\kappa$  is an internal variable, v is the vehicle speed and the parameter  $\epsilon$  is used for tuning the influence of the vehicle speed to high wheel slip and tyre slip angles. For the Dugoff tyre model, only the three parameters  $C_x$ ,  $C_y$  and  $\epsilon$  need to be calibrated. The drawback of the Dugoff model is the absence of the restoring moment  $M_z^w$ , which makes the simulation more accurate. The restoring moment  $M_z^w$  can be added by the calculation in [39]. The interconnection of longitudinal and lateral wheel forces is missing from [39], but can be included with the friction circle [36] limitation. The Dugoff tyre model describes a wide operating range for automotive vehicles, but the force calculations are not correct for extreme driving situations with combined longitudinal and lateral wheel force generation. The Dugoff model is also numerically unstable for low velocities, as is every wheel slip-based model.

<sup>&</sup>lt;sup>2</sup>Highway Safety Research Institute

Name	factor	$F_{x,front}$	$F_{y,front}$	$M_{z,front}$	$F_{x,rear}$	$F_{y,rear}$	$M_{z,rear}$
Stiffness factor	B	39.7	40.7	10	39.7	44.7	10
Shape factor	C	1.57	1.20	1.05	1.57	1.20	1.05
Peak factor	D	0.95	0.94	0.05	0.95	0.94	0.05
Curvature factor	E	0.96	0.88	-3	0.96	0.80	-3
Horizontal shift	$S_h$	0	0	0	0	0	0
Vertical sift	$S_v$	0	0	0	0	0	0

Table 2.2: Pacejka model parameters

#### Pacejka tyre model

The Pacejka tyre model [40], [36] is named after the scientist Hans Peter Pacejka. The Pacejka tyre model is also called the "Magic Formula model" because no physical laws are used. This model uses special equations which fit very well with the forces measured by various tests. The longitudinal force  $F_x^w$ , the lateral force  $F_y^w$  and the restoring moment  $M_z^w$  of the wheel are calculated with 18 parameter, as

$$F_x^w = (D \cdot \sin(C \cdot \arctan(B \cdot X_1 - E(B \cdot X_1 - \arctan(B \cdot X_1))))) + S_v$$
(2.32)

$$F_y^w = (D \cdot \sin(C \cdot \arctan(B \cdot X_2 - E(B \cdot X_2 - \arctan(B \cdot X_2))))) + S_v \qquad (2.33)$$

$$M_z^w = (D \cdot \sin(C \cdot \arctan(B \cdot X_2 - E(B \cdot X_2 - \arctan(B \cdot X_2))))) + S_v$$
(2.34)

$$X_1 = \lambda + S_h \tag{2.35}$$

$$X_2 = \alpha + S_h. \tag{2.36}$$

Parameters B, C, D and E are tuning parameters and  $S_h$  and  $S_v$  are chassis-based parameters. The inputs of the model are the longitudinal slip  $\lambda$ , the wheel slip angle  $\alpha$ and the vertical force  $F_z^w$ . Including the camber and inclination angle further improves the accuracy of the force calculation. Parameters  $B, C, D, E, S_h$  and  $S_v$  vary for the forces and moment calculations, and are listed in Table 2.2. The Pacejka model is widely used in the automotive industry for driving simulations because it is relatively fast and accurate. As a matter of fact, most race-car simulations use the Pacejka model to calculate type forces. The drawbacks of the model become evident at low velocities because the calculation of wheel slip and sideslip angle are numerically unstable for low velocities. The accuracy of the model is improved if parameters  $B, C, D, E, S_h$  and  $S_v$  depend on the longitudinal vehicle velocity  $v_x$ , the vertical load  $F_z$ , the road surface conditions  $\mu$  and the tyre inclination angle  $\iota$ .

### 2.2.2 Propulsion system

An electric vehicle has two sorts of actuators to change the longitudinal velocity of the vehicle: hydraulic brakes and electric motors. With electric motors, the dogma "hydraulic brakes to slow down - motor to accelerate" no longer applies. An electric motor, or more precisely an electric machine, can generate the same drive and brake torque. The only difference is that for acceleration the battery has to provide electric energy to the electric machines. The machines act as motors and convert electrical energy to mechanical energy. In the case of electrical deceleration, the electric machines act as generators and convert mechanical energy into electrical energy. The electrical energy generated is routed to the battery and charges the battery. The electric braking process is referred to as recuperation.

**Electric machine and inverter** The electric machine with the inverter converts electrical energy into mechanical energy. In contrast to a motor, the machine also converts mechanical to electrical energy. The amount of acceleration depends on various factors. The mechanical design of the motor defines the maximal torque that can be generated. The supply voltage from the inverter affects the electrical energy and energy losses. If the supply voltage is low, the current has to be high to get the same electrical power  $P_e$  because

$$P_e = U \cdot I, \tag{2.37}$$

where U is the supply voltage and I the current. With higher currents, power losses  $P_l$  are higher, and can be estimated as

$$P_l = R \cdot I^2, \tag{2.38}$$

where R is internal resistance. Power losses are converted to dissipative heat, which raise the temperature of the electric machine. Simultaneously, the mechanical power  $P_m$ generated by

$$P_m = P_e - P_l \tag{2.39}$$

is reduced and results in a lower machine torque

$$T = \frac{P_m}{\omega},\tag{2.40}$$

where T is torque generated by the machine and  $\omega$  the angular velocity of the machine.

The field of electric machines and their control is wide and complex, and it is not discussed generally here. For the application to torque vectoring, electric machines are treated as black boxes where certain amounts of torque  $T_{\rm req}$  are requested and particular torques  $T_{\rm real}$  are applied to the wheels. In normal operation mode, the requested and the applied torques are the same. However, electric machines are physical systems and torque output is limited by the maximal torque  $T_{\rm max}$ , the power limit  $P_{\rm max}$  and the torque slew rate limitation  $\dot{T}_{\rm max}$ . The machine torque plot in Figure 2.5 is helpful as a summary of these constraints. In addition to these static limits, the performance of an electric machine is limited by thermal, mechanical and communication constraints. These constraints are complex and time-varying.



Figure 2.5: Electric machine torque

## 2.3 Model Calibration

To calibrate the simulation models, test drives are performed using special measurement equipment. Several vehicle properties, such as vehicle mass, moment of inertia and tyre radius are directly measured. Other parameters, such as tyre cornering stiffness, cannot be measured directly, and these values are heuristically tuned. In the tuning process, the inputs of the real vehicle are used as inputs to the simulation model. The outputs of the simulation are compared with the real output signals. The unknown parameters are heuristically tuned until the vehicle state signals are consistent. For this task, three different vehicle models are used:

- A non-linear STM is used for the torque vectoring controller synthesis. The singletrack model is the simplest vehicle dynamics model and includes a linear tyre force model to calculate wheel forces.
- A DTM with a Dugoff tyre model is used for tuning the torque vectoring controller for various driving tests.
- The "virtual validation" is performed in a driving simulator, which is shown in Figure 2.6. The virtual prototype includes all of the developed software functions and a model of the real vehicle. The driving simulator is a 3D driving simulator allowing the vehicle model to move in three dimensions in space and also rotate about all three axes. A Pacejka tyre model is used to calculate the horizontal tyre forces. The electric drive train, including batteries, inverters, electric machines and its limitations, is also modelled. This "Full" model is used to test the interaction of all vehicle functions, and the interaction of the driver with the electric vehicle.

The inputs for the heuristic tuning process are the electric motor torques  $T_{\rm eMot}$  and the steering wheel angle  $\delta$ . The outputs compared (Figure 2.9 - Figure 2.17) are the



Figure 2.6: Driving simulator usage during software tests

longitudinal velocity  $v_x$ , the lateral velocity  $v_y$  and the yaw rate r. These signals are the most important ones for describing the vehicle's movement, but many more signals are inspected during the tuning process. All these signals are calculated using the STM, the DTM and the Full vehicle model. In the following section, a general driving manoeuvre and an extreme driving scenario are compared.

## 2.3.1 General driving

The first test manoeuvre is an average, unspecific driving scenario. The driver drives from the parking lot to the test field and does not perform any aggressive driving actions.

## Steering angle

The steering input of the driver is shown in Figure 2.7. The steering wheel angle varies



Figure 2.7: Model calibration with normal driving - steering angle

between -290 ° and 210 °. The rate of change for the steering command at the steering wheel  $\dot{\delta}_s$  is in a normal operation range  $|\dot{\delta}_s| < 8.73 \text{ rad/s}$ .
#### Motor torque

The torque of the two electric machines is displayed in Figure 2.8. The acceleration



Figure 2.8: Model calibration with normal driving - motor torque

request (which is not shown here, but the torque request can be used as reference)  $a_{x,req}$  is in a normal operation range  $|a_{x,req}| < 1 \text{ m/s}^2$  and does not lead to the maximal motor torque of 775 Nm. The left and right motor torques are different between 14 s and 78 s. During this time span, the torque vectoring function is active and distributes the torque.

#### Longitudinal velocity

Figure 2.9 compares the longitudinal velocity of the measurement (Meas), the STM, the DTM and the three-dimensional model (Full). The test track for this manoeuvre is not



Figure 2.9: Model calibration with normal driving - longitudinal velocity

completely flat, so a PID controller is included, which regulates the longitudinal velocity using the external model forces  $F_{x,ext}$ . With this controller, all three simulation models

calculate the vehicle velocity accurately. Small differences in the longitudinal velocity are visible at around 15 s, but the difference is below  $3 \text{ kph}^{-3}$ .

#### Lateral velocity

Figure 2.10 shows the measured and the simulated lateral vehicle velocity. The error



Figure 2.10: Model calibration with normal driving - lateral velocity

between the simulated and measured values is negligible. The measured lateral velocity is strongly corrupted with measurement noise. The increased noise is related to the optical measurement process of the Correvit sensor [41]. Small signals  $|v_y| < 0.4 \text{ m/s}$ are more corrupted than large signals because the signal-to-noise ratio is lower for small signals. The noise is less disturbing for stronger lateral manoeuvres.

#### Yaw rate

The yaw rates of the measurement and the simulation are shown in Figure 2.11. The yaw rates of the measurement and the three simulation models are similar. In sum, for an average driving manoeuvre, all three simulation models are sufficiently accurate. For normal driving, the single-track model is the most suitable model, because it is simple, fast and accurate enough.

#### 2.3.2 Extreme driving manoeuvre

A double lane change (DLC) [42] illustrates the model validity for extreme driving manoeuvres. The test is described in detail in Section 7.3. For the moment, it is sufficient to state that the DLC is an extreme lateral manoeuvre, which drives the vehicle and its types into the non-linear operation regime. In addition, the vertical dynamics such as rolling and pitching also have an effect on maximal type forces.

<sup>&</sup>lt;sup>3</sup>The author tries to use SI units. However, the average reader is used to certain units like "kph" for longitudinal velocity and "degrees" for the steering wheel angle. This may cause some inconveniences but many readers feel more comfortable with these units.



Figure 2.11: Model calibration with normal driving - yaw rate

#### Steering angle

The steering angle input of the driver is shown in Figure 2.12. At the beginning of the



Figure 2.12: Model calibration using extreme driving - steering wheel angle

test, the vehicle stops at the start location and the driver must steer at an angle of 65 ° to enter the test track. Form 1 s to 15 s the driver steers straight to the cone setting for the DLC. At 15 s the driver reaches the test setting and steers the vehicle through the cones with a left, right, left steering manoeuvre, which is shown in Figure 2.12. The steering angle varies between -212 ° and 197 °. The rate of change of the steering angle rises to 17.7 rad/s, which shows the extreme driver reaction.

#### 2.3.3 Motor torque

The electric motor torques are shown in Figure 2.13. From 0.1 s to 7.7 s the driver accelerates strongly to reach the desired test velocity. The maximal motor torque of 775 Nm is applied to both motors between 1.6 s and 2.6 s. Between 2.6 s and 7.3 s the motor torque is limited by the maximal power limit of 40 kW. Between 8 s and 14 s, the driver does not request strong motor torques because the desired test velocity is reached, and the driver maintains the velocity until entering the cone setting. At 14 s, the driver engages the neutral gear for the DLC.



Figure 2.13: Model calibration with extreme driving - motor torque

#### Longitudinal velocity

Figure 2.14 shows the longitudinal velocity of the measured and the simulated vehicles. The vehicle starts from a standstill and accelerates until it reaches a velocity of



Figure 2.14: Model calibration with extreme driving - longitudinal velocity

50 kph. Once this velocity is reached the driver maintains it until entering the DLC. The measured and simulated vehicle behaviours are similar, so all models have the same conditions for lateral vehicle dynamics.

#### Lateral velocity

The lateral vehicle velocity  $v_y$  is displayed in Figure 2.15. The sensor signal of the Correvit sensor is strongly corrupted with measurement noise, and it is not possible adequately to validate the quality of the simulation models. Nonetheless, it is possible to see that before 15 s no major lateral movement occurs. After 15 s, the vehicle performs the lateral movement.



Figure 2.15: Model calibration with extreme driving - lateral velocity

#### Lateral acceleration

The measurement of the lateral velocity  $v_x$  is strongly corrupted by measurement noise in this test. Another way to analyse the quality of the simulation models is to examine the lateral acceleration  $a_y$  in Figure 2.16. The time span of the Figure is now limited



Figure 2.16: Model calibration with extreme driving - lateral acceleration

to 15-20 s because the lateral movement is performed during this time. The STM is not as accurate as the other two models. During the time spans 15.5-16 s and 17.5-18 s the DTM and the Full model are relatively close to one another. However, the DTM seems to match the measurement data slightly better. By contrast, the Full vehicle model achieves better results in the time spans 16.4-16.8 s, 18.2-18.6 s and 19.2-19.6 s.

#### Yaw rate

Along with the lateral acceleration  $a_y$ , the yaw rate r is an important measurement of the lateral movement and is shown in Figure 2.17. The yaw rate is accurately simulated



Figure 2.17: Model calibration using extreme driving - yaw rate

using the Full model. Between 15.5-16 s and 18.3-19 s the Full simulation model is close to the measured yaw rate. The STM calculates the highest differences between simulation and measurement, and in particular the peak yaw rates at 15.8 s, 17.7 s and 18.7 s are overestimated by the STM. The accuracy of the DTM somewhere between that of the STM and the Full model.

# 2.4 Automotive Vehicle: Conclusion

In this chapter, several vehicle models and tyre models are described. Equations for vehicle movement and force generation are provided. The various models are calibrated and validated with measurement data from the eFuture project. In Chapter 4, the STM will be used for the controller synthesis because it is a simple and accurate model. The DTM is used with a Dugoff tyre model for the tuning of the basic torque vectoring controller, because the DTM is more accurate and includes additional physical constraints, such as tyre slip and so on. The final controller validation is performed with the full vehicle model, in which the Pacejka tyre model is implemented. This model is the slowest model computationally, but the Full model simulates vehicle behaviour very accurately, and tests with human drivers are performed with this model and the virtual driving simulator.

# **3** Torque Vectoring

The basic idea of torque vectoring is to distribute driving and braking torques to the wheels of the vehicle as shown in Figure 3.1. The wheel torques generate longitudinal



Figure 3.1: General torque vectoring concept

wheel forces  $F_{\rm FL}$ ,  $F_{\rm FR}$ ,  $F_{\rm RL}$  and  $F_{\rm RR}$ , which move the vehicle. For torque vectoring, it does not matter how the wheel torques are created, as long as the torques can be distributed individually to the wheels. The distributed wheel torques are used to increase vehicle safety [43], vehicle performance [44] and vehicle agility [45]. The driver feeling of the vehicle [46] is modified, increasing the "fun to drive aspect" [47]. Energy consumption is reduced [48], and driving offroad [49] or in bad road conditions [50] is improved.

This chapter provides a comparison of different forms of torque vectoring. The history of active safety is reviewed in Section 3.1 to clarify the "family background" of torque vectoring. Section 3.2 reviews actual implementations of torque vectoring in serial production vehicles. Actual control strategies from torque vectoring research are discussed in Section 3.3. Control properties of the vehicle model are analysed in Section 3.4. Requirements for torque vectoring are discussed in Section 3.5. A conclusion regarding torque vectoring is given in Section 3.6.

# 3.1 History

Torque vectoring is considered an active safety function. Active safety systems change the vehicle behaviour in such a manner as to make it less likely that the driver will experience an accident. Active safety systems can be divided into two categories. The first category improves the behaviour of the driver: these are described as advanced driver assistance systems (ADAS). Such systems enhance driver commands and include systems such as adaptive cruise control (ACC), emergency brake assistant (EBA), near object detection system (NODS), lane departure warning (LDW), lane keeping assistance system (LKAS) and many more.

The second active safety direction improves the behaviour of the vehicle. Such systems seek to keep the vehicle drivable for as long as possible, considering the vehicle's physical limitations. These systems include the anti-lock braking system (ABS), the traction control system (TCS), electronic stability control (ESC), active roll stabilisation (ARS) and the active suspension system (ASS). ABS, TCS and ESC are briefly reviewed because torque vectoring is related to these systems.

#### 3.1.1 Active safety functions

Vehicle behaviour is critical for most drivers if the vehicle leaves its linear attitude. For example, normal drivers are used to the fact that the vehicle turns more if the steering wheel is turned more. Now, if the front tyres reach their physical limits for lateral force generation, more steering does not turn the vehicle more strongly, it may even turn the vehicle less. This behaviour disturbs the driver and often results in dangerous accidents [51]. To improve vehicle behaviour in tyre force saturation regimes, several functions have been developed to make the vehicle more manageable for the driver.

#### ABS

ABS was the first active safety function to have been introduced for serial production vehicles in the 1970s [35], [51], [52]. ABS solves the problem of locked wheels caused by braking. If the driver brakes strongly, a high braking pressure is created which results in high braking forces acting on the wheels. Excessive braking forces lock the wheels. Locked wheels inhibit lateral wheel forces, so it becomes impossible to turn the vehicle. If the rear wheels are locked, the vehicle turns more, as expected, and even becomes unstable, which results in strong skidding. The ABS monitors the angular velocity of the wheels, and if one wheel has a tendency to lock, the brake pressure on the associated brake is reduced. With reduced brake pressure, the braking force acting on the wheel is reduced. The tendency of the wheel to lock is decreased, and therefore it becomes possible to generate lateral tyre forces.

#### TCS

The next active safety function development was TCS [35], [51]. TCS was introduced into serial production vehicles in the 1980s. ABS solves the problem of locked wheels during braking. TCS solves the problem of spinning wheels during acceleration. The general problem for the driver is the same. If the front wheels are spinning, no lateral forces can be generated and it is impossible to turn the vehicle with the steering wheel. If the rear wheels are spinning, no lateral rear wheel forces are generated and the vehicle becomes unstable and skids. When TCS recognises a spinning wheel it reduces the propulsion power of the engine, and, in some versions of TCS, actuates the hydraulic brakes of the spinning wheel.

### ESC

During the 90s ESC [35], [51], [52] was introduced. ABS and TCS deal with tyre force limitations for longitudinal requests like braking and accelerating. ESC deals with lateral drive requests during steering of the vehicle. ESC uses the steering angle of the driver to calculate how much the driver wants to turn the vehicle. The desired turning motion of the vehicle is compared with the actual turning motion. If the vehicle does not turn as much as desired, this is regarded as understeering. For an understeering vehicle, ESC generates a braking force on the inner wheel. This braking force generates a yaw moment  $M_z$  which increases the turning motion of the vehicle. Normally, the inner rear wheel is braked because in an understeering vehicle the front tyre forces are at their friction limits. Additional longitudinal forces would also reduce the lateral force capacity, as discussed in [53]. The vehicle is considered to be oversteering if it turns more than expected. For an oversteering vehicle, braking forces are applied to the outer wheels. Normally, the outer front wheel is braked because in an oversteering vehicle the rear wheels are at their saturation limits. Advanced ESC versions also use the steering angle of the wheels to modify the lateral performance of the vehicle [51].

#### 3.1.2 Interaction of active safety functions

Thus far, individual functions for active safety have been discussed. All of these functions aim to improve vehicle movement given the physical constraints arising from limited tyre forces. In most modern vehicles, many functions are included to improve vehicle behaviour while braking, accelerating and steering. However, each of these functions is only activated if certain driving limits are exceeded. The maximal tyre forces are described by the friction circle [53]. The active safety functions for improving vehicle behaviour can be graphically combined with this circle, as shown in Figure 3.2a. To visualise the activation strategy of these active safety functions, ABS is active during strong braking manoeuvres and ESC is active during strong lateral requests. Torque vectoring can be activated throughout most operation ranges and is not used only in safety critical situations, as indicated in Figure 3.2b. Additionally, ABS, TCS and ESC can be activated later if torque vectoring is integrated.

# 3.2 Vehicles

Torque vectoring is related to vehicle performance and helps to stabilise the vehicle, which improves driving safety. The key idea of torque vectoring is generating a force difference between the left and right wheels for improved cornering performance. The torque distribution is performed using the propulsion system and the brakes of the vehicle. In an actual standard vehicle, this is an internal combustion engine, including



Figure 3.2: Active safety activation strategy

an active differential, and hydraulic brakes. In future vehicle architectures, the actuators will be the hydraulic brakes and probably two or four electric machines. Torque vectoring can be divided into three classes: passive, active and electric torque vectoring systems.

#### Passive torque vectoring systems

Passive torque vectoring systems create the vehicle yaw moment with individual brake torque distribution. Passive torque vectoring is often described as "differential braking" because a braking difference is generated. A mechanical version of this function was introduced in 1997 for the Formula One McLaren Mercedes. However, the system was banned after protests from competitors. Newer systems use electronics which sense the steering angle and the yaw rate. This information is used to control the brake pressure for the individual wheels. The advantage of passive torque vectoring (TV) is a simple implementation because only the control software has to be changed if the vehicle is equipped with ESC. No additional parts need to be introduced, so the weight is constant, and the costs are increased only for additional software. The disadvantages of a passive TV are a reduced vehicle velocity and brake abrasions during activation.

#### Active torque vectoring systems

Active torque vectoring distributes the engine torque to the wheels of the vehicle. In vehicles, the mechanical differential splits the torque of the ICE to the left and right wheels of the vehicle. New types of differentials, so-called active differentials, have additional clutches and gears to distribute the drive torque to the left and right wheels [54]. Active torque vectoring uses driving torques to influence the movement and performance of the vehicle. It is rumoured that Mitsubishi introduced this system into the EVO II at

the World Rally Championship in 1994. The first serial production vehicle with active torque vectoring was the Mitsubishi Lancer Evolution IV in 1996. Active systems can be divided into front-wheel, rear-wheel, and four-wheel based systems. Front- or rearwheel based, active differentials are easier to build because only one active differential is required. Three active differentials must be implemented to control four wheels. The possibilities for influencing the vehicle behaviour are more intense with four-wheel based systems. The advantages of the active TV system are improved agility, the effectiveness of the system, reduced steering effort and no velocity losses. The disadvantage of the active TV is the introduction of additional parts for the active differential. These components increase the cost and the weight of the vehicle. Additionally, torque vectoring is only available if the vehicle is accelerating.

#### Electric torque vectoring systems

Electric torque vectoring systems are suitable for electric vehicles of the  $2^{nd}$  generation. Second generation electric vehicles are equipped with two or four electric machines, driving the wheels independently. These vehicles are considered to contribute an additional torque vectoring class because the electric machines generate positive and negative torques. High yaw moments can be generated because an electric TV is a combination of active and passive TV systems. The control of electric machines is fast and accurate, which implies an efficiently controlled vehicle. No active differential is necessary, so no additional hardware costs or weight are introduced. In the end, no differential is needed at all. As a result, torque vectoring is "for free" in an electric vehicle with two or four electric motors.

The advantages of fast, strong and accurate yaw moment generation create the drawbacks of the electric TVS in terms of functional safety [22]. It is important to guarantee that no undesireable yaw moment is generated that makes the vehicle unstable and risks serious accidents. Some considerations on this topic are discussed in Section 3.4. However, this problem is considered in detail in [20], [55].

A list of serial production vehicles with different torque vectoring systems is given in Table 3.1. At present, only one 2<sup>nd</sup> generation electric vehicle is available as a serial product for sale, the Mercedes AMG SLS electric drive [16]. This vehicle is equipped with four in-chassis electric machines, which drive all four wheels independently.

# 3.3 Controller Design

Several prototypes have been equipped with torque vectoring algorithms. Different systems have been developed which control the brakes, the propulsion system and the steering of the wheels. Individual actuation or use a combination of these systems is possible. Various control strategies are discussed in the literature and frequently-used concepts are summarised here. Before analysing the controller algorithm, it is useful to review the inputs and outputs of different torque vectoring strategies.

	*	
Vehicle	System	Year
Mitsubishi EVO II, for WRC	Active	1994
Mitsubishi EVO IV-X [56]	Active & Passive	1996
Honda/Acura RLX, MDX, RDX [57]	Active	2005
Audi A4, A5, S4, S5, Q5, A8 [58]	Active	2008
BMW X6, X6M, X5M [59]	Active	2008
Mercedes AMG S63, S65 [60]	Passive	2009
Mitsubishi Outlander GT [61]	Active & Passive	2009
Porsche 911 Turbo, Cayenne, Macan, Cayman [62]	Active	2009
Mercedes CL, CLS, GLA, S, SL, SLK [63]	Passive	2011
McLaren MP4-12C [64]	Passive	2011
Ford Focus [65]	Passive	2012
Nissan JUKE [66]	Active	2012
Range Rover Sport [67]	Passive	2013
Holden Special Vehicles Gen-F GTS [68]	Passive	2013
Mercedes AMG SLS electric [16]	Electric	2013
Cadillac XTS [69]	Active	2014
Lexus RC F [70]	Active	2014

Table 3.1: Torque vectoring implemented in serial production vehicles

#### 3.3.1 Input for the torque vectoring controller

Most torque vectoring systems, such as [27], [28], [44], [71], [72], [73], [74], [75], [76], [77], [78], [79], [80], [81], [82] control the yaw rate r of the vehicle. Fewer TV systems [43], [83] control the sideslip angle  $\beta$  of the vehicle or a combination of yaw rate and sideslip angle [34], [78], [84], [85], [86], [87], [88], [89], [90]. Other control designers try to minimise tyre forces in longitudinal and lateral directions [84], [87]. Some concepts [71], [78] use the longitudinal velocity  $v_x$ , the tyre slip  $\lambda$  [90] and the physical limitations of the propulsion system  $T_{\text{max}}$  [48] to improve controller operation.

#### 3.3.2 Output from the torque vectoring controller

The control output of most systems is the yaw moment, generated around the vertical axis of the vehicle. This yaw moment is generated by wheel force differences between the left and right side of the vehicle. This force difference can be created by different actuators. Active differentials are used in [86], [76], [75], [82]. Individual braking requests are applied in [76], [85], [72], [73], [74], [78], [91], [75], [77], [87], [89], [28], [90]. Different torque requests for electric machines are used in [84], [83], [78], [78], [44], [27], [78], [34], [79], [80], [81], [88], [89]. A combination of different wheel torques and active front steering, as in [91], [92], [73], [75], [85], [72], [74], [77], [87], [87], [82], is often used for further improvement of lateral vehicle performance.

#### 3.3.3 Control laws

Different control laws are used for torque vectoring. Frequently-used control strategies are briefly discussed here.

#### Feed forward control

The torque distribution between the left and right sides can be operated in an openloop scheme. The steering angle  $\delta$  is the input for the controller, and the output of the controller is the desired yaw moment  $M_z$ . The desired yaw moment is often described as

$$M_z = g(v_x)\delta,\tag{3.1}$$

where the nonlinear function g depends on longitudinal velocity  $v_x$ . Several concepts for the function  $g(v_x)$  are available.

- The function  $g(v_x)$  is a parameter-dependent gain [84], [93], [94]. This mapping is influenced by vehicle parameters and the longitudinal velocity.
- The dynamic behaviour is further improved if an additional time constant is included in the parameter-dependent gain [93].
- A look-up table is used for the mapping from the steering angle to the yaw moment [48], [34]. Additional limitations are introduced into the look-up table to cope with motor saturations and lateral boundaries.
- A flatness based feed-forward controller is used in [88] to improve the dynamic behaviour of the controller during dynamic vehicle manoeuvres.
- The dynamic model inversion technique, as in [91], is similar to the flatness based feed-forward controller and this technique is also used to improve the dynamic behaviour of the vehicle.

The advantages of feed-forward solutions are low computation effort and simple implementation of the controller, with accurate and fast performance for an appropriate model. The drawback of feed-forward solutions is sensitivity to noise and model variance. A feed-forward controller improves the performance of the closed-loop system but it is beneficial to include a feedback controller for parameter variations. In combined solutions, the feed-forward controller is used to improve the performance of the closedloop, and the feedback controller is introduced to deal with varying system parameters or disturbances.

#### **PID** control

The proportional-integral-derivative (PID) controller is the classic control structure and the most commonly used controller in practical applications [95]. The PID controller is well known, straightforward to implement, and detailed tuning rules are available [95]. Torque vectoring is designed with PID-controllers in [92], [78], [43], [80], [81], [96].

#### Sliding mode control

Sliding mode control is a non-linear control technique that forces the system to slide along a defined surface. Sliding mode control is appropriate for automotive applications because of its robustness and low computation effort. Different sliding mode control concepts are proposed for torque vectoring in [27], [75], [78], [86], [90], [92], [97].

### Predictive control

Some authors propose interesting results with model predictive control algorithms [33], [74], [87], [98], [99]. In model predictive control, the controller estimates the future behaviour of the system from specific inputs and minimises a given performance index to find the best control input.

#### Fuzzy control

Fuzzy control is widely used in control applications where human operators control the system with a set of rules for several inputs. Torque vectoring is not a typical application for fuzzy control, but [28] and [85] showed interesting results.

#### **Optimal and robust control**

Advanced linear control design methods from optimal and robust control are also applied to torque vectoring. For example, [34], [88], [90], [100], [89] use linear-quadratic-Gaussian (LQG) controllers.  $H_{\infty}$  controllers are used by [101], [102].

## LPV control

An extension of optimal and robust control techniques is LPV control. The ideas of  $H_{\infty}$  and  $H_2$  control are extended from linear time-invariant (LTI) systems to a certain class of non-linear systems.

- Interesting results are obtained by [77], [73], [103] where scheduling signals are used to choose between active front steering and brake force distribution.
- The major non-linearities in vehicle dynamics are related to non-linear tyre forces, described in Section 2.2.1. Baslamish [104] and Baslamish et al. [72] used tyre states like tyre slip  $\lambda$  and tyre slip angle  $\alpha$  as scheduling parameters.
- The surface condition of the road has a major effect on type force generation. Therefore Fallah et al. [50] used the maximal adhesion coefficient of the road  $\mu$  as a scheduling parameter.
- Palladino et al. [71], [105], Liu et al. [79], Kaiser et al. [106], [107] and Bartels et al. [108] define scheduling signals related to the longitudinal velocity of the vehicle. These approaches govern the non-linear behaviour of the single-track vehicle model.

# 3.4 Vehicle Behaviour

It is beneficial to review the vehicle dynamics before choosing a control method for a torque vectoring controller. The non-linear single-track model (2.13 - 2.15) is extended with air-drag forces acting on the vehicle. The resultant model is linearised [23] around certain operating points to analyse the dynamics of the system. Here, the non-linear state-space model

$$\dot{\boldsymbol{x}} = \boldsymbol{f}\left(\boldsymbol{x}, \boldsymbol{u}\right) \tag{3.2}$$

$$\boldsymbol{y} = \boldsymbol{x} \tag{3.3}$$

is considered with states  $\boldsymbol{x} = [v_x, v_y, r]^T$ , inputs  $\boldsymbol{u} = [\delta, F_x, M_z]^T$ , and outputs  $\boldsymbol{y} = [v_x, v_y, r]^T$ . The linearised model is considered in terms of steady-state operating points  $\boldsymbol{x}_0, \boldsymbol{u}_0$  and the deviation variables  $\Delta \boldsymbol{x}, \Delta \boldsymbol{u}$ . States  $\boldsymbol{x}$  and inputs  $\boldsymbol{u}$  are estimated as

$$\boldsymbol{x}(t) = \boldsymbol{x}_0 + \Delta \boldsymbol{x}(t) \tag{3.4}$$

$$\boldsymbol{u}(t) = \boldsymbol{u}_0 + \Delta \boldsymbol{u}(t). \tag{3.5}$$

Outputs  $\boldsymbol{y}$  are defined in (3.3). The state-space system is linearised, using the partial derivative of the non-linear functions  $\boldsymbol{f}$  in the operating point (marked as  $|_0$ ) as

$$\Delta \dot{\boldsymbol{x}} = \underbrace{\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}}}_{A} \underbrace{\Delta \boldsymbol{x}}_{A} + \underbrace{\frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}}}_{B} \underbrace{\Delta \boldsymbol{u}}_{B} (3.6)$$

The Jacobians of the non-linear functions f are the state-space matrices A and B. To analyse vehicle behaviour, vehicle movement is linearised around a given longitudinal velocity  $v_{x_0}$  and a given front wheel angle  $\delta_0$ . These two operating points are used to define the steady-state lateral velocity  $v_{y_0}$  and the steady-state yaw rate  $r_0$ . The states of the linearised single-track model are defined as  $\Delta x = [\Delta v_x, \Delta v_y, \Delta r]^T$  where  $\Delta v_x$ is the longitudinal velocity variation around the steady state longitudinal velocity  $v_{x_0}$ . The same principle is used for the lateral velocity  $\Delta v_y$  with steady-state  $v_{y_0}$  and yaw rate  $\Delta r$  and  $r_0$ . The system input is defined as  $\Delta u = [\Delta \delta, \Delta F_x, \Delta M_z]^T$  where  $\Delta \delta$  is the angle of the front wheels around the operating point  $\delta_0$ . The longitudinal force  $\Delta F_x$  acts around the steady state longitudinal force  $F_{x_0}$  and the yaw moment  $\Delta M_z$  acts around the steady-state yaw moment  $M_{z_0}$ . The Jacobians A and B are defined in Appendix A.1.

#### 3.4.1 Pole zero analysis

The location of poles and zeros define the behaviour of linear systems. The linearised single-track model (3.3, 3.6) has no transmission zeros, as  $C = I^{3\times3}$ ,  $D = 0^{3\times3}$  and  $det(B) \neq 0$  [109]. Figure 3.3 shows the poles of the linearised single-track model around various operating points. The operating points vary from 10 kph to 130 kph for the longitudinal velocity, and from -270 ° to 270 ° for the steering wheel angles. Figure 3.3



Figure 3.3: Linearised single-track model with location of poles for various operation points

shows that the poles vary in the left half plane, which does not guarantee stability of the system [110]. Moreover, the controller design must be robust against plant variations that degrade the performance. Furthermore, it is interesting to separate the effects of the longitudinal velocity and the steering wheel angle. Figure 3.4a shows the location of poles for straight line steering ( $\delta_0 = 0^\circ$ ) and a change of longitudinal velocity from 10 kph to 130 kph, in steps of 10 kph. The arrows in Figure 3.4a indicate the movement of the poles for an increasing vehicle velocity. For low velocities, two poles (related to the lateral dynamics) are located far in the left half plane. One pole (related to the longitudinal velocity) is located close to the imaginary axis. At higher velocities, the pole close to the imaginary axis moves further into the left half plane. The two other poles move closer to the imaginary axis and end up as complex conjugate poles for high velocities. The general shape of the movement of the poles is similar to that of the overall pole locations in Figure 3.3. Figure 3.4b shows the pole locations for a longitudinal velocity of 70 kph and a steering angle variation from  $-270^{\circ}$  to  $270^{\circ}$ , at the steering wheel, in steps of  $30^{\circ}$ . The location of the poles is relatively constant. The lateral movement of the vehicle has minor effects on the pole location, as compared to the longitudinal velocity.

#### 3.4.2 Frequency domain

The linearised single-track model is a multi input multi output (MIMO) system. Singular value plots are used to analyse the frequency behaviour of the system, instead of bode plots, used for single input single output (SISO) systems. Figure 3.5 shows the frequency behaviour of the system for varying longitudinal velocities and a steering angle of  $0^{\circ}$ .



Figure 3.4: Location of poles - separate velocity and steering angle variations

Three singular values are visible which are related to three input signals and three output signals. The absolute magnitudes of the signals are not significant because the system is not normalised. For example, a yaw moment change of 1 Nm has nearly no effect and a front wheel change of 1 rad/s of the steering angle has a major effect. Here, the maximum singular value and the minimum singular value are not important. The main fact is the impact of the longitudinal velocity on the singular values. The magnitude of one singular value rises with higher velocities; the magnitude of another singular value lowers with higher velocities. This implies that the vehicle reacts differently to the same inputs but for different longitudinal velocities of the vehicle. The frequency responses for a constant longitudinal velocity and varying steering angles are not shown here because these differences are relatively low, compared to the longitudinal velocity variation.

#### 3.4.3 Time domain

Besides the pole-zero location and the frequency response, time domain behaviour is also of interest. Figure 3.6 shows the responses due to different longitudinal velocities. Figure 3.6a shows the response of longitudinal velocity  $\Delta v_x$  to a step in the longitudinal force  $\Delta F_x$ , for different initial velocities  $v_{x_0}$ . The magnitude of these steps is reduced with increased velocity, but settling time is reduced with higher longitudinal velocities. The response is weaker but faster for higher velocities. Figure 3.6b shows the response from the yaw moment  $\Delta M_z$  to the yaw rate  $\Delta r$ . For lower velocities, the magnitude of the response is reduced but the settling time is shorter. The response is weaker but faster for lower velocities.



Figure 3.5: Linearised single-track model with pole zero location for various operation points

# 3.5 Requirements

Beyond the various implementations and control strategies, some constraints are valid for all torque vectoring controllers because these constraints are related to the physical properties of the vehicle or the safety margin for reliable operation of the vehicle. If the vehicle is slow, the lateral dynamics are negligible. The influence of a yaw moment  $M_z$  on yaw rate r is low, as shown in Figure 3.6b. The negative effects of tyre abrasion and energy consumption outweigh the positive effects of the yaw control. If the vehicle is fast, it is more important to control the lateral movement of the vehicle to reduce skidding and loss of vehicle control. Lateral control is achieved by the yaw moment  $M_z$ . For torque vectoring the following behaviour is desired:

- If the vehicle is slow, torque vectoring
  - should accelerate strongly,
  - should brake strongly,
  - can allow high control requests in a longitudinal direction,
  - can ignore yaw rate error, because skidding is not a issue,
  - must minimise yaw moment, because  $M_z$  does not change yaw movement and only harms the tyres.
- If the vehicle is fast, torque vectoring
  - should accelerate moderately,



Figure 3.6: Step responses for the major input output channels

- should brake strongly,
- should limit actuator requests to a moderate level,
- must be sensitive to the yaw rate tracking error, because skidding is a major issue,
- should allow high yaw moment requests in order to maintain the desired yaw rate.

# 3.6 Torque Vectoring: Conclusion

This chapter describes the historical background for torque vectoring and active safety functions. Various torque vectoring implementations in vehicles are reviewed and grouped into active, passive and electric torque vectoring systems. Different control strategies were developed for torque vectoring controllers, and a summary of these strategies is given. The vehicle dynamics themselves are examined to explain the general behaviour of the vehicle from a "control point of view". General vehicle dynamics are slightly altered for different steering angles. Longitudinal velocity has a major effect on vehicle behaviour. The main requirements for torque vectoring, relating to vehicle dynamics, are summarised. As well as vehicle dynamics, vehicle safety is a major concern for torque vectoring. This topic is omitted here, but discussed in [20], [55].

Finally, an LPV controller is chosen as the torque vectoring controller for the prototype vehicle. This controller type deals with non-linear and velocity-dependent vehicle dynamics. The controller guarantees stability and performance. The output of the controller is continuous, so switching effects can have no negative effects on the comfort of the ride. The tuning of the controller is systematic and saves development time. The real-time implementation of an LPV controller on an automotive microcontroller should be possible if this point is taken into consideration during the design of the controller.

# 4 LPV Modelling and Control of the Vehicle

As seen in Chapter 2, the vehicle model for torque vectoring is non-linear and parameterdependent. Non-linear control theory offers ways to design controllers, but these tend to be complex in synthesis and implementation, and are often difficult to tune for good performance. For this reason, in practice linear control techniques are often used even for non-linear systems, from simple proportional (P) controllers, to PID controllers to linear-quadratic regulator (LQR) and LQG or more advanced  $H_{\infty}$  or  $H_2$  controllers. To apply linear control techniques to non-linear plants, several strategies are possible:

- The controller is designed to be robust. The non-linear characteristics are treated as time-varying uncertainties. The controller copes with variations and achieves a stable closed-loop system. This approach sometimes leads to unsatisfactory performance because one controller has to handle all non-linear characteristics. The controller has to guarantee stability for a wide range of parameter variations, which normally reduces the performance of the closed-loop system.
- Another possibility is linearising the non-linear model at certain points in the operation range and calculating appropriate controllers for these points. A switching sequence, interpolation scheme or lookup table is used to control the non-linear system between its defined linearisation points. This method is called (heuristic) gain-scheduling control and is often used in practical applications. Gain-scheduled controllers benefit from a simple application, good performance and low computation effort. A drawback of gain-scheduled control is the cumbersome search for linearisation points. The design of the scheduling process is tedious as it requires time to find an acceptable switching strategy. The most serious drawback of gain-scheduling control is the loss of stability and performance guarantees for the controller design. In practice, many simulations are performed to ensure stability, but the danger of losing stability in an unfortunate switching sequence remains a concern.
- A way of extending well-tried linear techniques, such as H<sub>∞</sub> control, to non-linear systems is to use LPV gain scheduling. Conditions for stability and performance of the closed-loop systems are expressed as convex optimisation problems. The convex optimisation problem is defined as linear matrix inequality (LMI). Several LMIs can be combined for more tuning requirements, and the LMIs are solved with numerical tools like [111], [112]. The advantage of linear parameter-varying control design is that LPV guarantees stability and performance of the closed-loop system.

It is possible to include robustness, bandwidth and time-domain constraints. The design of LPV controllers is potentionally conservative, which means that a controller with better performance might be available; various techniques are available to reduce this conservatism.

General properties of LPV control are discussed in Section 4.1. In Section 4.2, the synthesis procedures for linear fractional transformation linear parameter-varying (LFT-LPV) and polytopic linear parameter-varying (polytopic LPV) controllers are described. In Section 4.3, the vehicle model is converted into both an LFT-LPV and a polytopic LPV model. In Section 4.4 shaping filters and a generalised plant are defined for the LFT-LPV controller and the polytopic LPV controller. Additional constraints are integrated in the designs for practical implementation and numerical robustness. Both LPV controllers are tuned and simulated in Section 4.4. Section 4.5 provides conclusions about LPV control for torque vectoring.

# 4.1 General LPV Control Synthesis

The strength of LPV control is that it allows the usage of well-known LTI control techniques for non-linear systems. One of these LTI techniques is the mixed-sensitivity controller design. Frequency-dependent shaping filters are used to penalise the sensitivity, complementary sensitivity or control sensitivity. Constraints on the closed-loop pole region, the H<sub>2</sub> norm and the H<sub> $\infty$ </sub> norm can be introduced.

For closed-loop systems configured as in Figure 4.1, Chilali and Gahinet [113] showed how to find  $H_{\infty}$  optimal LTI controllers using LMIs. Since the 1990s, efficient numerical



Figure 4.1: LTI closed-loop system with shaping filters

solvers for LMIs are available. These solvers made it possible for control designers to describe closed-loop goals as convex LMI expressions. The tuning of the closed-loop behaviour is performed with frequency-dependent shaping filters and results in  $H_{\infty}$  or  $H_2$  optimal controllers.

If a non-linear system is expressed by differential equations, and the parameters of the differential equations are variable but measurable in real time, it is appropriate to use LPV control (instead of LTI control for constant parameters). A general state-space description [114], [115] of an LPV system  $G(\rho)$ 

$$G(\rho): \begin{cases} \dot{x} = A(\rho)x + B(\rho)w\\ z = C(\rho)x + D(\rho)w \end{cases}$$

$$(4.1)$$

shows the close relation to LTI state-space models. The signals x, x, w, z, and  $\rho$  depend on the time t which is not shown here for an simplified expression. The state vector is defined as  $x \in \mathbb{R}^n$ , the input vector as  $w \in \mathbb{R}^m$  and the output vector as  $z \in \mathbb{R}^l$ . For LPV systems, the state-space matrices  $A(\rho), B(\rho), C(\rho)$  and  $D(\rho)$  are continuous functions of  $\rho \in \mathbb{R}^p$  and describe the varying behaviour of the model. The scheduling parameters  $\rho$  are directly measurable. However,  $\rho$  is assumed to be restricted to the admissible scheduling parameter set

$$\mathcal{P} \subset \mathbb{R}^p : \rho \in \mathcal{P}, \forall t > 0, \tag{4.2}$$

which is assumed to be compact. The scheduling signals  $\rho$  are often considered as external signals, which means that the system is linear in input and states. Non-linear systems can be represented by allowing the model matrices of an LPV system to depend on state variables, inputs or outputs. In this case the model is referred to as quasi-LPV system. Quasi-LPV systems are more difficult to analyse because the bounds of the scheduling parameters cannot be limited a priori.

Closed-loop stability is established by Lyapunov stability [116], [114], [117]. Performance of the closed-loop system is expressed by the induced  $\mathcal{L}_2$ -norm [118]. In [118], the L<sub>2</sub>-gain is defined as follows:

**Definition 4.1** Let  $\gamma \geq 0$ . The system is said to have  $\mathcal{L}_2$  less than or equal to  $\gamma$  if

$$\int_{0}^{T} \|z(t)\|^{2} dt \leq \gamma^{2} \int_{0}^{T} \|w(t)\|^{2} dt$$
(4.3)

for all  $T \ge 0$  and all  $w \in L_2(0,T)$ , with ... initial state  $x(0) = x_0$ .

The induced  $\mathcal{L}_2$ -norm of a system expresses the maximum amplification from the input w(t) to the output z(t) in terms of the  $\mathcal{L}_2$  signal norm.

Typical fields for LPV controller applications are aerospace [119], [120], [121], [122], robotics [123], [124], [125] and automotive engineering [126], [127], [128]. In practise, there are three major approaches to LPV control of non-linear plants. These approaches are gridding-based linear parameter-varying (gridding-based LPV) control, LFT-LPV control and polytopic LPV control.

#### 4.1.1 Polytopic LPV control

The polytopic LPV design [114], [129] is an extreme form of gridding-based LPV control, where only gridding points at the vertices of a convex polytope  $\mathcal{P}$  that covers the admissible parameter range are checked. Checking only the vertices is possible if the model depends affinely on the parameters. For polytopic LPV control, a quadratic Lyapunov function is used, which guarantees stability across all vertex coordinates. In the synthesis process, a stable controller is searched for every vertex point of the convex hull and the resulting controllers are linearly interpolated to control the entire parameter range. An LPV model

$$\dot{x} = A(\theta)x + B(\theta)w$$
  

$$z = C(\theta)x + D(\theta)w,$$
(4.4)

where  $\theta(t) \in \mathcal{P}$ ,  $\forall t \ge 0$  for a given compact set  $\mathcal{P} \subset \mathbb{R}^{n_{\theta}}$  is polytopic if it satisfies the following two conditions.

1. The set  $\mathcal{P}$  is a polytope, i.e. it can be expressed as a convex hull

$$\mathcal{P} = \operatorname{Co}\left\{\theta_{v1}, \theta_{v2}, ..., \theta_{vr}\right\},\tag{4.5}$$

where the  $\theta_{vi} \in \mathbb{R}^{n_{\theta}}$  are the vertices of the polytope and r is the number of vertices. The representation (4.5) implies that

$$\mathcal{P} = \left\{ \theta \in \mathbb{R}^{n_{\theta}} | \theta = \sum_{i=1}^{r} \alpha_i \theta_{vi}, \quad \sum_{i=1}^{r} \alpha_i = 1, \quad \alpha_i \ge 0, \quad i = 1, ..., r \right\}.$$
 (4.6)

The coefficients  $\alpha_i$  are referred to as convex coordinates.

2. The model (4.4) depends affinely on the parameter vector  $\theta$ . In this case, the set of admissible LTI systems can be represented by

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} = \operatorname{Co} \left\{ \begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix}, i = 1, ..., r \right\} \quad \forall \theta \in \mathcal{P},$$
(4.7)

where

$$(A_i, B_i, C_i, D_i) = (A(\theta_{vi}), B(\theta_{vi}), C(\theta_{vi}), D(\theta_{vi})), \quad i = 1, ..., r$$
(4.8)

are the vertex plants. More generally, if

$$\theta(t) = \sum_{i=1}^{r} \alpha_i \theta_{vi}, \tag{4.9}$$

then

$$\begin{bmatrix} A(\theta) & B(\theta) \\ C(\theta) & D(\theta) \end{bmatrix} = \sum_{i=1}^{r} \alpha_i \begin{bmatrix} A_i & B_i \\ C_i & D_i \end{bmatrix},$$
(4.10)

where the  $\alpha_i$  are convex coordinates.

Minimum values  $\underline{\theta}_i$  and maximum values  $\overline{\theta}_i$  limit the scheduling parameters to define the polytope. Not every trajectory in  $\mathcal{P}$  may be physically possible, which introduces conservatism into the design [130]. A mathematical representation of the polytopic LPV



Figure 4.2: Closed-loop polytopic LPV system

system in Figure 4.2 is expressed by the closed-loop system  $T_{cl}$  including the generalised plant model

$$P(\theta) \begin{cases} \dot{x} = A(\theta)x + B_w(\theta)w + B_u(\theta)u \\ z = C_z(\theta)x + D_{zw}(\theta)w + D_{zu}(\theta)u \\ v = C_v(\theta)x + D_{vw}(\theta)w + D_{vu}(\theta)u \end{cases}$$
(4.11)

and the controller

$$K(\theta) \begin{cases} \dot{\xi} = A_K(\theta)\xi + B_K(\theta)v \\ u = C_K(\theta)\xi + D_K(\theta)v. \end{cases}$$
(4.12)

The signal w represents exogenous inputs (reference signals, disturbance signals, sensor noise, etc.) and u represents the controller input to the plant model. The output of the closed-loop system is the performance channel z. The input to the controller is v. Advantages of polytopic LPV control are guaranteed stability and performance properties of the closed-loop system inside the convex hull  $\mathcal{P}$ . A drawback of polytopic LPV is its conservatism, because stability is guaranteed for the whole parameter range, even if it is impossible to reach certain sectors. As well, the non-linear plant model must be converted into an affine or polytopic representation. Also, the number of vertex controllers grows exponentially with the number of scheduling parameters  $n_{\theta}$  as  $n_K = 2^{n_{\theta}}$ . Hence, polytopic LPV is limited to a small number of scheduling parameters to implement the controller. For example, if the system is affinely described using six scheduling parameters, this leads to  $2^6 = 64$  vertex controllers.

# 4.1.2 LFT control

LFT-LPV control [131], [132], [133] emerged from robust control theory and separates the parameter-varying plant into a dynamic, parameter-independent part P(s) and a static, parameter-dependent part  $\Theta$  [131]. The dependence of the scheduling parameters must be rational. Using the Full-block S-Procedure [132] reduces the conservatism of



Figure 4.3: Closed-loop system in LFT form

earlier methods. The introduction of parameter-dependent Lyapunov functions [133] can further reduce conservatism and improve the performance of the closed-loop system. An LFT-LPV representation of the plant model and the controller is given in Figure 4.3. The closed-loop system  $T_{cl}$  consists of the plant model and the controller. The parameterdependent block  $\Theta$  of the controller is modelled simultaneously with the parameterdependent part of the plant model where  $\Theta$  is a diagonal matrix

$$\Theta(t) = \begin{bmatrix} \Theta_1(t)I_{r_1} & \dots & 0\\ \vdots & \ddots & \vdots\\ 0 & \dots & \Theta_{n_{\Theta}}(t)I_{r_{n_{\Theta}}} \end{bmatrix}, \qquad (4.13)$$

where the diagonal elements depend on the time-varying parameters and can be measured online.

The exogenous input w and the control input u are the inputs for the plant. The performance channel z and the controller input v are output of the generalised plant. The signals p and q are input and output of the parameter-dependent block  $\Theta$ . For the controller, these signals are  $\bar{q}$  and  $\bar{p}$ ; here it is assumed that the scheduling blocks of the plant and the controller are the same. The generalised plant model P is defined as

$$P:\begin{cases} \dot{x} = Ax + B_w w + B_u u + B_q q\\ z = C_z x + D_{zw} w + D_{zu} u + D_{zq} q\\ v = C_v x + D_{vw} w + D_{vu} u + D_{vq} q\\ p = C_p x + D_{pw} w + D_{qu} u + D_{pq} q \end{cases}$$
(4.14)

with

$$q = \Theta p. \tag{4.15}$$

To form the closed-loop system  $T_{cl}$ , the LFT-LPV plant P is combined with a controller

K. The interconnection is shown in Figure 4.3 and the controller is defined as

$$K: \begin{cases} \xi = A_{K}\xi + B_{Kv}v + B_{Kq}\bar{q} \\ u = C_{Ku}\xi + D_{Kuv}v + D_{Kuq}\bar{q} \\ \bar{p} = C_{Kp}\xi + D_{Kpv}v + D_{Kpq}\bar{q} \end{cases}$$
(4.16)

with

$$\bar{q} = \Theta \,\bar{p}.\tag{4.17}$$

The states of the controller are defined as  $\xi$ . The controller inputs are v from the generalised plant and  $\bar{q}$  for the uncertainty. The outputs are the control input u and the uncertainty input  $\bar{p}$ . The linear fractional transformation approach [132], [133] applies  $H_{\infty}$  design methods for a wide range of parameter varying systems. The use of parameter-dependent Lyapunov functions reduces the conservatism of the design and is recommended by [133].

For the application considered here, the polytopic LPV controller design and the LFT-LPV controller design appear to be promising. In the next section, both synthesis procedures are reviewed and then the polytopic LPV and the LFT-LPV torque vectoring controllers are compared.

# 4.2 LPV Controller Synthesis

For the LPV controller synthesis, two different approaches are summarised. First, the design method of Wu and Dong [133] is shown for an LFT-LPV controller with the Full-block S-Procedure and a quadratic, parameter-dependent Lyapunov function. The conservatism of this design is reduced, as compared with previous designs, such as [131], [132]. Next, the design of a polytopic LPV controller [129], [114] is shown. Before calculating the controller, the generalised plant is normalised as described in [23]. The modification of the plant signals improves the numerical condition of the LMI solvers [111], and it is an important step for a numerically successful controller calculation.

#### 4.2.1 LFT - controller synthesis

To obtain the LFT controller (4.16, 4.17), Wu [133] proposes the following procedure:

1. The generalised plant (4.14, 4.15) must be converted into a generalised plant that absorbs the parameter-dependent components as

$$P(\Theta): \begin{cases} \dot{x} = \mathcal{A}(\Theta)x + \mathcal{B}_{w}(\Theta)w + \mathcal{B}_{u}(\Theta)u \\ z = \mathcal{C}_{z}(\Theta)x + \mathcal{D}_{zw}(\Theta)w + \mathcal{D}_{zu}(\Theta)u \\ v = \mathcal{C}_{v}(\Theta)x + \mathcal{D}_{vw}(\Theta)w + \mathcal{D}_{vu}(\Theta)u \end{cases}$$
(4.18)

using

$$\begin{bmatrix} \mathcal{A}(\Theta) & \mathcal{B}_{w}(\Theta) & \mathcal{B}_{u}(\Theta) \\ \mathcal{C}_{z}(\Theta) & \mathcal{D}_{zw}(\Theta) & \mathcal{D}_{zu}(\Theta) \\ \mathcal{C}_{v}(\Theta) & \mathcal{D}_{vw}(\Theta) & \mathcal{D}_{vu}(\Theta) \end{bmatrix} = \begin{bmatrix} A & B_{w} & B_{u} \\ C_{z} & D_{zw} & D_{zu} \\ C_{v} & D_{vw} & D_{vu} \end{bmatrix} + \begin{bmatrix} B_{q} \\ D_{zq} \\ D_{vq} \end{bmatrix} \Theta \left(I - D_{pq}\Theta\right)^{-1} \begin{bmatrix} C_{q} \\ D_{qw} \\ D_{qu} \end{bmatrix}^{T}.$$

$$(4.19)$$

The new system is used for controller synthesis. Additionally, the following assumptions are introduced by Wu:

- The triple  $(\mathcal{A}(\Theta), \mathcal{B}_u(\Theta), \mathcal{C}_v(\Theta))$  is parameter-dependent, stabilisable and detectable for all  $\Theta \in \mathcal{P}$ .
- The matrices  $[\mathcal{C}_v(\Theta), \mathcal{D}_{vw}(\Theta)]$  and  $[\mathcal{B}_u^T(\Theta), \mathcal{D}_{zu}^T(\Theta)]$  have full row rank for all  $\Theta$ .
- The matrices  $\mathcal{D}_{zw}(\Theta) = 0$  and  $\mathcal{D}_{vu}(\Theta) = 0$ .
- 2. For the synthesis, quadratic, parameter-dependent Lyapunov functions  $R(\Theta)$  and  $S(\Theta)$  need to be constructed. These matrices are defined as

$$R(\Theta) = T_R^T(\Theta) P T_R(\Theta)$$
(4.20)

$$S(\Theta) = T_S^T(\Theta) Q T_S(\Theta).$$
(4.21)

The parametrization factors  $T_R(\Theta)$ ,  $T_S(\Theta)$  are used for tuning. The positive definite matrices P and Q are determined by solving LMIs [133]. The controller performance and stability depend on the choice of  $T_R$  and  $T_S$ . These factors are not unique, and different structures can be used for  $T_R$  and  $T_S$ . Bartels et al. [108] obtained the best results by considering the structure of the vehicle model's Amatrix, given in (4.48). The LFT-LPVs are defined as

$$T_R = I_7 \tag{4.22}$$

$$T_{S}(\Theta) = \begin{bmatrix} \Theta_{1}I_{2} & 0\\ 0 & \Theta_{2}I_{2} \end{bmatrix} \begin{bmatrix} & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 1 & 0\\ 0 & 1 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0\\ 0 & I_{2} & 0 & 0\\ 0 & 0 & 0 & I_{7} \end{bmatrix},$$
(4.23)

$$P = \begin{bmatrix} 0 & & 0 & P_1 & 0 \\ 0 & & P_2 & 0 & 0 \\ \hline 0 & P_2^T & & & \\ P_1^T & 0 & & P_0 \\ 0 & 0 & & & \end{bmatrix},$$
(4.24)

with the matrices  $P_1, P_2 \in \mathbb{R}^{2 \times 2}$  and  $P_0, Q_0 \in \mathbb{R}^{7 \times 7}$ ; the matrix blocks  $\Theta_1$  and  $\Theta_2$  are defined in (4.46). The torque vectoring controller uses the configuration

$$R(\Theta) = R = Q_0; \tag{4.25}$$

$$S(\Theta) = P_0 + \begin{pmatrix} 0 & 0 & 0 \\ 0 & P_1 + P_1^T & 0 \\ 0 & 0 & 0 \end{pmatrix} \Theta_1 + \begin{pmatrix} P_2 + P_2^T & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \Theta_2.$$
(4.26)

- 3. The parameter-dependent matrices  $R(\Theta)$  and  $S(\Theta)$  are chosen separately. For practical applications, some limitations are recommended for  $T_S$  and  $T_R$  (4.22 -4.24). The derivative  $\dot{R}(\Theta)$  must be calculated for controller synthesis. The design process becomes numerically more robust by defining  $\dot{R} = 0$ , which results in a constant R matrix.
- 4. Wu proposed to define additional matrices  $M(\Theta)$ ,  $N(\Theta)$  with

$$M(\Theta)N^{T}(\Theta) = I - R(\Theta)S(\Theta)$$
(4.27)

to calculate the controller. The factorisation into  $M(\Theta)$  and  $N(\Theta)$  is not unique. For the implementation, it is advisable to choose a constant matrix M because the derivative of M is used in the controller calculation. A suitable choice is

$$M := I,$$
  

$$N^{T}(\Theta) := I - R S(\Theta).$$
(4.28)

5. Using the proposed simplifications, the final controller K is achieved in the form

$$K: \begin{cases} \dot{\xi} = \mathcal{A}_K(\Theta)\xi + \mathcal{B}_K(\Theta)v \\ u = \mathcal{C}_K(\Theta)\xi & . \end{cases}$$
(4.29)

The controller matrices are defined as

$$\mathcal{A}_{K}(\Theta) = -N^{-1}(\Theta) \left( \mathcal{A}^{T}(\Theta) + S(\Theta) \left[ \mathcal{A}(\Theta) + \mathcal{B}_{u}(\Theta)F(\Theta) + L(\Theta)\mathcal{C}_{v}(\Theta) \right] \right. \\ \left. + \frac{1}{\gamma}S(\Theta) \left[ \mathcal{B}_{w}(\Theta) + L(\Theta)\mathcal{D}_{vw}(\Theta) \right] \mathcal{B}_{w}^{T}(\Theta) \right.$$

$$\left. + \frac{1}{\gamma}\mathcal{C}_{z}^{T}(\Theta) \left[ \mathcal{C}_{z} + \mathcal{D}_{zu}F(\Theta) \right] R \right)$$

$$\left. + \frac{1}{\gamma}\mathcal{C}_{z}^{T}(\Theta) \left[ \mathcal{C}_{z} + \mathcal{D}_{zu}F(\Theta) \right] R \right)$$

$$\left. + \frac{1}{\gamma}\mathcal{C}_{z}^{T}(\Theta) \left[ \mathcal{C}_{z} + \mathcal{D}_{zu}F(\Theta) \right] R \right)$$

$$\left. + \frac{1}{\gamma}\mathcal{C}_{z}^{T}(\Theta) \left[ \mathcal{C}_{z} + \mathcal{D}_{zu}F(\Theta) \right] R \right)$$

$$\left. + \frac{1}{\gamma}\mathcal{C}_{z}^{T}(\Theta) \left[ \mathcal{C}_{z} + \mathcal{D}_{zu}F(\Theta) \right] R \right)$$

$$\left. + \frac{1}{\gamma}\mathcal{C}_{z}^{T}(\Theta) \left[ \mathcal{C}_{z} + \mathcal{D}_{zu}F(\Theta) \right] R \right)$$

$$\mathcal{B}_K(\Theta) = N^{-1}(\Theta)S(\Theta)L(\Theta) \tag{4.31}$$

$$\mathcal{C}_K(\Theta) = F(\Theta)R,\tag{4.32}$$

where the additional parameters  $F(\Theta)$  and  $L(\Theta)$  are defined as

$$F(\Theta) = -\left(\mathcal{D}_{zu}^{T}(\Theta)\mathcal{D}_{zu}(\Theta)\right)^{-1} \times \left(\gamma \mathcal{B}_{u}^{T}(\Theta)R^{-1} - \mathcal{D}_{zu}^{T}(\Theta)\mathcal{C}_{z}(\Theta)\right)$$
(4.33)

$$L(\Theta) = -\left(\gamma S^{-1}(\Theta)\mathcal{C}_v^T(\Theta) + \mathcal{B}_w(\Theta)\mathcal{D}_{vw}^T(\Theta)\right) \times \left(\mathcal{D}_{vw}(\Theta)\mathcal{D}_{vw}^T(\Theta)\right)^{-1}.$$
 (4.34)

#### 4.2.2 Polytopic synthesis procedure

The polytopic LPV controller  $K(\theta)$  in (4.12) uses the controller input v and the scheduling parameter  $\theta$  to generate the controller output u. If the LPV model is in affine form, the generalised plant can be converted into the polytopic representation. The limits of the scheduling parameters  $\theta$  are given by the convex polytope  $\mathcal{P}$ . The controller synthesis is based on the work of [114], [129] and uses a polytopic LPV representation for the controller design. The polytopic LPV design assumes that:

- 1. The matrix  $D_{vu,i} = 0$  for i = 1, ..., r.
- 2. The matrices  $B_{u,i}$ ,  $C_{v,i}$ ,  $D_{zu,i}$ ,  $D_{vw,i}$  are parameter-independent for i = 1, ..., r.
- 3. The pairs  $(A_i, B_{u,i})$  and  $(A_i, C_{v,i})$  are quadratically stabilisable and quadratically detectable, respectively, in the polytope  $\mathcal{P}$ .

The matrices of the generalised plant  $P(\theta)$  are defined in (4.11) and the index *i* indicates that the system is calculated for the vertex coordinate i = 1, ..., r. The polytopic LPV design guarantees stability and performance in the defined parameter range  $\theta \in \mathcal{P}$ . To calculate a polytopic LPV controller, the following result is proposed in [114] and can be used.

**Theorem 4.1** Let  $\mathcal{N}_R$  and  $\mathcal{N}_S$  denote bases for the the null space of  $[C_v, D_{vw}, 0]$  and  $[B_u, D_{zu}, 0]$ , respectively. There exists an LPV controller guaranteeing stability and  $L_2$ -gain performance  $\gamma$  along all parameter trajectories in the polytope mathcal P (4.5) if there exist a pair of symmetric matrices (R,S) in  $\mathbb{R}^{n \times n}$  satisfying the system of 2r + 1 LMIs:

$$\begin{bmatrix} \mathcal{N}_{R} & 0 \\ - & \mathbf{x} & \bar{I} \end{bmatrix}^{T} \begin{bmatrix} A_{i}R + RA_{i}^{T} & RC_{z,i}^{T} & B_{w,i} \\ - & - & \mathbf{x} & - & -\mathbf{\gamma}I & D_{zw,i} \\ - & - & \mathbf{x} & - & -\mathbf{\gamma}I & D_{zw,i} \\ - & - & \mathbf{x} & - & -\mathbf{\gamma}I \end{bmatrix}^{T} \begin{bmatrix} \mathcal{N}_{R} & 0 \\ - & \mathbf{x} & \bar{I} & \bar{I} \end{bmatrix}^{T} < 0, \quad i = 1, ..., r \quad (4.35)$$

$$\begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} \begin{bmatrix} A_{i}^{T}S + SA_{i} & SB_{w,i} & C_{z,i}^{T} \\ - & - & \mathbf{v} & -\gamma I \\ - & - & \mathbf{v} & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} \end{bmatrix}^{T} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} \end{bmatrix}^{T} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} \end{bmatrix}^{T} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} \end{bmatrix}^{T} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} \end{bmatrix}^{T} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v} \end{bmatrix}^{T} \end{bmatrix}^{T} \end{bmatrix}^{T} = \begin{bmatrix} \mathcal{N}_{S} & 0 \\ - & \mathbf{v}$$

$$\begin{bmatrix} R & I \\ I & S \end{bmatrix} \ge 0. \tag{4.37}$$

Moreover, there exists k-th order LPV controllers solving the same problem if and only if R, S further satisfy the rank constraint

$$Rank[I - SR] \le k. \tag{4.38}$$

The controller synthesis process is defined as:

1. A constant Lyapunov matrix  $\mathcal{X}$  is searched for the closed-loop system using (4.35-4.37). Additional matrices M, T are defined as

$$MN^T = I - RS. (4.39)$$

The closed-loop Lyapunov matrix  $\mathcal{X}$  is constructed as

$$\mathcal{X} = \begin{bmatrix} S & I \\ N^T & 0 \end{bmatrix} \begin{bmatrix} I & R \\ 0 & M^T \end{bmatrix}^{-1}.$$
 (4.40)

2. If the positive definite Lyapunov matrix  $\mathcal{X}$  is found, one can proceed as follows. The closed-loop matrices  $\mathcal{A}(\theta)$ ,  $\mathcal{B}(\theta)$ ,  $\mathcal{C}(\theta)$  and  $\mathcal{D}(\theta)$  can be calculated with the bounded real lemma (BRL). The closed-loop system T is defined as

$$T: \begin{cases} \dot{x}_{cl} = \mathcal{A}(\theta)x_{cl} + \mathcal{B}(\theta)w_{cl} \\ z_{cl} = \mathcal{C}(\theta)x_{cl} + \mathcal{D}(\theta)w_{cl} \end{cases}$$
(4.41)

including the closed-loop states  $x_{cl}$ , the closed-loop input  $w_{cl}$  and the closed-loop output  $z_{cl}$ . With the given Lyapunov matrix  $x_{cl}$ , one can obtain r LTI controllers for every vertex

$$\begin{bmatrix} \mathcal{A}_{i}^{T}\mathcal{X} + \mathcal{X}\mathcal{A}_{i} & \mathcal{X}\mathcal{B} & \mathcal{C}^{T} \\ \star & -\gamma I & \mathcal{D}^{T} \\ \star & \star & -\gamma I \end{bmatrix} < \gamma \quad i = 1, ..., r.$$
(4.42)

These controllers are linearly interpolated to form the polytopic LPV controller. The controller matrices  $A_{K,i}$ ,  $B_{K,i}$ ,  $C_{K,i}$  and  $D_{K,i}$  can be extracted from the closed-loop system using

$$\begin{aligned}
\mathcal{A}_{i} &= A_{0,i} + B_{1,i} \Omega_{i} C_{1,i} \\
\mathcal{B}_{i} &= B_{0,i} + B_{1,i} \Omega_{i} D_{2,i} \\
\mathcal{C}_{i} &= C_{0,i} + D_{1,i} \Omega_{i} C_{1,i} \\
\mathcal{D}_{i} &= D_{wz,i} + D_{1,i} \Omega_{i} D_{2,i},
\end{aligned} \tag{4.43}$$

where the controller is defined as

$$\Omega_i := \begin{bmatrix} A_{K,i} & B_{K,i} \\ C_{K,i} & D_{K,i} \end{bmatrix}$$
(4.44)

Additional matrices are defined as

$$A_{0,i} = \begin{bmatrix} A_i & 0\\ 0 & 0 \end{bmatrix}, \quad B_{1,i} = \begin{bmatrix} 0 & B_{u,i}\\ I & 0 \end{bmatrix}, \quad C_{1,i} = \begin{bmatrix} 0 & I\\ C_{v,i} & 0 \end{bmatrix}, \quad (4.45)$$
$$B_{0,i} = \begin{bmatrix} B_{w,i}\\ 0 \end{bmatrix}, \quad C_{0,i} = \begin{bmatrix} C_{z,i}\\ 0 \end{bmatrix}^T, \quad D_{1,i} = \begin{bmatrix} 0\\ D_{zu,i} \end{bmatrix}^T, \quad D_{2,i} = \begin{bmatrix} 0\\ D_{vw,i} \end{bmatrix}.$$

# 4.3 Vehicle Model in LPV Form

Before calculating an LPV controller, it is essential to generate an appropriate LPV plant model. The eFuture prototype [20] is equipped with two electric machines that drive the front wheels of the vehicle. With two actuators, it is possible to control a maximum of two states independently. Here, these two vehicle states are the longitudinal velocity  $v_x$  and the yaw rate r.

As described in Section 2.1.3, a non-linear STM is sufficient to describe the vehicle movement for most driving scenarios. The model is defined in (2.13 - 2.15). The STM model is valid for a positive longitudinal velocity  $v_x > 0$  and unsaturated tyre forces as described in Section 2. An LPV representation is not unique, and the model can be derived using mathematical tools [130], [134] for complex models. Here, the LPV model is based on the nonlinear single-track model from Section 2.1.3 and a manual choice of scheduling signals  $\theta$ . The advantage of the manual description is the physical representation of the states x and the scheduling signals  $\theta$ . The superscript <sup>v</sup> is introduced to the vehicle matrices to indicate the relation to the vehicle.

#### 4.3.1 LFT vehicle model

LFT-LPV controllers for longitudinal and lateral vehicle dynamics are intensively studied by Baslamish [104] and Poussot-Vassal [103]. These works show promising results. One (not unique) choice for the decomposition is

$$\begin{aligned}
x^{v} &= [v_{x}, v_{y}, r]^{T} \quad u^{v} &= [F_{x}, M_{z}]^{T} \quad v^{v} &= [v_{x}, r]^{T} \\
\Theta_{1} &= \frac{1}{v_{x}} \qquad \Theta_{2} &= r \qquad d^{v} &= \delta,
\end{aligned} \tag{4.46}$$

based on which the LFT-LPV vehicle model in [108] is constructed at

$$\begin{aligned} A_{1}^{v} &= \begin{bmatrix} 0 & 0 \\ a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad A_{2}^{v} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \qquad B_{d}^{v} = \begin{bmatrix} 0 \\ \frac{L_{y,F}}{L_{z}} \\ \frac{L_{y,F}l_{F}}{L_{z}} \end{bmatrix} \\ B_{u}^{v} &= \begin{bmatrix} \frac{1}{m} & 0 \\ 0 & 0 \\ 0 & \frac{1}{I_{z}} \end{bmatrix} \qquad C^{v} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad D_{d}^{v} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad D_{u}^{v} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \qquad (4.47) \\ a_{22} &= -\frac{C_{y,F}+C_{y,R}}{m} \qquad a_{23} = \frac{C_{y,R}l_{R}-C_{y,F}l_{F}}{m} \\ a_{32} &= \frac{C_{y,R}l_{R}-C_{y,F}l_{F}}{I_{z}} \qquad a_{33} = -\frac{C_{y,F}l_{F}^{2}+C_{y,R}l_{R}^{2}}{I_{z}}, \end{aligned}$$

with  $Rank(A_1) = Rank(A_2) = 2$ , leading to

$$A^{v}(\theta) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 & 0 \\ a_{22} & a_{23} & -1 & 0 \\ a_{32} & a_{33} & 0 & 0 \end{bmatrix}}_{B^{v}_{p}} \underbrace{\begin{bmatrix} \theta_{1} & 0 & 0 & 0 \\ 0 & \theta_{1} & 0 & 0 \\ 0 & 0 & \theta_{2} \end{bmatrix}}_{\Theta^{v}} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}}_{C^{v}_{q}}.$$
(4.48)

For a robust analysis, the norm of  $\Theta^v$  must be smaller than one, which is achieved with the scaling of  $B_p$  and  $C_q$  to guarantee  $||\Theta|| < 1$ . The LFT-LPV generalised plant is obtained by including the shaping filters as shown later in Section 4.4.2.

# 4.3.2 Polytopic vehicle model

An affine LPV representation of the single-track model is defined as an affine LPV model with the scheduling parameters

$$\begin{aligned}
x^{v} &= [v_{x}, v_{y}, r]^{T} \quad u^{v} &= [F_{x}, M_{z}]^{T} \quad v^{v} &= [v_{x}, r]^{T} \\
\theta_{1} &= \frac{1}{v_{x}} \qquad \theta_{2} &= r \qquad d^{v} &= \delta.
\end{aligned} \tag{4.49}$$

E.

The system is defined using the state-space matrices as

$$A^{v}(\theta) = \begin{bmatrix} 0 & \theta_{2} & 0 \\ -\theta_{2} & a_{22}\theta_{1} & a_{23}\theta_{1} \\ 0 & a_{32}\theta_{1} & a_{33}\theta_{1} \end{bmatrix} \quad B^{v}_{u} = \begin{bmatrix} \frac{1}{m} & 0 \\ 0 & 0 \\ 0 & \frac{1}{I_{z}} \end{bmatrix}$$

$$B^{v}_{d} = \begin{bmatrix} 0 \\ \frac{C_{y,F}}{I_{z}} \\ \frac{C_{y,F}l_{F}}{I_{z}} \end{bmatrix} \quad C^{v} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad D^{v}_{u} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \quad D^{v}_{d} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$a_{22} = -\frac{C_{y,F} + C_{y,R}}{m} \quad a_{23} = \frac{C_{y,R}l_{R} - C_{y,F}l_{F}}{m}$$

$$a_{32} = \frac{C_{y,R}l_{R} - C_{y,F}l_{F}}{I_{z}} \quad a_{33} = -\frac{C_{y,F}l_{F}^{2} + C_{y,R}l_{R}^{2}}{I_{z}}.$$

$$(4.50)$$

The vehicle model from (4.50) is in an affine form, with a parameter-dependent matrix

$$A^{v}(\theta) = \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{A_{0}} + \theta_{1} \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{bmatrix}}_{A_{1}^{v}} + \theta_{2} \underbrace{\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{A_{2}^{v}}.$$
 (4.51)

The matrices  $B_d^v$ ,  $B_u^v$ ,  $C^v$ ,  $D_d^v$  and  $D_u^v$  are parameter-independent as defined in (4.50). The matrix  $A^v(\theta)$  is converted into the polytopic representation of the form

$$A^{v}(\theta) = \alpha_{1} \underbrace{\begin{bmatrix} 0 & \underline{\theta}_{2} & 0 \\ -\underline{\theta}_{2} & a_{22}\underline{\theta}_{1} & a_{23}\underline{\theta}_{1} \\ 0 & a_{32}\underline{\theta}_{1} & a_{33}\underline{\theta}_{1} \end{bmatrix}}_{A^{v}} + \alpha_{2} \underbrace{\begin{bmatrix} 0 & \underline{\theta}_{2} & 0 \\ -\underline{\theta}_{2} & a_{22}\overline{\theta}_{1} & a_{23}\overline{\theta}_{1} \\ 0 & a_{32}\overline{\theta}_{1} & a_{33}\overline{\theta}_{1} \end{bmatrix}}_{\bar{A}^{v}_{3}} + \alpha_{4} \underbrace{\begin{bmatrix} 0 & \overline{\theta}_{2} & 0 \\ -\overline{\theta}_{2} & a_{22}\overline{\theta}_{1} & a_{23}\overline{\theta}_{1} \\ 0 & a_{32}\overline{\theta}_{1} & a_{33}\overline{\theta}_{1} \end{bmatrix}}_{\bar{A}^{v}_{4}},$$
(4.52)

where the scheduling signals  $\theta_i$  are defined in the range  $\left[\underline{\theta}_i, \overline{\theta}_i\right]$  and the scheduling variables  $\alpha$  are defined as

$$\theta: \left\{ \sum_{i=1}^{n} \alpha_i \theta_i : \alpha_i \ge 0; \sum_{i=1}^{n} \alpha_i = 1 \right\}.$$
(4.53)

**Remark 4.1** Using the lateral velocity  $v_y$  for the STM, instead of the sideslip angle

 $\beta$ , has the advantage that the matrix B is parameter-independent. Only the A matrix depends on  $\theta$ .

**Remark 4.2** The lateral velocity  $v_y$  or sideslip angle  $\beta$  are not considered as controlled outputs because it is not possible to control the yaw-rate and the sideslip angle independently, using only the yaw moment  $M_z$ . Trying to control both properties leads to a functionally uncontrollable [23] system with uncontrollable directions. Controlling the lateral velocity (or the sideslip angle) and the yaw rate is possible only by including an additional device like an active steering system.

# 4.4 Torque Vectoring Controller Design

A mixed-sensitivity loop shaping design is chosen for the torque vectoring controller design. Even though for time-varying systems transfer functions and poles or zeros are not defined, it is meaningful to shape the frequency response of sensitivity functions when the parameters are frozen. Dynamic shaping filters are designed in Section 4.4.1 and combined with the vehicle model from Section 4.3 to the generalised plant in Section 4.4.2. The synthesis procedure from Section 4.2 is applied to the generalised plant. Additional synthesis constraints are discussed in Section 4.4.3, and these constraints are included in the design. All steps are performed both for the LFT-LPV and the polytopic LPV design.

#### 4.4.1 Shaping filter

Shaping filters are used in mixed-sensitivity designs to realise frequency-dependent performance requirements. Different priorities between control error and control input are realised. As in [135], the sensitivity of the closed-loop system is shaped using a low-pass filter  $W^S$  to reduce the steady state error and to obtain good tracking performance for low frequencies. The control-sensitivity is shaped using the high-pass filter  $W^C$  to reduce the control effort, which reduces the energy consumption of the system and is used to deal with actuator limitations.

#### Shaping filters - LFT

For the torque vectoring controller in LFT-LPV form, the sensitivity filter  $W^S$  and the control-sensitivity filter  $W^C$  are defined as first order filters which for frozen values of

 $\theta_1$  can be represented as

$$W^{S}(s) = \begin{bmatrix} \frac{1}{M_{1}^{S}} \frac{\Theta_{1}}{\frac{s}{\omega_{1}^{S}} + 1} & 0\\ 0 & \frac{1}{M_{2}^{S}} \frac{1}{\frac{s}{-s} + 1} \end{bmatrix}$$
(4.54)

$$W^{C}(s) = \left(\frac{1}{M^{C}} \frac{\frac{s}{\omega^{C}} + 1}{\frac{s}{c \,\omega^{C}} + 1}\right) \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix}.$$
(4.55)

The scheduling parameter  $\Theta_1$  can be used for parameter-dependent shaping filters because the requirements for longitudinal and lateral response varies strongly with the velocity of the vehicle, as described in Section 3.5.

Often, shaping filters are parameter-independent as in [104], [103], but parameterdependent shaping filters reduce conservatism and improve the torque vectoring controller, as shown by Bartels et al. [108]. The shaping filters  $W^{S}(\theta)$  and  $W^{C}(\theta)$  can be represented as lower LFT-LPVs (definition in [133]) in the form

$$\begin{bmatrix} \dot{x}^{S} \\ \bar{z}^{S} \\ q^{S} \end{bmatrix} = \begin{bmatrix} A^{S} & B^{S} \\ \bar{C}^{S} & \bar{D}^{S} \\ \bar{C}^{S} & \bar{D}^{S} \\ C^{S} & D^{S} \\ Q^{S} \end{bmatrix} \begin{bmatrix} x^{S} \\ \bar{w}^{S} \\ p^{S} \end{bmatrix}$$
(4.56)

$$p^{S} = \Theta^{S} q^{S} \tag{4.57}$$

$$\begin{bmatrix} \dot{x}^{C} \\ z^{\overline{C}} \\ q^{C} \end{bmatrix} = \begin{bmatrix} A^{C} + B^{C}_{w} - B^{C}_{p} \\ \overline{C}^{\overline{C}} - \overline{D}^{\overline{C}}_{zw} - \overline{D}^{\overline{C}}_{zp} \\ C^{C}_{z} + D^{C}_{zw} - D^{C}_{zp} \\ C^{C}_{q} + D^{C}_{qw} - D^{C}_{qp} \end{bmatrix} \begin{bmatrix} x^{C} \\ \overline{w}^{\overline{C}} \\ p^{C} \end{bmatrix}$$
(4.58)

$$p^{C} = \Theta^{C} q^{C}.$$

$$(4.59)$$

#### Shaping filters - polytopic LPV

In most polytopic LPV controllers, such as [129], [136] shaping filters are independent of scheduling parameters  $\theta$ . The transfer functions of the filters are pure gains or first order systems. For torque vectoring, the requirements change with the longitudinal velocity of the vehicle, as described in 3.5. The shaping filters penalise the yaw rate error at high velocities and relax the lateral constraints for low velocities. The number of scheduling parameters r must be as small as possible to minimise the computational effort of the microcontroller. The filters are designed with a parameter-dependent A-matrix  $A(\theta)$  and constant B, C and D matrices.

As recommended in [23], dynamic components of filters are realised using first order, low-pass filters for sensitivity and first order, high-pass filters for control sensitivity. The
sensitivity shaping filter  $W^S$  is defined as

$$W^{S}(\theta) = C^{S}(sI - A^{S}(\theta))^{-1}B^{S} + D^{S}, \qquad (4.60)$$

where

$$A^{S}(\theta) = \begin{bmatrix} -\omega_{1}^{S} & 0\\ 0 & -\theta_{1}\omega_{2}^{S} \end{bmatrix}, \quad B^{S} = \begin{bmatrix} 1 & 0\\ 0 & 1 \end{bmatrix},$$
(4.61)  
$$C^{S} = \begin{bmatrix} \frac{\omega_{1}^{S}}{M_{1}^{S}} & 0\\ 0 & \frac{\omega_{2}^{S}}{M_{2}^{S}} \end{bmatrix}, \quad D^{S} = \begin{bmatrix} 0 & 0\\ 0 & 0 \end{bmatrix}.$$

The control sensitivity filter  $W^C$  is defined as

$$W^{C}(\theta) = C^{C}(sI - A^{C}(\theta))^{-1}B^{C} + D^{C}, \qquad (4.62)$$

where

$$A^{C}(\theta) = \begin{bmatrix} -c\,\omega_{1}^{C} & 0 \\ 0 & -c\,(0.995 + 0.001\theta_{1})\,\omega_{2}^{C} \end{bmatrix}, \quad B^{C} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$
(4.63)  
$$C^{C} = -\begin{bmatrix} \frac{c\,\omega_{1}^{C}\,(c-1)}{M_{1}^{C}} & 0 \\ 0 & \frac{c\,\omega_{2}^{C}\,(c-1)}{M_{2}^{C}} \end{bmatrix}, \quad D^{C} = \begin{bmatrix} \frac{c}{M_{1}^{C}} & 0 \\ 0 & \frac{c}{M_{2}^{C}} \end{bmatrix}.$$

With these structures, parameter-dependent shaping filters can be constructed that follow the assumptions of the polytopic LPV controller design of Apkarian et al. [129].

#### 4.4.2 Generalised plant

For LPV control synthesis, a generalised plant, as shown in Figure 4.1, must be defined. The generalised plant contains the vehicle model and shaping filters for closed-loop tuning. Inputs to the generalised plant are exogenous signal w, such as disturbances, reference and noise signals, and the control input u from the controller. Outputs of the generalised plant are the performance signal z, and the input of the controller v. The controller input v may consist of control error signals, measured signals or disturbance signals, which should be routed to the controller. Generalised plants are presented for LFT-LPV and polytopic LPV controllers.

The mixed-sensitivity design penalises the control error e and the control input u with the parameter-dependent shaping filters  $W^{S}(\theta)$  and  $W^{C}(\theta)$ . The filters are used to shape closed-loop performance and to find an appropriate trade-off between tracking performance and control effort. Here, a disturbance term d is introduced to handle the change of the front tyre steering angle.



Figure 4.4: Generalised plant in LFT form

## Generalised plant - LFT

A graphical representation of the LFT-LPV closed-loop system is given in Figure 4.4. The system is mathematically defined as lower LFT-LPV

$$\begin{bmatrix} \dot{x} \\ - \ddot{z} \\ - \ddot{y} \\ - \ddot{p} \end{bmatrix} = \begin{bmatrix} A & B_{w} & B_{u} & B_{q} \\ - \ddot{C}_{z} & D_{zw} & D_{uz} & D_{zq} \\ - \ddot{C}_{v} & D_{yw} & D_{vu} & D_{vq} \\ - \ddot{C}_{p} & D_{pw} & D_{pu} & D_{pq} \end{bmatrix} \begin{bmatrix} x \\ - \ddot{w} \\ - \ddot{u} \\ - \ddot{v} \\ - \ddot{v$$

The representation of the generalised plant is given as

$$\begin{bmatrix} \dot{x} \\ \dot{x}^{S} \\ \dot{x}^{C} \\ -\frac{z^{S}}{z^{S}} \\ -\frac{z^{C}}{z^{S}} \\ -\frac{z^{C}}{e^{v}} \\ -\frac{d^{v}}{p^{v}} \\ -\frac{d^{v}}{p^{v}} \\ p^{S} \\ p^{C} \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} A^{v} & 0 & 0 & | & 0 & B^{v}_{d} & | & B^{v}_{u} & | & B^{v}_{u} & | & B^{v}_{d} & 0 & 0 \\ -B^{S}C^{v} & A^{S} & 0 & | & B^{S} & -B^{S}D^{v}_{d} & | & -B^{S}D^{v}_{v} & B^{S}_{d} & 0 \\ -D^{S}C^{v} & -C^{S} & 0 & | & D^{S} & -D^{S}D^{v}_{d} & | & -D^{S}D^{v}_{v} & -D^{S}D^{v}_{v} & D^{S}_{d} & 0 \\ -D^{S}C^{v} & -C^{S} & 0 & | & D^{S} & -D^{S}D^{v}_{d} & | & -D^{S}D^{v}_{v} & -D^{S}D^{v}_{v} & D^{S}_{d} & 0 \\ -D^{S}C^{v} & -C^{S} & 0 & -C^{C} & | & 0 & 0 & | & D^{C} & | & 0 & 0 & D^{C}_{zq} \\ 0 & -C^{C} & -C^{v} & -0 & 0 & -C^{C} & -D^{v}_{d} & | & -D^{v}_{d} & | & -D^{v}_{vq} & -D^{S}D^{v}_{q} & -D^{S}D^{v}_{q} & -D^{S}D^{v}_{q} & -D^{S}D^{v}_{q} & 0 \\ 0 & -C^{C}_{p} & -0 & 0 & | & 0 & -D^{v}_{pd} & | & -D^{v}_{pu} & -D^{v}_{pq} & 0 & 0 \\ -D^{S}_{pu}C^{v} & C^{S}_{p} & 0 & | & D^{S}_{pu} & -D^{S}_{pu}D^{v}_{u} & | & -D^{S}_{pu}D^{v}_{v} & D^{S}_{pq} & 0 \\ 0 & 0 & C^{C}_{p} & | & 0 & 0 & | & D^{v}_{pu} & | & 0 & 0 & D^{C}_{pq} \end{bmatrix} \right] \begin{bmatrix} x^{v} \\ x^{S} \\ x^{C} \\ -\frac{x^{v}}{r} \\ -\frac{x^{v}}$$

including the shaping filters in LFT form. The states x of the generalised plant are defined as  $x = [x^v, x^S, x^C]^T$ . The input, as  $u = [r^v, d^v, u^v, p^v, p^S, p^C]^T$ , the output as  $y = [z^S, z^C, e^v, d^v, q^v, q^S, q^C]^T$ . The exogenous input w is separated into the reference signals  $r^v$  and the disturbance signal  $d^v$ . The shaping filters are included in the uncertainty block  $\Theta$  as

$$\begin{bmatrix} q^{v} \\ q^{S} \\ q^{C} \end{bmatrix} = \underbrace{\begin{bmatrix} \Theta^{v} & 0 & 0 \\ 0 & \Theta^{S} & 0 \\ 0 & 0 & \Theta^{C} \end{bmatrix}}_{\Theta} \begin{pmatrix} p^{v} \\ p^{S} \\ p^{C} \end{pmatrix}$$
(4.67)

#### Generalised plant - polytopic LPV

A generalised plant  $P(\theta)$  [23] is necessary for the mixed-sensitivity loop-shaping design [135] of the polytopic LPV controller. The basic structure of the generalised plant is shown in Figure 4.5. The LPV vehicle model  $G(\theta)$  is combined with shaping filters  $W^{S}(\theta)$ 



Figure 4.5: Generalised plant in affine LPV form

and  $W^{C}(\theta)$  to the generalised plant  $P(\theta)$ . The filter  $W^{S}(\theta)$  modifies the sensitivity  $S(\theta)$ of the closed-loop by penalising the control error in defined frequency regions. Filter  $W^{C}(\theta)$  penalises the control sensitivity  $K(\theta)S(\theta)$  and is implemented to tune the control effort of the controller. The measured disturbance d and the control error e are routed as v to the controller, to improve the behaviour of the closed-loop system. The generalised plant  $P(\theta)$  from (4.11) is defined as

$$\begin{bmatrix} \dot{x}^{v} \\ \dot{x}^{S} \\ \dot{x}^{C} \\ \ddot{z}^{S} \\ z^{C} \\ \vec{z}^{S} \\ z^{C} \\ \vec{v}^{v} \\ d^{v} \end{bmatrix} = \begin{bmatrix} A^{v}(\theta) & 0 & 0 & 0 & B^{v} \\ -B^{S}C^{c} & A^{S}(\theta) & 0 & B^{S} & -B^{S}D^{v}_{d} & -B^{S}D^{v}_{u} \\ 0 & 0 & A^{C}(\theta) & 0 & 0 & B^{C} \\ -\overline{D^{S}}\overline{C^{v}} & -\overline{C^{S}} & 0 & 0 & B^{C} \\ 0 & 0 & C^{S} & -\overline{D^{S}}\overline{D^{v}} & -\overline{D^{S}}\overline{D^{v}} \\ 0 & 0 & 0 & 0 & D^{C} & D^{C} \\ -\overline{C^{C}} & 0 & 0 & 0 & D^{C} \\ 0 & 0 & 0 & 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} x^{v} \\ x^{S} \\ x^{C} \\ \overline{r^{v}} \\ d^{v} \\ \overline{u^{v}} \end{bmatrix}.$$
(4.68)

Shaping filters are often parameter-independent, as in [105], [120], [129]. As discussed in the LFT-LPV controller design, it can be advantageous to use parameter-dependent shaping filters.

### 4.4.3 Additional constraints

The LFT-LPV and the polytopic LPV controller synthesis procedures, described in 4.2, generate continuous-time controllers for the LPV torque vectoring controller which guarantee stability and performance. To implement the control algorithm, some additional constraints should be included in the controller design.

#### Spectral radius constraint

Controller poles (for frozen parameters) may turn out to be located far in the left half plane (LHP). Far LHP poles are not problematic in continuous-time simulations, but the continuous-time controller design must be converted into a discrete-time design for practical implementation. In fact, it is not possible to represent excessively fast controller poles correctly with a defined sample time  $T_d$  and a quantization step h. The mapping of pole locations from continuous-time to discrete-time is given by

$$z = e^{sT_d} \tag{4.69}$$

where s is the location in the continuous complex plane. The sampling time is defined as  $T_d$ , and the location in the discrete complex plane is defined as z. The microcontroller used limits the system to 32 bits and a sample time of 0.01s. Including a safety factor sF of 10, the maximum pole location in continuous-time is given as

$$s = \ln\left(sF \cdot z\right) \frac{1}{T_d} \tag{4.70}$$

$$s = \ln \left(10 \cdot 2^{-31}\right) \frac{1}{0.01} = -1918.5. \tag{4.71}$$

The distance of the controller poles from the origin must not be greater than  $R_{sp} =$  1918.5. The spectral radius of the controller can be limited by

$$R_{sp}(\theta) = \begin{bmatrix} \alpha I & A^{C^{T}}(\theta) \\ A^{C}(\theta) & I \end{bmatrix} > 0$$
(4.72)

 $\alpha < R_{sp,lim}.\tag{4.73}$ 

This be used as an additional LMI for the controller synthesis process from Section 4.2. A detailed explanation is given in [134]. However, the spectral radius chosen should not be too small because, at least for this example, this would make the controller unstable, which would undermine a safe vehicle operation.

#### Strictly proper controller

The LFT-LPV controller design from Section 4.2.1 generates a controller without direct feed-through term  $D_K$ . For the polytopic LPV design from Section 4.2.2, this is not required. However, the tuning process shows that the numerical solvers [111], [112] more often find feasible and stable solutions if the controller feed-through term  $D_K$  is manually set to **0** for the polytopic LPV controller. This step limits the dynamics of the controller but helps to cope with sensor noise in the control signal v. Even if the solver finds a solution with a non-zero  $D_K$ , the matrix is here very close to **0**.

#### Operation range of the LPV control

To reduce conservatism, it is beneficial to use a trapezoidal shaped operation range  $\mathcal{P}$ , as shown in Figure 4.6. The trapezoidal polytope is convex but less conservative than a



Figure 4.6: Scheduling parameters and vertex coordinates

rectangular design. The controllers lead to an improved closed-loop performance, as they need not guarantee stability for unreachable trajectories in space. The upper limit of  $\theta_1$  is given by the vehicle model because the STM, which is the basis of the LPV controller, is not valid for standstill or reverse driving. Additionally, the effect of a yaw moment  $M_z$  is weak for low velocities but increases for higher velocities, as shown in Figure 3.6b.

The lower limit for  $\theta_1$  is given by the propulsion system, which limits the longitudinal velocity  $v_x$  of the vehicle to a maximum of 110 kph. Finally, the longitudinal operation range of torque vectoring from 12 kph to 130 kph is defined to include uncertainties like tailwinds and so on. The range of the second scheduling parameter  $\theta_2 = r$  is determined by simulations and driving tests. So far, the vehicle does not exceed the boundaries of  $\theta_2$ , which are shown in Figure 4.6.

For the polytopic LPV controller design, it is theoretically sufficient to use four vertices at the corners of the convex polytope. However, simulations show that the performance of the closed-loop system is improved if two additional operating points are used, as shown in Figure 4.6. These two points are placed at the minimal and maximal value of  $\theta_1$ , and  $\theta_2$  is set to 0 rad/s.

#### 4.4.4 Tuning and simulation

The tuning of the LFT-LPV and the polytopic LPV controller designs use slightly different shaping filters because the controller designs are different, and the best solution should be found for each controller. For the LFT-LPV controller, the tuning parameters are given in [108]. The tuning parameters for the polytopic LPV controller are documented in [106].

#### Tuning and simulation - LFT control

filter	parameter	value	parameter	value
sensitivity $S(\theta)$	$M_1^S$	$0.7 \cdot 10^{-3}$	$M_2^S$	0.9
	$\omega_1^S$	$1.4 \cdot 10^{-3}$	$\omega_2^S$	60
control sensitivity $K(\theta)S(\theta)$	$M_1^C$	10	$M_2^C$	10
	$\omega_1^C$	$1 \cdot 10^3$	$\omega_2^C$	$1 \cdot 10^3$

The tuning parameters of the shaping filters are shown in Table 4.1. To achieve proper

Table 4.1: LFT control - shaping filters

high-pass filters  $W^{C}(\theta)$ , the static constant c is set to c = 1000.

**Frequency response** - **LFT control** In the frequency domain, the behaviour of the closed-loop system is described using the output sensitivity  $S(\Theta)$  and the output control sensitivity  $K(\Theta) S(\Theta)$ . The controller must keep the induced  $\mathcal{L}_2$  norm of the closed-loop system  $T_{cl}(\Theta)$  below the performance index  $\gamma$ , i.e.

$$\|T_{cl}(\Theta)\|_{\mathcal{L}_2} = \left\| \begin{array}{c} W^S(\Theta)S(\Theta) \\ W^C(\Theta)K(\Theta)S(\Theta) \end{array} \right\|_{\mathcal{L}_2} < \gamma.$$

$$(4.74)$$

The sensitivity shaping filter  $W^{S}(\Theta)$  and control sensitivity shaping filter  $W^{C}(\Theta)$  are used to penalise undesirable closed-loop system attributes. For visualisation, the scheduling signals are fixed to  $\Theta_0$ . Here, the scheduling parameters are frozen as  $\Theta_{1,0} = \frac{1}{60} \frac{1}{\sqrt{p}}$  and  $\Theta_{2,0} = 0$  rad/s. Figure 4.7 shows sensitivity and control sensitivity plots in the



Figure 4.7: Sensitivity and control sensitivity of the LFT controlled system

frequency domain. The filters are normalised with  $1/\gamma$ , so sensitivity and closed-loop sensitivity are below the shaping filters. From optimal control of LTI systems [23], it is known that the system response should "touch" the inverse shaping filters response at some point. This is also true for LPV systems, but in LPV control the system and the filters touch at one specific point in the convex polytope  $\mathcal{P}$ . Everywhere else the responses  $S(\theta)$  and  $K(\theta)S(\theta)$  are below the inverse filters, which is the case here.

**Time response - LFT control** For the time domain, the closed-loop system's behaviour is analysed using the step responses. At the beginning of the simulation, the vehicle moves straight with an initial velocity of 60 kph. After 1 s, a step is applied to the yaw request. After 6 s, the velocity request is changed. After 12 s, a step request is applied to both signals to analyse the lateral, longitudinal and combined vehicle behaviour.

The yaw request is generated from the steering angle, given by the driver. This process is explained in detail in Section 6.2.3. For now, it is sufficient to state, that the steering wheel is turned with a step input to 45°, which results in a yaw rate request of 0.18 rad/s. The steering step input is shown in Figure 4.8. After 1 s, the steer step from 0° to 45° is applied to the steering wheel angle. After 3.5 s, the steering wheel is turned back to 0°. The modified steering angle results in a desired yaw rate  $r_{des}$  change. The comparison between the desired and real yaw rate is given in Figure 4.9. After 1 s, the desired yaw rate rises to 0.18 rad/s. After 3.5 s, the desired yaw rate lowers to 0 rad/s. The yaw rate tracking of the LFT-LPV torque vectoring controller is sufficient and follows the desired yaw rate. The overshoots at 1.2 s and 3.7 s of the desired yaw rate are related to generation of the desired yaw rate. Nonetheless, the torque vectoring controller follows these requests.



Figure 4.8: LFT controller, steering input



Figure 4.9: LFT controller, yaw rate

The desired velocity and the actual vehicle velocity are displayed in Figure 4.10. The initial vehicle velocity is 60 kph. During the step steer manoeuvre, vehicle velocity is lowered to 59.8 kph but the torque vectoring controller reaches 60 kph after the steer manoeuvre. At 6 s, the desired velocity is lowered by 1 kph. The torque vectoring controller tracks this velocity request without any steady state error.

After 12 s, a combined longitudinal and lateral step is applied. The steering wheel angle is changed to  $45^{\circ}$  as shown in Figure 4.8. The steering angle change results in a yaw rate request of 0.18 rad/s, which is tracked by the torque vectoring controller accurately. At the same time, the velocity request is changed to 60 kph. The torque vectoring controller tracks the yaw rate accurately and fast. The longitudinal velocity tracking is flawed by a steady state error.

#### Tuning and simulation - polytopic LPV control

The polytopic LPV controller  $K(\theta)$  is tuned using the parameter-dependent shaping filters  $W^{S}(\theta)$  and  $W^{C}(\theta)$  from [106]. The filter parameters are defined in Table 4.2. To



Figure 4.10: LFT controller, longitudinal velocity

filter	parameter	value	parameter	value
sensitivity $S(\theta)$	$M_1^S$	0.01	$M_2^S$	2
	$\omega_1^S$	0.02	$\omega_2^S$	10
control sensitivity $K(\theta)S(\theta)$	$M_1^C$	10	$M_2^C$	20
	$\omega_1^C$	7	$\omega_2^C$	9

Table 4.2: Polytopic LPV control - shaping filters

achieve proper high-pass filters  $W^{C}(\theta)$ , the static constant c is set to c = 1000.

**Frequency response - polytopic LPV control** As for the LFT-LPV controller design, the singular value plot in Figure 4.11 is used to compare the shaping filter constraints and the closed-loop frequency response for frozen parameters. To visualise the system behaviour, the scheduling parameters are fixed to  $\theta_0$ , and the induced  $\mathcal{L}_2$  norm of the inverse filters and the closed-loop system are plotted.  $\theta_0$  is defined as  $\theta_{1,0} = \frac{1}{60} \, \frac{1}{\text{kph}}$  and  $\theta_{2,0} = 0 \, \frac{\text{rad}}{\text{s}}$ .

**Time response - polytopic LPV control** The closed-loop system behaviour is simulated with the same step responses as for the LFT-LPV design in Section 4.1.2. At the beginning, the vehicle moves straight with an initial velocity of 60 kph. After 1 s, a steer step from 0° to  $45^{\circ}$  is applied to the steering wheel angle which is displayed in Figure 4.8. After 3.5 s, the steering wheel is turned back to 0°. The modified steering angle results in a desired yaw rate  $r_{des}$  change. The comparison between the desired, the LFT-LPV controlled, and the LPV controlled yaw rate is shown in Figure 4.12. After 1 s, the desired yaw rate rises to 0.18 rad/s. After 3.5 s, the desired yaw rate lowers to  $0^{\text{ rad/s}}$ . The yaw rate tracking of the LFT-LPV and the polytopic LPV torque vectoring controller is good and follows the desired yaw rate. The overshoots at 1.2 s and 3.7 s of the desired yaw rate as mall difference between LFT-LPV and LPV control is visible. The LFT-LPV controlled vehicle does not reach the desired values. The polytopic LPV



Figure 4.11: Polytopic LPV control - sensitivity and control sensitivity



Figure 4.12: Yaw rate response of the LFT-LPV and polytopic LPV controlled vehicle

controlled vehicle has a stronger yaw movement as required. However, these differences are minor, and both controllers are acceptable for the movement of the vehicle.

The velocity and desired velocity of the vehicle are displayed in Figure 4.13. The initial vehicle velocity is 60 kph. During the step steer manoeuvre, the velocity of the LFT-LPV controlled vehicle is lowered to 59.79 kph. The velocity of the polytopic LPV controlled vehicle is lowered to 59.94 kph. Both controllers reach the desired longitudinal velocity after the step steer manoeuvre. At 6 s, a step request is used, lowering the desired velocity to 59 kph. Both controllers reach the new desired velocity. The LPV controller reaches the request at 7.2 s and the LFT-LPV controller at 8.6 s.

After 12 s, steps for longitudinal and lateral vehicle movement are applied. The steering wheel angle is changed to  $45^{\circ}$ , which is shown in Figure 4.8. The steering step results in a yaw rate request of 0.18 rad/s, which is shown in Figure 4.12. At the same time, the



Figure 4.13: Longitudinal velocity of the LFT-LPV and polytopic LPV controlled vehicle

velocity request is changed to 60 kph. Both controllers reach the desired yaw rate, but the longitudinal velocities are different. The polytopic LPV controller reaches a velocity of 59.94 kph and the LFT-LPV controller reaches 59.79 kph.

Figure 4.14 shows the motor torque requests and explains the differences between the two controllers. The torque requests of the polytopic LPV controller vary from -170 to 245 Nm, whereas the LFT-LPV requests are more moderate, with a range of -90 to 100 Nm. The polytopic LPV controller is not in general better for longitudinal request tracking; the LFT-LPV controller is just "more smoothly" tuned. The different tunings are related to the diverse designs with different implementations of the parameter-dependent shaping filters.



Figure 4.14: Longitudinal velocity of the LFT-LPV and polytopic LPV controlled vehicle

# 4.5 LPV Control: Conclusion

For torque vectoring controller design, different LPV controller design methods are reviewed. The synthesis methods of LFT-LPV and polytopic LPV are explained and both controllers are constructed for torque vectoring. The single-track vehicle model from Section 2.1.3 is converted into LPV form and constitutes the basis for both controller designs. The LFT-LPV design is used by [104], [103] and [108] and shows promising results. The closed-loop system is stable, and the tracking of the longitudinal velocity and the yaw rate are acceptable. The trade-off between tracking error e and control effort u depends on the tuning of the parameter-dependent shaping filters  $W^S(\Theta)$  and  $W^C(\Theta)$ , and is a compromise between tracking performance and energy consumption.

The drawback of the LFT-LPV controller design of Wu [133], using a parameterdependent Lyapunov matrix, is the necessity of an online matrix inversion. This inversion must be performed in real-time and with a fixed-point arithmetic. The matrix inversion is crucial because this operation is numerically expensive and may lead to tremendous quantification errors. Authors, such as Bausch [137] discuss this problem and propose solutions. Nonetheless, the matrix operation can still cause problems for the microcontroller. Thus a controller without matrix inversion seems to be preferable. It is possible to use a constant Lyapunov matrix for the LFT-LPV controller design, and for this controller no online matrix inversion is required. However, Baker [138] reported conservatism resulting from a design for LFT-LPV controllers with a constant Lyapunov matrix.

On the other hand, the polytopic LPV design is suitable because the vehicle model can be described with an affine representation using two scheduling parameters  $\theta$ . With parameter-dependent shaping filters included, simulations showed no major drawback to polytopic LPV controller design, compared to the LFT-LPV controller design. Small differences exist, but these discrepancies are related to different tuning parameters of the controllers. Also, it seems that LFT-LPV controllers need more decision variables for this problem, and the numerical solver [111] has more problems in finding an appropriate LFT-LPV controller. In the rest of this work, the torque vectoring controller is based on the polytopic LPV controller design. Here, only a cursory comparison is given. Recently, Hoffmann et al. [139], [140], [141] explored LFT-LPV based LPV control with models that are affine in the scheduling parameters, in which case the on-line matrix inversion is avoided; this may lead to a much more efficient LFT-LPV based design.

# 5 Torque and Slip Limiter

The LPV controller from Chapter 4 achieves acceptable simulation results for various driving conditions. However, the vehicle performance is inadequate if the vehicle has to accelerate or brake strongly or if extreme steering manoeuvres are executed. If such manoeuvres are performed, the vehicle oscillates, or even worse, becomes unstable. The cause of this problem is the physical limitation of the drivetrain.

As mentioned in Section 2.2.2, the maximum torque, the maximum torque slew rate, and the maximum power of the electric machines are limited. If the controller requests more motor torque than is physically possible, the actuators reach the limits of saturation and the controller starts to "windup". Controller windup degrades the performance of the controller and can make the closed-loop system unstable. Windup effects have been discussed since the 1930s and controller windup is probably the root cause of many catastrophes, such as the nuclear accident in Chernobyl in 1986 [142] or the crash of jet fighters [143].

In the eFuture prototype, the motor torque and wheel slip limiter (TSL) is implemented to improve the performance and safety of the electric vehicle and to prevent the torque vectoring controller from experiencing windup. Section 5.1 gives an overview of standard strategies for coping with actuator limitations. In Section 5.2, anti-windup strategies are applied to the torque vectoring controller. In Section 5.3, the anti-windup scheme is extended to limit tyre slip and enhance the anti-windup compensator for the TSL algorithm. Simulation results are discussed in Section 5.4, and conclusions are given in Section 5.5.

# 5.1 Anti-Windup Compensator - Overview

The problem of windup for proportional-integral (PI) controllers was practically recognised in the 1930s, and has been theoretically discussed since the 1950s. One solution to the windup problem is to reconfigure the controller, to make the controller less aggressive. The control input u does not reach physical saturation and the closed-loop system behaves as designed. This solution is acceptable for torque vectoring if the vehicle drives from one place to another with constant, low performance requirements. However, personal automobiles are driven in various conditions, and sometimes the operator wants to drive with high power consumption, which is impossible with a smooth controller.

## 5.1.1 Anti-windup compensator - the classical scheme

The classical solution to the PI controller windup problem is the so-called anti-windup compensator (AWC) [144]. The AWC uses the control input difference  $\Delta u$  between the

desired control input u and the measured control input  $u_m$ . The difference  $\Delta u$  forms the input to the AWC and is used to suppress the windup of the integral controller part. The general assembly is given in Figure 5.1, showing the signal interconnection between the controller, the plant and the actuator limit. The actuator saturation is modelled as



Figure 5.1: Anti-windup compensator PID

$$u_m = \begin{cases} \overline{u} & \text{for } u > \overline{u} \\ u & \text{for } \underline{u} \le u \le \overline{u} \\ \underline{u} & \text{for } u < \underline{u}, \end{cases}$$
(5.1)

where the maximum control input  $\overline{u}$  and the minimum control input  $\underline{u}$  are constant. The AWC achieves the following properties:

- If the desired u and the applied control input  $u_m$  are equal, the AWC has no effect on the control-loop system. This is an important property for maintaining all controller specifications, such as performance and robustness, in the unsaturated case.
- If the actuator is saturated, the real control input is different from the desired control input. The AWC then limits the control error of the integral controller part, and the integrator does not windup. This operation suppresses overshoots and oscillations. The system is stabilised in case of a saturated control input.

#### 5.1.2 Anti-windup compensator - modern control theory

In modern control theory, such as optimal  $H_{\infty}$ ,  $H_2$  or LPV control, every controller is calculated on the basis of a generalised plant. The controller synthesis guarantees stability and performance properties that are configured using shaping filters. When the actuator is saturated, none of these properties are guaranteed, because the plant model assumptions are no longer correct. Several methods [142], [145], [146], [147] have been developed to regain stability and performance.

In general, modern anti-windup compensators are divided into two classes [142]. The first class is based on a one-step procedure, where the input saturation is part of the controller, and the AWC is included in the controller synthesis. The controller and the anti-windup compensator are designed simultaneously. The second class is based on a two-step approach, where in the first step the controller is calculated on the basis of unlimited actuators, and in the second step, the AWC is calculated on the basis of the plant, the designed controller, and the saturation limit. In the two-step approach the controller itself is not modified. The AWC is an additional system that modifies the input, the output, and sometimes the states of the controller. Modern anti-windup de-



Figure 5.2: Anti-windup compensator for LTI MIMO-systems, see [148]

signs are mostly based on full order AWCs, where the order of the AWC is equal to the sum of the plant and controller orders. With the limited resources of an automotive microcontroller, it is advantageous to use zero- or first-order AWCs to keep the computational effort as low as possible. During the eFuture project, several anti-windup concepts, such as [79], [106], [108], have been tested. Modified versions of the approach in [148] obtained the most promising results in simulations.

Weston and Postlethwaite [149] showed that the closed-loop system in Figure 5.2 can be transformed into a system with nominal linear dynamics and a non-linear controlloop. Turner and Postlethwaite [148] designed AWCs for LTI systems on the basis of this transformed system, as shown in Figure 5.3. The detailed definition of the matrix



Figure 5.3: Dead-zone based representation of the actuator saturation, see [148]

M and the plant feedback matrix  $G_{fb}$  is given in [148], and the saturation is converted

to a dead-zone function using

$$d_{\mathbf{z}}(u) = u - u_m,\tag{5.2}$$

see Figure 5.2 and Figure 5.3. The transformed representation emphasises the fact that the system operates as a nominal system without saturation. When the actuator limits are reached, the non-linear loop is activated. Turner and Postlethwaite [148] minimise the  $\mathcal{L}_2$  gain of the non-linear system to improve stability and performance in case of saturation. [148] considers: "purely static anti-windup compensators, which are, from a practical point of view, most desirable. Then these ideas are extended to the suboptimal 'low-order' compensators, which are often feasible for problems for which static compensators are not." Turner and Postlethwaite [148] propose the anti-windup problem as solved if:

- 1. when no saturation occurs, the non-linear control-loop has no effect,
- 2. when saturation occurs, the non-linear system is  $\mathcal{L}_p$  gain bounded for  $p \in [1, \infty)$ .

Furthermore, Turner and Postlethwaite [148] consider the anti-windup in Figure 5.3 as "strongly" solved by the anti-windup compensator if the operator  $\tau : u_{\text{lin}} \mapsto \zeta_y$  is well-defined.

The design [148] deals with the feedback parts of the plant G and the controller K. Also, it assumes that the plant G is asymptotically stable, the controller K is stabilisable and detectable, and the saturation input  $u_m$  is given. The closed-loop feedback system, containing G and K, must be stable and mathematically well-posed.

Turner and Postlethwaite [148] present LMI conditions for the synthesis of the antiwindup compensator  $\Gamma(s)$ . In their design, they partition  $\Gamma(s)$  as  $\Gamma(s) = \Gamma_d(s)\Gamma_s$ , where  $\Gamma_d(s)$  is a heuristically tuned low-pass filter.  $\Gamma_s$  is a static gain and found by the solution of a set of LMIs. Here, the LTI anti-windup compensator is extended to a gainscheduled approach to use the advantages of the LPV design. With the LPV concept, the compensator achieves improved results for non-linear vehicle behaviour. The AWC is designed for every vertex  $\theta_i$  as an LTI-AWC. The LTI-AWCs are gain-scheduled with the scheduling parameters  $\alpha$  from the polytopic LPV controller. From a theoretical point of view, the system does not guarantee stability or performance for varying parameters, but this configuration works well in practice and is based on the idea of gain-scheduled control.

# 5.2 Anti-Windup Compensator - Torque Vectoring

The anti-windup compensator for torque vectoring must regain stability when the assumptions of the non-linear single-track model are no longer valid. The model is erroneous when the electric motors reaches their operation limit. It is not possible to define one classical saturation function for the eFuture application. The electric motors are physically limited by the maximal torque  $T_{\text{max}}$ , the maximal torque slew rate  $\dot{T}_{\text{max}}$  and the maximal power  $P_{\text{max}}$ . These limits are defined in Section 2.2.2 and depend on the angular wheel velocities, the temperature of the motors and the condition of the electric battery, so no unique saturation limit is defined. Nevertheless, it is possible to calculate the control input difference  $\Delta u$ , which is the difference between the desired control input u and the measured control input  $u_m$ . Thus, the main information for the AWC is available from the control input difference  $\Delta u$ .

For torque vectoring, the controller outputs are the longitudinal force  $F_x$  and the yaw moment  $M_z$  acting on the chassis. The measured actuator outputs are the electric motor torque  $T_{m,\text{FL}}$  and  $T_{m,\text{FR}}$  of the front left and front right motors. These signals are aligned, using the wheel radius  $r_{\text{wheel}}$  and the width of the front axle  $w_F$  as

$$\Delta u = \begin{bmatrix} F_x \\ M_z \end{bmatrix} - \begin{bmatrix} \frac{1}{r_{\text{wheel}}} & \frac{1}{r_{\text{wheel}}} \\ -\frac{w_F}{2r_{\text{wheel}}} & \frac{w_F}{2r_{\text{wheel}}} \end{bmatrix} \begin{bmatrix} T_{m,\text{FL}} \\ T_{m,\text{FR}} \end{bmatrix}.$$
(5.3)

Here,  $\Delta u$  is the input force difference between the desired control inputs  $[F_x, M_z]^T$  and the measured electric motor torques  $[T_{m,\text{FL}}, T_{m,\text{FR}}]^T$  of the vehicle. The force difference  $\Delta u$  is converted using the operator  $\Gamma$  to a control error modification signal  $\zeta_y$  and a control input modification signal  $\zeta_u$ . As recommended in [148],  $\Gamma(\theta) = \Gamma_d(s) \Gamma_s(\theta)$  and is defined as

$$\begin{bmatrix} \zeta_{u} \\ \overline{\zeta_{y}} \end{bmatrix} = \Gamma(\theta) \Delta u$$

$$\begin{bmatrix} \zeta_{u} \\ \overline{\zeta_{y}} \end{bmatrix} = \underbrace{\begin{bmatrix} \Gamma_{d,u}(s) & 0 \\ 0 & \overline{\zeta_{y}} \end{bmatrix}}_{\Gamma_{d}} \underbrace{\begin{bmatrix} \Gamma_{s,11}(\theta) & \Gamma_{s,12}(\theta) \\ \Gamma_{s,21}(\theta) & \overline{\Gamma_{s,22}(\theta)} \\ \overline{\Gamma_{s,31}(\theta)} & \overline{\Gamma_{s,32}(\theta)} \end{bmatrix}}_{\Gamma_{s,32}(\theta)} \Delta u, \qquad (5.4)$$

$$\Gamma_{d,u} = \begin{bmatrix} \frac{1}{M_{\Gamma_{u1}}} \frac{1}{\omega_{\Gamma_{u1}}} & 0 \\ 0 & \frac{1}{M_{\Gamma_{u2}}} \frac{1}{\omega_{\Gamma_{u2}}} - 1 \\ 0 & \frac{1}{M_{\Gamma_{u2}}} \frac{1}{\omega_{\Gamma_{u2}}} - 1 \\ 0 & \frac{1}{M_{\Gamma_{u2}}} - 1 \\ \frac{1}{\omega_{\Gamma_{u2}}} - 1 \\ \frac{1}{\omega_{\Gamma_{u2}}$$

$$\Gamma_{d,y} = \begin{bmatrix} \frac{1}{M_{\Gamma_{y1}}} \frac{1}{\omega_{\Gamma_{y1}}} & 0\\ 0 & \frac{1}{M_{\Gamma_{y2}}} \frac{1}{\omega_{\Gamma_{y2}}} \end{bmatrix}$$
(5.6)

where  $\Gamma_{s,j}(\theta)$  is calculated using a linear interpolation for the vertex coordinates. The tuning parameters  $M_{\Gamma_i}$  and  $\omega_{\Gamma_i}$  are defined as in Table 5.1. The final structure of

Parameter	value	Parameter	value
$M_{\Gamma_{u1}}$	30	$\omega_{\Gamma_{u1}}$	100
$M_{\Gamma_{u2}}$	50	$\omega_{\Gamma_{u2}}$	100
$M_{\Gamma_{y1}}$	1.5	$\omega_{\Gamma_{y1}}$	100
$M_{\Gamma_{y^2}}$	8	$\omega_{\Gamma_{y2}}$	100

Table 5.1: Parameters of the dynamic anti-windup compensator filter

the torque vectoring anti-windup compensator is given in Figure 5.4, where the general closed-loop system is extended using the AWC  $\Gamma$  and the controller modification signals  $\zeta_u$  and  $\zeta_y$ .



Figure 5.4: Structure of the AWC for electric motor saturation

# 5.3 Wheel Slip Limitation

Simulations and virtual driving tests with the driving simulator [150] showed improved vehicle behaviour due to use of the anti-windup compensator. Strong acceleration, braking or steering requests are no longer problematic. However, the torque vectoring controller shows unsatisfying results for "bad" road conditions, like icy roads or very wet roads. Acceleration is not optimal. The lateral performance of the vehicle is reduced and even unstable in certain cases.

#### 5.3.1 Wheel slip limitation - standard applications

The unwanted vehicle behaviour is related to the excessive wheel slip of the front tyres. Wheel slip is necessary to generate longitudinal tyre forces but with increased longitudinal wheel slip, the potential lateral tyre forces are reduced. Reduced tyre forces lead to unstable lateral movement of the vehicle. As described in Section 3.1.1, this phenomena is well known and resolved in serial vehicles with systems like ABS and TCS. The main idea of ABS is to limit the braking forces if the wheels slow down too much, and the wheel slippage (or the angular acceleration of the wheels) exceeds defined thresholds. In principle, TCS operates like ABS, but TCS limits excessive accelerations of the wheels by reducing the motor torques or employing the hydraulic brakes. Detailed explanations are given by Bosch [35] and Isermann [151]. Nonetheless, standard ABS and TCS systems have not been developed for electric vehicles and would need additional work to extend the systems for an electric drivetrain. In a standard automotive vehicle, these functions are separated. Here, it is shown that these functions can be integrated into a single torque vectoring controller.

#### 5.3.2 Wheel slip limitation - torque vectoring

As Bosch [35] proposes, wheel slip must be limited for safe and efficient driving. Several design approaches are possible to deal with wheel slip requirements:

- It is possible to include the wheel slip of the driven wheels in the vehicle model. The vehicle states can be the longitudinal velocity  $v_x$ , the lateral velocity  $v_y$ , the yaw rate r and the wheel slippage of the driven wheels  $\lambda_{FL}$ ,  $\lambda_{FR}$ . These states lead to a dual-track model instead of a single-track model. Unfortunately, the dual-track model does not perfectly match eFuture requirements. Two actuators are available, but four system outputs  $(v_x, r, \lambda_{FL}, \text{ and } \lambda_{FR})$  should be controlled. Having more control outputs than actuators leads to an underactuated system. These systems are defined by [23] as functionally uncontrollable, so the system is controllable [135] but not every trajectory in the state-space can be reached. For example, it is not possible to generate positive slip values at both wheels while slowing down the vehicle. To avoid uncontrollable directions, a switching algorithm between different controllers might be used. However, switching control may lead to additional stability problems [110]. Here, only poor results could be achieved with this method.
- Using LPV control, it is possible to define additional scheduling parameters  $\theta$  that are related to the wheel slip of the driven wheels. With varying wheel slip conditions, the performance of the controller is modified as proposed by Poussot-Vassal [103]. For this concept, the number of scheduling parameters is increased. More scheduling parameters result in increased computational effort and a more complicated tuning process for the controller. The one-step anti-windup approach is a good solution for simulation purposes but is not beneficial for the implementation of the controller within the prototype.
- Wheel slip limitations can be treated as actuator saturations. Wheel slip need not be controlled, but must be limited. For the here proposed controller design, the wheel dynamics are not included in the vehicle plant model. Instead, wheel slip limitations are treated as actuator saturations.

#### Wheel slip limitation as actuator property

The relation between the type slip  $\lambda$  and the longitudinal type force  $F_x$  is described in Section 2.2.1. The relation is non-linear and depends on factors like temperature, road surface, type pressure, type profile and so on. A simple, linear type force approximation is defined by the linear type model as

$$F_{\rm lin} = C_x \lambda. \tag{5.7}$$

The linearised, longitudinal type force  $F_{\text{lin}}$  is calculated using the wheel slip  $\lambda$  and the type stiffness factor  $C_x$ . This approximation is accurate until a threshold of  $\lambda_0 \approx 0.15$  is reached. If the wheel slip is further increased, the linear type force generation (5.7) is

no longer valid. The force difference  $\Delta F_w$  between the linear type force  $F_{\text{lin}}$  and the real type force  $F_r$  is defined as

$$\Delta F_w = F_{\rm lin} - F_r \tag{5.8}$$

and the relation is shown in Figure 5.5a. The force difference  $\Delta F_w$  can be approximated with a dead-zone function

$$\Delta F_w = \begin{cases} C_x(\lambda - \lambda_0) & \text{for } \lambda > \lambda_0 \\ 0 & \text{for } |\lambda| < \lambda_0 \\ C_x(\lambda + \lambda_0) & \text{for } \lambda < -\lambda_0, \end{cases}$$
(5.9)

where  $\Delta F_w$  is related to the type slip  $\lambda$ , the threshold  $\lambda_0$  and the longitudinal type stiffness  $C_x$ . The dead-zone function is shown in Figure 5.5b.



Figure 5.5: Longitudinal tyre force - force limit and dead-zone

## 5.3.3 Combination of actuator and wheel slip limitation

In the design of Turner and Postlethwaite [148], the actuator limitation is described using a dead-zone function, and the tyre slip limitation can be also expressed as a dead-zone function. Thus, the electric motor force difference  $\Delta u$  and the tyre force difference  $\Delta F_w$ have the same properties. The requested force cannot be supplied because of physical limitations, defined on the one hand by the electric motor, and on the other hand by the stability condition for lateral movement. The idea is to use the maximum

$$\Delta F = \max(\Delta u, \Delta F_w) \tag{5.10}$$

of both signals for the anti-windup compensator, and treat both problems equally. Combining the anti-windup compensator  $\Gamma$  for the electric motors with the wheel slip limitation creates the feature "motor torque and wheel slip limiter" (TSL). The architecture of TSL is shown in Figure 5.6. TSL is useful for eliminating windup effects of the controller,



Figure 5.6: Torque slip limiter including motor saturation and slip limitation

which are related to the limitations of the electric motors. Additionally, TSL is utilised to limit the wheel slip to a defined threshold  $\lambda_0$ . Both tasks improve the safety of the vehicle.

# 5.4 Torque and Slip Limiter: Simulation

The test sequence with the three steps from Chapter 4 is used again. Now however, the steering steps are increased to  $120^{\circ}$  at the steering wheel, and the velocity steps are requested to a difference of 20 kph. Furthermore, the road conditions are changed to a wet road with a road adhesion coefficient of 0.3. If the vehicle behaviour was linear, the intensity of the input steps would not alter the form of the response. With the increased input steps, the vehicle reaches the saturation limits of the actuator. The limited actuators change the system behaviour and indicate the necessity for the TSL algorithm.

Figure 5.7 shows the steering step request, given by the virtual driver. The time sequence is extended so the vehicle can reach the desired values. The left steer step is performed after 1s, the straight steer step after 7s and the combined step of a longitudinal and lateral request is executed after 20s.

#### Longitudinal velocity

Figure 5.8 shows the longitudinal request and the longitudinal velocity of the vehicle. In one test, the vehicle is controlled with the pure LPV controller, in another test the LPV controller is extended with TSL. During the first lateral test phase, the velocity of the LPV controlled vehicle drops 3 kph where the vehicle with TSL drops 1 kph. Both vehicles reach the reduced velocity of 40 kph at approximately at the same time of 14 s.



Figure 5.7: TSL step steer - steering angle



Figure 5.8: TSL step steer - longitudinal velocity

However, the pure LPV controlled vehicle performs an overshoot of 1.5 kph whereas the vehicle with TSL has a smoother operation and less overshoot.

For the combined step after 20 s, the pure LPV controlled vehicle increases the longitudinal velocity slightly faster at first. Neither configurations reaches the desired velocity of 60 kph but the LPV controller with the TSL extension has a steady-state error of 2 kph whereas the pure LPV controller has a steady state error of 5.5 kph.

#### Yaw rate

Figure 5.9 shows the yaw rate of the vehicle during the step sequence. The pure LPV controlled vehicle oscillates, which can be seen from 1 s to 5 s and from 20 s to 26 s. The tracking of the yaw rate request is not sufficient during the steering manoeuvres. The yaw response of the torque vectoring controller with TSL is better. There is an overshoot at 1.4 s with small oscillations, but the controller tracks the request accurately after 2.4 s. There is an overshoot of 0.05 rad/s at 6.1 s. For the combined request after 20 s the vehicle yaw rate oscillates from 20 s to 23 s, but the vehicle yaw rate is closer to the desired yaw rate and the oscillations are weaker, compared to the pure LPV controller.



Figure 5.9: TSL step steer - yaw rate

#### Wheel slip

To understand the effect of TSL, it is useful to look at the wheel slip of the driven wheels in Figure 5.10. The wheel slip values of the LPV controller with TSL are limited and do



Figure 5.10: TSL step steer - wheel slip

not exceed an absolute value of 0.25. Especially between 12 s and 14 s, the effect of TSL on wheel slip is visible. During this time span, the wheels tend to lock but TSL reduces the electric braking torque and suppresses the locking tendency of the wheel. Wheel slip oscillates during this phase, which is not ideal for ride comfort. Nonetheless, the safety of the vehicle is improved because it is possible to manoeuvre the vehicle. During the combined step, the wheel slip is again limited and oscillates around the desired threshold of 0.15.

The wheel slip data for the pure LPV controller reveals the drawback of this controller.

The wheel slip varies much more during the simulation. The front left wheel is locked from 4.7 s to 7.6 s and from 25.4 s to 30 s. This is problematic because locked wheels cannot generate lateral wheel forces, and this reduces lateral stability. Even worse, both wheels are locked between 12 s and 14.5 s. During this time span, the steering commands of the driver have no effect on the lateral movement of the vehicle. This condition is very safety critical.

# 5.5 Torque and Slip Limiter: Conclusion

Simulations of extreme driving manoeuvres with the pure LPV controller show unstable vehicle behaviour. This phenomenon is not related to LPV control; it is related to all controller designs where actuator limitations are neglected. In this chapter, an anti-windup compensator is developed to deal with saturation of the electric motors. Furthermore, the concept of the AWC is extended to deal with the wheel slip limitation requirement. Both problems are combined and solved by the motor torque and wheel slip limiter (TSL).

If the control input is not saturated or if the wheels are not in a critical slip region, the TSL does not influence closed-loop performance. If the actuators reach their limits or the wheel starts to lock or spin, the TSL improves closed-loop stability and reduces oscillations and overshoots of the controlled states. Different driving scenarios are simulated in MATLAB/Simulink and virtual test drives with the driving simulator [150] are performed. The positive effects of TSL are shown with the triple step test because this test analyses the lateral, the longitudinal and the combined vehicle behaviour.

# 6 Torque Vectoring Implementation

Before implementing the controller, the control strategy must be converted into machine code [152]. The normal procedure is to develop the controller in the continuous-time domain within a "user friendly" software, like MATLAB/Simulink [112]. The software is converted into machine code after a satisfactory controller tuning with simulation tests. These tests are straight line braking [153], straight line accelerating, constant radius turn [154], brake in bend [155], lift off oversteer [156], accelerate in bend, sinus with dwell manoeuvre [157] and random simulator drives. For the eFuture project and this thesis, several steps are performed to implement the torque vectoring controller in the prototype.

This chapter discusses the steps for a real implementation of the designed torque vectoring controller. Section 6.1 gives an overview of the functional architecture of the eFuture prototype and where the torque vectoring controller is integrated. Section 6.2 discusses additional components of a torque vectoring controller for operating its function in every driving scenario, and with critical failure cases. Section 6.3 describes the discretisation process of the developed controller, and Section 6.4 describes the quantisation requirements of the controller within the fixed-point representation of the microcontroller.

# 6.1 Drivetrain Controller

The structure of the drivetrain controller is given in Figure 6.1. The torque vectoring



Figure 6.1: Architecture - eFuture drivetrain controller

system (TVS) gets input signals from the vehicle observer (VehObs) and the command decision unit (DU1). The output signals of the torque vectoring system are torque requests T. These requests are sent to the drive-train decision unit (DU2), where the requests are checked for additional safety concerns and sent, as  $T_{\text{safe}}$ , to the electric motors. All developed vehicle functions are described in the eFuture project [20]. The functions closely related to the torque vectoring system are described briefly:

- The VehObs uses available sensor signals and filters these signals. Furthermore, the VehObs estimates non-measurable or missing vehicle signals and corrects corrupted measurements.
- The DU1 combines several requests from the driver and the ADAS and selects the best request for the drivetrain. For example, the driver requests a certain acceleration and the ACC requests a different acceleration. The DU1 decides which one is suitable and sends the chosen acceleration request to torque vectoring.
- The torque vectoring system receives the drive request from the DU1 and generates torque requests while considering the desired and real vehicle movement.
- The DU2 receives torque requests from torque vectoring and decides if these requests are acceptable for the actuators or if the requests must be limited. Additionally, the DU2 selects the actuators to execute these requests. For example, it is energy efficient generating a braking torque with the electric machines because the energy can be stored in the battery. However, with a fully loaded battery, recuperation is prohibited, and the hydraulic brakes must be applied.

A detailed explanation of these functions can be found at the eFuture project [20].

# 6.2 Torque Vectoring System

The structure of the torque vectoring system is shown in Figure 6.2. The torque vectoring



Figure 6.2: Torque vectoring algorithm with safety switch

system consists of the vehicle dynamics LPV controller, from Chapter 4, and the TSL,

from Chapter 5. Additional constraints are considered for implementing the torque vectoring system in a real vehicle. The LPV controller design is based on the STM (2.13)-(2.15). This model is not adequate for reverse driving, and the model is not numerically defined for standstill. Additionally, the lateral moment  $M_z$  has no positive effect for low velocities and does not improve the control of the vehicle. If the LPV controller cannot be used, the equal torque (ET) controller is activated.

#### 6.2.1 Equal torque distribution

The basic idea of equal torque distribution is to copy the mechanical differential [158], [159] of a standard drivetrain. Within a differential, the torque of the combustion engine is equally split to the driven wheels. Two independently controlled motors are integrated in the eFuture prototype, so no mechanical differential is necessary. However, the software component ET emulates a mechanical differential. Within the ET, the acceleration request is multiplied with a constant gain  $C_T$  and the resultant torque request is the output. The gain  $C_T$  is defined as

$$C_T = \frac{m\,R}{2},\tag{6.1}$$

where m is the vehicle mass and R the effective tyre radius. As with a classical mechanical differential, no torque distribution and no control laws are introduced.

## 6.2.2 Torque vectoring activation

The proposed torque distribution algorithm is activated above 18 kph and deactivated below 15 kph. A time dependent, linear interpolation is implemented for the transition between torque vectoring and equal torque distribution. The smooth transition suppresses discontinuous torque requests that would lead to an uncomfortable vehicle operation. During linear interpolation, the stability of the system is not guaranteed. No unstable situation has yet been caused by the linear interpolated switching.

Besides longitudinal vehicle velocity, several other factors may disable the torque vectoring controller. Safety checks are defined in the eFuture project and documented in [20]. Torque vectoring is deactivated if lateral vehicle control with the electric machines may be unreasonable. For example, if the yaw rate sensor is inoperable, no torque distribution is allowed, and the driver is informed about the malfunction of the vehicle. If the error is more severe, the torque request is set to 0 Nm. This occurs if the temperature of the battery exceeds the critical thermal limit. Finally, the torque vectoring function has three general states:

- 1. The torque request is set to 0 Nm if a serious error occurs and driving is inhibited.
- 2. Equal torque distribution is used if uncritical errors occur or if the vehicle drives slowly in reverse.
- 3. Torque distribution is activated if the vehicle drives faster than 18 kph and no error is detected.

## 6.2.3 Desired value generator

The desired value generator (DVG) is used to convert the steering request and the acceleration or velocity request from DU1 into a velocity request and a yaw rate request.

#### Longitudinal request

The longitudinal request is sent by DU1 as a velocity request  $v_{req}$  or an acceleration request  $a_{req}$ . Additionally, the status of both signals is sent to determine which request should be taken. If the velocity request should be accepted, the DU1 velocity request is routed to the LPV controller. If the acceleration request should be accepted, the acceleration request is converted to a virtual velocity request using

$$v_{req} = v_{meas} + f \cdot a_{req},\tag{6.2}$$

where f is a tuning factor and  $v_{meas}$  the measured velocity of the vehicle.

#### Lateral request

For the conversion from steering request to yaw rate request, several transformations are possible:

- The simplest conversion is a non-linear gain operation [34], [74], [80], [85], [90], [96] or a lookup-table [81], [82]. Using static gains neglects the dynamic time response of the vehicle, which is physically generated between the steering angle changes and the yaw rate changes. Ignoring the time delay leads to overshoots in the closed-loop behaviour for yaw control.
- Another strategy is to use a non-linear gain and a first-order filter [46], [72], [78] or a second-order filter [44], [100] for the vehicle dynamics. These methods require an excessive tuning process and do not provide information about the desired lateral velocity.
- The third method uses a linear single-track model to generate the desired yaw rate. The input signals are the steering wheel angle and the vehicle velocity. This strategy is used by [73], [86], [87], [88], [103]. The desired lateral motion of the vehicle is tuned using physical properties like vehicle mass or cornering stiffness, which are known for different vehicles. For example, the desired vehicle behaviour can be designed for the properties of a small sports car or a comfortable van even if the vehicle does not change its physical properties.

Here, the single-track model method is used for the DVG. The function is tuned to emulate an automobile that is 12% lighter with a 16.5% lower front cornering stiffness. The emulated vehicle understeers less and reacts faster to steering inputs. The desired

vehicle dynamics are defined as

$$\dot{\hat{\beta}} = -\frac{\hat{C}_{y,F} + \hat{C}_{y,R}}{\hat{m}v_{x,m}}\hat{\beta} + \left(\frac{-\hat{l}_F\hat{C}_{y,F} + \hat{l}_R\hat{C}_{y,R}}{\hat{m}v_{x,m}^2} - 1\right)\hat{r} + \frac{\hat{C}_{y,F}}{\hat{m}v_{x,m}}\delta_m \tag{6.3}$$

$$\dot{\hat{r}} = \frac{-\hat{l}_F \hat{C}_{y,F} + \hat{l}_R \hat{C}_{y,R}}{\hat{l}_z} \hat{\beta} - \frac{\hat{l}_F^2 \hat{C}_{y,F} + \hat{l}_R^2 \hat{C}_{y,R}}{\hat{l}_z v_{x,m}} \hat{r} + \frac{\hat{l}_F \hat{C}_{y,F}}{\hat{l}_z} \delta_m.$$
(6.4)

The desired distance from the front axle to the CoG is  $\hat{l}_F$ , and the desired distance from the CoG to the rear axle is  $\hat{l}_R$ . The desired cornering stiffness for the front axle  $\hat{C}_{y,F}$ , and the rear axle  $\hat{C}_{y,R}$ , the desired vehicle mass  $\hat{m}$  and the desired moment of inertia  $\hat{l}_z$  are chosen by the function designer. The desired sideslip angle  $\hat{\beta}$  and the desired yaw rate  $\hat{r}$  are calculated using (6.3, 6.4), given the measured steering angle  $\delta_m$  and the measured longitudinal velocity  $v_{x,m}$ .

**Remark 6.1** The sideslip angle is used to describe the desired vehicle behaviour, rather than the lateral velocity. Both states are interchangeable, but it is easier to compare different vehicle manoeuvres (with varying longitudinal velocities) if the sideslip angle is used.

Several studies ([32], [73], [160]) recommend limiting the vehicle states to request a controllable vehicle movement without excessive tyre saturation. Therefore, the desired vehicle states are limited using

$$|\hat{r}| < s_M \frac{\mu g}{v_{x,meas}} \tag{6.5}$$

$$|\hat{\beta}| < \tan^{-1}(0.02\mu g), \tag{6.6}$$

where  $\mu$  defines the maximum road adhesion coefficient and g the Earth's gravity. Rajamani [160] recommends a safety margin  $s_M$  between 0.85 and 1.5 for (6.5). Following real test drives, the safety margin  $s_M$  is set empirically to the value of 1.27.

# 6.3 Discrete-Time LPV Controller Design

The behaviour of physical systems is precisely described using differential equations in continuous-time representation because most physical systems operate continuously. For example, a vehicle moves in space in continuous-time and does not perform discontinuous movements. Physical equations are developed to describe this behaviour. By contrast, most controllers for regulating physical systems are implemented on microcontrollers that operate in the discrete-time domain. There are two different approaches to deal with the different representations of the time domain:

- Convert the physical continuous-time model of the system into a discrete-time model and calculate a discrete-time controller based on the discrete-time plant model.
- Calculate a continuous-time controller for the continuous-time plant model and

convert the continuous-time controller into a discrete-time controller. This discretisation process is called *emulation*.

A rule of thumb for LTI systems with single input / single output behaviour is that continuous-time controller synthesis can be used if the bandwidth of the microcontroller is about 20-30 times higher [161], [162] than the bandwidth of the system. For the discrete-time controller synthesis, the bandwidth of the controller should be at least 4-10 times higher [161], [162] [163] than the bandwidth of the system. No general rule of thumb is yet available for discrete-time LPV controller designs. Toth et al. [164], [165] reported some interesting results. The present study uses knowledge of LTI systems heuristically extended to discrete-time control of LPV systems. The LPV system is linearised at certain operation points and the maximum bandwidth of the system is calculated. The ratio of the sampling bandwidth to the system bandwidth is calculated for these points and compared with known discretisation rules.

• In the eFuture project [20], the sampling time  $T_S$  of the microcontroller is limited to 0.01 s. This results in a sampling frequency  $\omega_S$  of 628 rad/s using

$$\omega_S = \frac{2\pi}{T_S}.\tag{6.7}$$

• Linearised single-track models, from Section 3.4, are used to estimate the bandwidth of LTI models in the scheduling space. This step is performed for several operating points inside the polytope  $\mathcal{P}$ . The -3 db cut-off frequency  $\omega_C$  [166] is calculated using

$$\omega_C = 2|\sigma_P|,\tag{6.8}$$

where  $\sigma_P$  represents the real part of the complex pole. Poles close to the origin are responsible for low bandwidths, poles far in the LHP generate higher bandwidth. These far LHP poles are therefore called *fast poles*. These fast poles of the system must be considered for the bandwidth analysis.

• Dividing the sampling frequency  $\omega_S$  by the highest bandwidth  $\omega_C$  results in the sampling factor  $F_{\text{sample}}$ .

$$F_{\text{sample}} = \frac{\omega_S}{\omega_C}.$$
(6.9)

The factor  $F_{\text{sample}}$  is calculated for the vehicle model that is linearised in certain operation points. The results are linearly interpolated and shown in Figure 6.3 as level curves.

As discussed, the sampling factor  $F_{\text{sample}}$  should be at least 4 for the discrete-time controller synthesis and at least 20 for the emulation process. The abscissa of Figure 6.3 is the inverse scheduling signal  $1/\theta_1$ , which results in a simplified representation. It is observable that the sampling factor  $F_{\text{sample}}$  depends more on the longitudinal velocity and less on the yaw rate of the vehicle.



Figure 6.3: Sampling factor  $F_{\text{sample}}$  inside the operation range

The bandwidths of the LTI systems decrease with increased velocities. At 43 kph, the minimal bandwidth is reached. At higher velocities, the bandwidths of the linearised systems rise again. At low velocities, the poles related to the lateral dynamics are relatively fast, as discussed in Section 3.4.1. For higher velocities, the pole related to the longitudinal dynamics is the fastest pole. Consequently, the discrete-time controller design, based on the discrete-time plant model, should be used for the eFuture controller design. Longitudinal velocities below 15 kph might be problematic because the sampling factor is below 10.

#### 6.3.1 Discrete-time controller synthesis

The discrete-time controller synthesis must be used for controller implementation. Therefore, a discrete-time plant model is necessary. Several methods are available for converting a continuous-time model to a discrete-time model with sampling time  $T_S$ . Frequentlyused conversion techniques for LTI systems are documented in Table 6.1, including the transformation instruction.

**Remark 6.2** The complex variables s and z of the Laplace transform and z-transform, respectively, cannot be used here because these transforms cannot be applied to time-varying systems. Here, the continuous-time differential operator is denoted by p and the discrete-time delay operator is denoted by q. However, the general idea of the discretisation process is the same for LTI and LPV systems.

Method	s Approximation	z Approximation
Euler-forward	$s \to \frac{z-1}{T_S}$	$z \rightarrow 1 + T_S s$
Euler Backward	$s \to \frac{z-1}{T_S z}$	$z \to \frac{1}{1 - T_S s}$
Bilinear	$s \to \frac{2}{T_S} \frac{z-1}{z+1}$	$z \to \frac{2 + T_S s}{2 - T_S s}$

Table 6.1: List of LTI conversion methods for continuous-time and discrete-time models

For LPV state-space representations, a continuous-time system

$$G(\theta) : \begin{cases} \dot{x}(t) = A(\theta)x(t) + B(\theta)w(t) \\ z(t) = C(\theta)x(t) + D(\theta)w(t) \end{cases}$$
(6.10)

must be converted into a discrete-time system

$$G_d(\theta) : \begin{cases} x(k+1) = \Phi(\theta)x(k) + \Gamma(\theta)w(k) \\ z(k) = \Upsilon(\theta)x(k) + \Omega(\theta)w(k). \end{cases}$$
(6.11)

The basic conversion for the state-space matrices is summarised in Table 6.2, where the explicit term  $(\theta)$  is omitted for improved readability. The discrete-time polytopic LPV

Discrete	Euler forward	Euler backward	Bilinear transformation
$\Phi \rightarrow$	$I + T_S A$	$(I - T_S A)^{-1} T_S B$	$(I + \frac{T_S}{2}A)(I - \frac{T_S}{2}A)^{-1}$
$\Gamma \rightarrow$	$T_S B$	$(I - T_S A)^{-1} T_S B$	$(I - \frac{T_S}{2}A)^{-1}B\sqrt{T_S}$
$\Upsilon \to$	C	$C(I - T_S A)^{-1}$	$\sqrt{T_S}C(I - \frac{T_S}{2}A)^{-1}$
$\Omega \rightarrow$	D	$D + C(I - T_S A)^{-1} B T_S$	$D + C(I - \frac{T_S}{2}A)^{-1}B\frac{T_S}{2}.$

Table 6.2: Continuous-time to discrete-time state-space conversion

design from Apkarian et al. [114], [129] requires certain properties of the discrete-time generalised plant. Therefore, the plant model and the shaping filters are separately discretised to realise a parameter-dependent generalised plant with a parameter-dependent vehicle model and parameter-dependent shaping filters.

# 6.3.2 Discrete-time LPV vehicle model

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Using the polytopic LPV controller design method from Apkarian et al. [129], the discrete-time LPV plant model must satisfy the requirements that the discrete-time plant matrices  $\Gamma_u$ ,  $\Upsilon_v$ ,  $\Omega_{zu}$ ,  $\Omega_{vw}$  are parameter-independent and that  $\Omega_{vu} = 0$ . To guarantee these assumptions, it is necessary to use the Euler-forward discretisation method. The non-linear single-track model (2.13 - 2.15) is discretised with a sampling time of 0.01 s and is defined as

$$v_x(k+1) = v_x(k) + T_S\left(v_y(k)r(k) + \frac{1}{m}F_x(k)\right)$$
(6.12)

$$v_{y}(k+1) = v_{y}(k) + T_{S} \left( -v_{x}(k)r(k) - \frac{C_{y,F} + C_{y,R}}{mv_{x}(k)}v_{y}(k) + \frac{l_{R}C_{y,R} - l_{F}C_{y,F}}{mv_{x}(k)}r(k) + \frac{C_{y,F}}{m}\delta(k) \right)$$
(6.13)

$$r(k+1) = r(k) + T_S \left( \frac{l_R C_{y,R} - l_F C_{y,F}}{I_z v_x(k)} v_y(k) - \frac{l_F^2 C_{y,F} + l_R^2 C_{y,R}}{I_z v_x(k)} r(k) + \frac{l_F C_{y,F}}{I_z} \delta(k) + \frac{1}{I_z} M_z(k) \right),$$
(6.14)

where  $T_s$  represents the sampling time. The sampling instances are defined as  $kT_S$  (or shorter k), and the physical parameters are defined in Table 2.1. The system is described as a discrete-time polytopic LPV model using

$$\theta_1 = \frac{1}{v_x}, \quad \theta_2 = r, \quad x^v = \begin{bmatrix} v_x \\ v_y \\ r \end{bmatrix}, \quad y^v = \begin{bmatrix} v_x \\ r \end{bmatrix}, \quad \begin{bmatrix} w^v \\ \bar{u^v} \end{bmatrix} = \begin{bmatrix} \delta \\ \bar{F_x} \\ M_z \end{bmatrix}$$
(6.15)

which results in a discrete-time model  $G_d(\theta)$  with the polytopic LPV form (6.11). The discrete-time matrices are defined as

$$\Phi^{v}(\theta) = I + T_{S} \begin{bmatrix} 0 & \theta_{2} & 0 \\ -\theta_{2} & -\frac{C_{y,F} + C_{y,R}}{m} \theta_{1} & \frac{C_{y,R}l_{R} - C_{y,F}l_{F}}{m} \theta_{1} \\ 0 & \frac{C_{y,R}l_{R} - C_{y,F}l_{F}}{I_{z}} \theta_{1} & -\frac{C_{y,R}l_{R}^{2} + C_{y,F}l_{F}^{2}}{I_{z}} \theta_{1} \end{bmatrix}, \quad \Upsilon^{v} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$
$$\Gamma^{v} = T_{S} \begin{bmatrix} B_{w}^{v}, B_{u}^{v} \end{bmatrix} = T_{S} \begin{bmatrix} 0 & | \frac{1}{m} & 0 \\ \frac{C_{y,F}l_{F}}{m} & | 0 & 0 \\ \frac{C_{y,F}l_{F}}{l_{z}} & | 0 & \frac{1}{l_{z}} \end{bmatrix}, \quad \Omega^{v} = \begin{bmatrix} 0 & | 0 & 0 \\ 0 & | 0 & 0 \end{bmatrix}. \quad (6.16)$$

## 6.3.3 Discrete-time LPV shaping filters

The vehicle model has to be discretised with the Euler-forward method to get an adequate discrete-time model. The shaping filters cannot be discretised with the Eulerforward method because the fast poles of the high pass filter  $W^C$  would be located outside the unit disc. Having poles outside the unit disc leads to unstable plants, which would result in a erroneous conversion technique. Apkarian et al. [114] showed that the maximum spectral radius of the generalised plant  $\overline{\lambda}(A(\theta))$  is limited by the sampling frequency, using

$$f_S > \frac{\bar{\lambda}(A(\theta))}{2}, \quad \forall \theta \in \Theta.$$
 (6.17)

This condition is not satisfied for the generalised plant, with the parameter-dependent shaping filters, from Section 4.4.1. The maximum sampling frequency for the eFuture project is limited to 100 Hz. A different discrete-time parameter-dependent filter strategy must be applied.

The filters  $W^S$  and  $W^C$  are designed in continuous-time as LTI filters, with frequency characteristics as determined in Chapter 4. In the next step, the filters are converted into discrete-time LTI filters with the standard zero-order-hold (ZOH) technique [112], [167]:

$$W(s) := \begin{bmatrix} A & B \\ \bar{C} & \bar{D} \end{bmatrix}, \quad W(s) \xrightarrow{ZOH} W_d(z), \quad W_d(z) := \begin{bmatrix} \Phi & \Gamma \\ \bar{\Upsilon} & \bar{\Omega} \end{bmatrix}.$$
(6.18)

After the discretisation step, the filters are modified to affine parameter-dependent shaping filters, using

$$W_d(\theta) := \begin{bmatrix} \Phi_0 + \theta_1 \Phi_1 + \theta_2 \Phi_2 & \Gamma \\ \bar{\Upsilon} & \bar{\Omega} \end{bmatrix}.$$
(6.19)

The discretisation step is executed for LTI filters and is well defined in the literature [167], [168]. The parameter-dependent tuning process of the filter is performed in discrete-time. For this torque vectoring controller, the discrete-time parameter-dependent and polytopic sensitivity filter  $W_d^S(\theta)$  is defined as

$$W_d^S(\theta) = \begin{bmatrix} 1 & 0 & | & 0.03125 & 0 \\ 0 & 2.7288\theta_1 & | & 0 & 0.0625 \\ -0.016 & 0 & | & 0.00025 & -0 & 0 \\ 0 & 0.08791 & | & 0 & 0.003021 \end{bmatrix}.$$
 (6.20)

The discrete-time parameter-dependent polytopic control sensitivity filter  $W_d^C(\theta)$  is de-

fined as

$$W_d^C(\theta) = \begin{bmatrix} 0.006738 & 0 & 0.25 & 0 \\ 0 & 0.2962 - 0.965\theta_1 & 0 & 0.5 \\ -0.1514 & 0 & 0.06311 & 0 \\ 0 & -0.6056 & 0 & 0.5049 \end{bmatrix}.$$
 (6.21)

After defining the plant model and the shaping filters, the scheme from Figure 4.5 is used to generate the parameter-dependent discrete-time generalised plant.

## 6.3.4 Discrete-time LPV generalised plant

The discrete-time generalised plant consists of the discrete-time vehicle model from Section 6.3.2 and the discrete-time, parameter-dependent shaping filters from Section 6.3.3. The system is illustrated graphically in Figure 6.4. Mathematically the generalised plant



Figure 6.4: Discrete-time generalised plant in affine LPV form

 $P_d(\theta)$  is defined as

$$\begin{bmatrix} \dot{x}^{v} \\ \dot{x}^{S} \\ \dot{x}^{C} \\ \ddot{z}^{S} \\ z^{C} \\ \vec{v}^{C} \\ \vec{v}^{V} \\ \vec{v}^{V} \\ \vec{v}^{V} \end{bmatrix} = \begin{bmatrix} \Phi^{v}(\theta) & 0 & 0 & 0 & \Gamma^{v}_{d} + \Gamma^{v}_{u} \\ -\Gamma^{S}C^{c} & \Phi^{S}(\theta) & 0 & \Gamma^{S} & -\Gamma^{S}\Omega^{v}_{d} + -\Gamma^{S}\Omega^{v}_{u} \\ 0 & 0 & \Phi^{C}(\theta) + 0 & 0 & \Gamma^{C} \\ -\overline{D^{S}}\bar{\Upsilon^{v}} - \bar{\Upsilon^{S}} & 0 & 0 & \Gamma^{S} \\ 0 & -\overline{D^{S}}\bar{\Upsilon^{v}} - \bar{\Upsilon^{S}} & 0 & 0 & 0 & \Gamma^{C} \\ 0 & 0 & 0 & 1 & \overline{D^{S}} & -\overline{D^{S}}\bar{\Omega}^{v}_{d} + -\overline{\Omega^{S}}\bar{\Omega}^{v}_{u} \\ 0 & 0 & 0 & 0 & 0 & \Omega^{C} \\ -\bar{\Upsilon^{C}} & 0 & 0 & 0 & 1 & \Gamma^{C} \\ 0 & 0 & 0 & 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} x^{v} \\ x^{S} \\ x^{C} \\ -\bar{r^{v}} \\ u^{v} \\ u^{v} \end{bmatrix}.$$
(6.22)

### 6.3.5 Discrete-time LPV controller synthesis

The discrete-time polytopic LPV controller is calculated using the generalised plant and the synthesis procedure described in [114], [129], [169]. First, the symmetrical matrices

 $R_d$ ,  $S_d$  are found for the closed-loop system. The matrices  $R_d$ ,  $S_d$  must satisfy LMIs

$$\begin{bmatrix} \mathcal{N}_{R_{di}} & 0 \\ -\frac{\sigma}{0} & 1 \end{bmatrix}^{T} \begin{bmatrix} \Phi_{i}R_{d}\Phi_{i}^{T} - R_{d} & \Phi_{i}R_{d}\Upsilon_{z,i}^{T} & \Gamma_{w,i} \\ -\frac{\sigma}{1} & -\frac{\sigma}{1} & -\frac{\sigma}{1} + \Upsilon_{z,i}R_{d}\Upsilon_{z,i}^{T} & \Omega_{wz,i} \\ -\frac{\sigma}{1} & -\frac{\sigma}{1} & -\frac{\sigma}{1} \end{bmatrix} \begin{bmatrix} -\mathcal{N}_{R_{di}} & 0 \\ -\frac{\sigma}{1} & 1 \end{bmatrix} < 0 \quad (6.23)$$

$$\begin{bmatrix} \mathcal{N}_{\underline{S}_{di}} & 0 \\ 0 & 1 \end{bmatrix}^{T} \begin{bmatrix} \Phi_{i}^{T} S_{d} \Phi_{i} - S_{d} & \Phi_{i}^{T} S_{d} \Gamma_{w,i} & \Upsilon_{z,i} \\ & \star & -\gamma I + \Gamma_{\underline{w}_{i}}^{T} S_{d} \Gamma_{w,i} & \Omega_{\underline{w}_{z,i}}^{T} \\ & & \star & -\gamma I \end{bmatrix} \begin{bmatrix} \mathcal{N}_{\underline{S}_{di}} & 0 \\ 0 & I \end{bmatrix} < 0 \quad (6.24)$$

$$\left[\begin{array}{cc} R_d & I\\ I & S_d \end{array}\right] \ge 0, \quad (6.25)$$

where  $\mathcal{N}_{R_{di}}$  and  $\mathcal{N}_{S_{di}}$  are the base of the null space from  $[\Gamma_{u,i}, \Omega_{zu,i}^T]$  and  $[\Upsilon_{v,i}, \Omega_{vw,i}^T]$ . Next, the Lyapunov matrix  $X_{d,cl}$  is defined as

$$M_d N_d^T = I - R_d S_d \tag{6.26}$$

$$X_{d,cl} = \begin{bmatrix} S_d & I \\ N_d^T & 0 \end{bmatrix} \begin{bmatrix} I & R_d \\ 0 & M_d^T \end{bmatrix}^{-1}.$$
 (6.27)

The discrete-time version of LMI conditions is defined as

$$X_{d,cl} = X_{d,cl}^T > 0$$
 (6.28)

The remaining procedure is similar to the continuous-time case, as described in Section 4.1.1.

## 6.3.6 Discrete-time motor torque and wheel slip limiter

The motor torque and wheel slip limiter from Chapter 5 is calculated in discrete-time with the discrete-time vehicle model from Section 6.3.2 and the discrete-time polytopic LPV controller from Section 6.3.5. The dynamic TSL is calculated as a low pass filter but converted using ZOH operation into the discrete-time representation. As for the continuous-time case in Section 5, Turner et al. [170] propose a similar theorem for discrete-time, where operator  $\tau$  maps the linear control input  $u_{\text{lin}}$  to the non-linear control output  $\zeta_d$  as in Figure 5.3. By solving the proposed LMI [170] for all six vertices, the discrete-time LTI anti-windup compensators are calculated for every corner point. The resulting gains  $\Xi_i$  are linearly interpolated to deal with the non-linear single-track model and the discrete-time polytopic LPV controller.
## 6.4 Quantisation of the Controller

The eFuture project uses the microcontroller Freescale Bolero MPC5607B. The torque vectoring function is calculated using a sampling time of 0.01 s. In addition, five major functions must be calculated by the same microcontroller. The automotive software standard AUTOSAR [21] must be used for this project, which creates additional overhead for the base software calculation. The final requirement for the execution time of the torque vectoring is that the compiled machine code must run within 1 ms on the microcontroller. To execute the generated code properly, the software must be converted to fixed-point representation [171].

The drawbacks of fixed-point representation are the risk of overflows and the possibility of inaccurate representation. Nonetheless, this process is required to reduce the online computation effort. To implement the fixed-point controller, it is useful to transform the controller matrices. A suitable fixed-point conversion strategy for an LTI state-space controller is explained in [172]. The general idea is that every value calculated by the controller should be below 1. To achieve this property, a transformation matrix T is applied to the controller. The design from Jerez et. al [172] is modified to meet the LPV controller design. Here, every vertex controller is transformed using

$$\Phi_i^f = T^{-1} \Phi_i T, \qquad \Gamma_i^f = T^{-1} \Gamma_i$$
  

$$\Upsilon_i^f = \Upsilon_i T, \qquad \Omega_i^f = \Omega_i.$$
(6.30)

The transformation matrix T is defined as the inverse square root matrix of the matrix M as recommended by [172]. The matrices T and M are defined as

$$T = M^{-\frac{1}{2}} \tag{6.31}$$

$$M_{k,k} := \max_{i \in r} \sum_{j=1}^{N} |A_{i,k,j}|, \qquad (6.32)$$

where  $M_{i,k,k}$  is the maximum of *i*-vertex elements. All column elements *j* are summed up for row *k*. The matrix *M* is a diagonal matrix with *N* elements. With the scaled matrices  $\Phi_i^f$ ,  $\Gamma_i^f$ ,  $\Upsilon_i^f$  and  $\Omega_i^f$ , the fixed-point representation of the vertex controller is improved. Every calculated value of the matrix multiplication  $\Phi_i^f \cdot x^f$  is within the limit  $\in (-1, 1)$ .

To suppress overflows and to maintain maximum accuracy, it is helpful to transform the controller and to use the scaling matrix T to make the system numerically stable while containing the system properties. The scaled vertex controllers  $K_i^f(s)$  are defined as

$$K_{i}^{f}(s): \begin{cases} \dot{x}^{f} = \Phi_{K_{i}}^{f} x^{f} + \Gamma_{K_{i}}^{f} v \\ u = \Upsilon_{K_{i}}^{f} x_{f} + \Omega_{K_{i}}^{f} v. \end{cases}$$
(6.33)

As well as scaling the control parameters, it is necessary to limit the bit size of all signals

for the Bolero microcontroller. The inputs to an accumulator can have a maximum of 32 bits. For efficient code execution, multiplications and memory storage functions are limited to 16 bit signals. Calculations such as square root or trigonometric functions must be replaced by suitable lookup tables. With all these modifications, the microcontroller executes the torque vectoring controller within 0.7 ms.

# 6.5 Implementation: Conclusion

In the eFuture project, several functions are directly connected to torque vectoring. These functions are briefly described. Within the torque vectoring software, additional functions must be implemented, beyond the LPV vehicle dynamics controller and the torque slip limiter. An equal torque distribution component is included for low velocities and reverse driving. This component is activated if system error prevents the use of torque vectoring. The desired value generator is described and is used to generate reference signals from given driving requests. The LPV controller, designed in Section 4.1.1, is calculated in a discrete-time representation. For this purpose, the vehicle model and the shaping filters are converted into a discrete-time representation. The discrete-time synthesis procedure for polytopic LPV systems from [114] is used to calculate the controller. The discrete-time anti-windup scheme from Turner et al. [170] is applied to calculate the discrete-time TSL. Then, the torque vectoring controller is converted into fixed-point representation for implementation in the automotive microcontroller.

The design process for the discrete-time parameter-dependent shaping filters is not very intuitive. However, this step is necessary to define a discrete-time generalised plant model that represents the continuous-time dynamics adequately and can be used for the polytopic synthesis process. An advantage of the discrete-time controller synthesis is the improved numerical stability of the LMI solver and the faster calculation time for the controller synthesis. In future, the fixed-point representation will be further analysed to generate a more automated design process. Results from Rotea and Williamson [173] and Roozbehani et al. [174] seem to be promising for this purpose.

# 7 Test Driving

In the course of the eFuture project [20], several tests were defined and performed with the prototype. Here, three tests for torque vectoring are discussed. In Section 7.1, a general driving scenario is reviewed. In Section 7.2, a constant radius turn test is performed to analyse steady-state lateral dynamics. A double lane change is discussed in Section 7.3. This test is used to analyse dynamic vehicle behaviour. Finally, a conclusion about the tests is given in Section 7.4.

# 7.1 General Driving

The effect of torque vectoring is analysed using a general driving scenario. The eFuture prototype has no official road approval, so the vehicle is driven on test-tracks. For the general driving test, a professional driver operated the vehicle at the emission circuit on the test track in Gaydon, Warwickshire, England. The layout of the emission circuit is shown in Figure 7.1. The driver starts from standstill and drives around the course with



Figure 7.1: Gaydon emission circuit

the purpose of analysing energy efficiency. This manoeuvre is used as a general driving scenario because the driver is not concerned with the vehicle dynamics during this test. The track itself has straight parts, slight curves and two sharp corners. The two closer corners are defined as turn one (T1) and turn two (T2).

## Emission circuit - steering angle

Figure 7.2 shows the angle of the steering wheel. At the beginning of the test, the steering angle varies between  $-80^{\circ}$  and  $128.3^{\circ}$ . Negative steering angles represent a steering to the right side (from the driver's point of view). Positive steering angles indicate a steering to the left side. The vehicle enters curve T1 at 73 s, where the driver steers up to  $-50^{\circ}$ . After 186 s, the vehicle reaches curve T2, where the driver performs



Figure 7.2: Emission circuit - driver steering request

a right turn with a maximum steering angle of  $-65^{\circ}$  and a duration of about 19 s. The vehicle reaches T1 again after 285 s.

### Emission circuit - longitudinal velocity

The longitudinal velocity of the vehicle is shown in Figure 7.3. The vehicle starts from



Figure 7.3: Emission circuit - longitudinal velocity

a standstill and enters the emission circuit. For torque vectoring, this test is interesting after 5.2 s because from this point on the vehicle exceeds a velocity of 18 kph and the active yaw moment distribution is activated. The vehicle runs at a maximum of 110 kph and most of the time drives around 106 kph. At curves T1 and T2 the vehicle slows down just before the turn and accelerates at the end of the turn. The other parts of the track are covered at maximum velocity.

#### Emission circuit - lateral acceleration

The lateral acceleration of the vehicle is measured with the yaw rate sensor and is shown in Figure 7.4. The turn T1 is visible from 73 s to 93 s and again from 285 s to 306 s.



Figure 7.4: Emission circuit - lateral acceleration

Turn T2 is recognisable from 186 s to 205 s. The maximum lateral acceleration reaches  $-6.1 \text{ m/s}^2$  at 203.3 s, where the vehicle exceeds the limit of linear, lateral vehicle behaviour of  $4 \text{ m/s}^2$  to  $5 \text{ m/s}^2$  [47], [175]. The two long straight lines of the track are visible during the time intervals from 120 s to 180 s and between 207 s and 251 s. The rest of the time the vehicle is driven through moderate curves.

#### Emission circuit - yaw rate

The yaw rate of the vehicle is measured with the yaw rate sensor and is shown in Figure 7.5. The general shape of the yaw rate looks similar to that of the lateral acceleration in



Figure 7.5: Emission circuit - yaw rate

Figure 7.4. Turns T1 and T2, the straight lines and the slight curves are again visible.

Figure 7.5 shows the desired yaw rate that is used by the torque vectoring controller. Figure 7.6 gives a detailed view of the yaw rate for T2 from 180s to 210s, to show differences between the desired and measured yaw rates. The measured yaw rate tracks



Figure 7.6: General driving - yaw rate for T2

the desired yaw rate closely, with a maximum difference of 0.03 rad/s at 203.7 s.

#### Emission circuit - motor torque

Besides the vehicle states, the control inputs are also important signals for classifying the performance of the controller. Here, the front left and front right motor torques are the control inputs and these signals are shown in Figure 7.7. The motor torques vary between



Figure 7.7: General - motor torque

-223 Nm and 317 Nm. The maximum difference between the left and the right side is 301 Nm at 106.45 s, but in general the signals do not show major differences. Figure 7.8 shows the motor torque values for the T2 time interval between 180 s and 210 s. At the beginning, the vehicle speed is reduced, which is visible in the negative torque requests. Between 181 s and 184 s, the vehicle performs a modest left turn before reaching the



Figure 7.8: General - motor torque zoomed

major right turn T2 from 190 s to 205 s. During this time interval, an average torque difference of about 100 Nm is noticeable. This torque difference generates a right-turning yaw moment. After 205 s the right turn is completed, and the motor torque values are similar.

The general drive on the emission circuit is not useful for analysing lateral dynamics of the prototype because a human driver always behaves differently. Special manoeuvres are performed to compare ET distribution with TV distribution. This comparison is used to evaluate the effects of the torque vectoring controller.

# 7.2 Constant Radius Turn

The constant radius turn (CRT) is defined by ISO 4138 [154] and is used as a lateral dynamics test. This test should be performed with a minimum circle radius of 30 m. The maximum available space at the test facility in Giebelstadt, Germany limits the driving circle to a radius of 15 m. Therefore, the CRT test has to be modified. The driver follows the 15 m radius circle while accelerating the vehicle moderately. After some time, the maximum velocity for this test is reached, and it is not possible to follow the circle with a higher velocity. At the end, the driver brakes and slows down the vehicle. Two different configurations are used for comparison. First, the ET distribution is used. Second, the test is performed with the activated torque vectoring controller. Because these are real tests with human drivers, the tests are not exactly the same. However, general vehicle behaviour and trends can be recognised.

### CRT - steering angle

The steering wheel angles for the CRT test are shown in Figure 7.9, where the ET and TV-configurations are compared. At the beginning, the measured steering angle with torque vectoring oscillates more and rises to a steering angle of  $-344^{\circ}$  at 24.9 s (a negative steering angles indicate a turn to the right side). The steering angle of the vehicle with



Figure 7.9: CRT - driver steering request

equal torque distribution oscillates less and rises faster until reaching the angle of -494  $^{\circ}$  at 19.2 s. To understand the steering angle changes after 20 s for ET (and after 25 s for TV) it is useful to review the vehicle velocity from Figure 7.10. After these two points in time, the driver slows down the vehicle and has to perform different steering commands to control the vehicle. For ET, the required steering difference is 441  $^{\circ}$  between 20 s and 22 s. For the TV-configuration, the steering difference is 249  $^{\circ}$  between 25 s and 27 s. The steering responses indicate that the ET-configuration requires powerful steering and the steering corrections are stronger. The steering requests of the TV-configuration are weaker, and fewer corrections are necessary. However, the TV-configuration shows a higher oscillation of the steering wheel angle at the beginning of the test.

#### **CRT** - longitudinal velocity

The longitudinal velocities for both configurations are shown in Figure 7.10. Both ve-



Figure 7.10: CRT - longitudinal vehicle velocity

hicle configurations show a similar velocity increase. The maximum velocity of the ET-configuration is 39.4 kph at 19.4 s. The maximum velocity of the TV-configuration

is 41 kph at 23.9 s. After reaching the maximum velocity, the driver slows down the vehicle. The velocities of both configurations appear comparable, but the TV-configuration achieves a 1.6 kph higher maximum velocity for the constant radius turn.

## **CRT** - lateral acceleration

The lateral acceleration of the vehicle is shown in Figure 7.11. Both vehicle configurations



Figure 7.11: CRT - lateral acceleration

show similar behaviour at the beginning of the test. The sharp lateral acceleration reduction of both configurations is related to the braking and steering operations of the driver. The maximum lateral acceleration with ET is  $8.7 \text{ m/s}^2$  at 20.6 s. For the TV-configuration, the maximum lateral acceleration is  $9.1 \text{ m/s}^2$  at 25.3 s. As for the longitudinal velocity, the lateral accelerations seem to be similar and the maximum values differ by about  $0.4 \text{ m/s}^2$ .

#### CRT - yaw rate

The desired yaw rate and the measured yaw rate are shown in Figure 7.12. Figure 7.12a shows the desired and measured yaw rates for the ET-configuration. The difference between the desired and measured yaw rate, the yaw rate error  $e_r$ , constantly increases and for the ET-configuration reaches a maximum error of 0.7 rad/s at 19.3 s. Figure 7.12b shows the yaw rate signals for the TV-configuration. For TV, the difference between desired and measured yaw rate is below 0.1 rad/s until 14.6 s. The maximum yaw rate error for the TV-configuration is 0.2 rad/s at 20.2 s.

## CRT - motor torque

Figure 7.12b raises to the question: Why is there a constant yaw rate error if the LPV controller controls the yaw rate? Better yaw rate tracking should be possible. Examining the motor torque, shown in Figure 7.13, suggests an explanation for why better tracking is impossible. For the TV-configuration, the front left motor operates at its physical limit



Figure 7.12: CRT - yaw rate

of 775 Nm. No additional yaw moment can be generated with a higher motor torque because the motor is at its torque boundary. The only way to increase the yaw moment is to reduce the motor torque of the front right motor. However, reduced torque would decrease the vehicle speed, which is also not desired. This case shows that the physical limits of the motor restrict the combination of longitudinal and lateral performance of the vehicle. Nonetheless, the vehicle is still controllable for the driver due to the TSL implementation, which creates a trade-off between longitudinal and lateral requests. The oscillations of the TV-configuration for the front right motor from 20.5 to 22.5 s and from 24.5 s to 27 s are not desired. During these time spans, the wheel starts to spin. The slip limiting component of TSL suppresses excessive wheel slip. The oscillating behaviour is unavoidable, but it is not as crucial as a spinning wheel, which would lead to lateral instability.

The left and right torque requests of the ET-configuration are identical most of the time, as expected. Between 15.7 s and 19.6 s the motor torques differ, up to 213 Nm at 16.9 s. This difference is not requested by the drivetrain controller but is related to the power limitations of the electric machines. The front right wheel starts to spin during this period and increases the angular velocity of the electric machine. According to the maximum torque characteristics from Figure 2.5, the motor cannot supply the requested torque for the increased angular velocity.



Figure 7.13: CRT - motor torque

#### **CRT** - sideslip angle

Reviewing the sideslip  $angles^1$  in Figure 7.14 shows a major difference in vehicle behaviour. Up to 10s, the sideslip angles of the ET and the TV-configuration are sim-



Figure 7.14: CRT - sideslip angle

ilar. After 10s the TV-configuration increases the torque difference (see Figure 7.13) to achieve the desired yaw rate request, shown in Figure 7.12. The generated yaw moment changes the sideslip angle. The sideslip angle of the ET-configuration reaches 0.082 rad for the maximum lateral acceleration at 20.6 s. The sideslip angle of the TV-configuration is 0.25 rad for the maximum lateral acceleration at 25.3 s. The torque vectoring controller increases the sideslip angle while it distributes the yaw moment actively. Some authors [43], [83] claim that a zero sideslip angle is advantageous, so

<sup>&</sup>lt;sup>1</sup>The sideslip angle is related to the lateral velocity by (2.16). It is advantageous to review the sideslip angle  $\beta$  because  $\beta$  is more easily comparable for different longitudinal velocities.

increasing the sideslip angle is not desirable. Here, a different interpretation [176] is applied to the vehicle behaviour.

#### CRT - vehicle behaviour

General terms to classify vehicle behaviour are "understeer", "neutral" and "oversteer". Understeer means that the vehicle does not turn more when the driver increases the steering wheel angle. Oversteer means that the vehicle turns more even if the steering wheel angle is constant (or even reduced). Neutral driving means that the lateral movement and the steering wheel angle behave linearly all the time. This classification can be expressed in terms of the wheel slip angle  $\alpha$  which is defined in (2.21). If the slip angles of the front and rear tyres are similar, the vehicle behaves neutrally. If the slip angles of the front wheels are much higher, the vehicle understeers. If the slip angles of the rear wheels are much higher, the vehicle oversteers [36], [177]. Figure 7.15 summarises these classifications. Today, a slightly understeering vehicle behaviour is accepted as the best



Figure 7.15: Vehicle behaviour with understeer and oversteer

vehicle behaviour [177] for safe driving by an average driver.

## CRT - wheel slip angle

Figure 7.16 shows the steering angle  $\delta$  and the calculated slip angles of the front  $\alpha_F$  and rear  $\alpha_R$  wheels for the ET- and TV-configurations. The wheel slip angles are calculated as

$$\alpha_F = \delta - \beta - \frac{l_F}{v_x} r \tag{7.1}$$

$$\alpha_R = -\beta - \frac{l_R}{v_x} r \tag{7.2}$$

using the steering angle  $\delta$  and the sideslip angle  $\beta$ , the distance from the front axle to the CoG  $l_F$ , the distance from the CoG to the rear axle  $l_R$  and the longitudinal velocity  $v_x$ . In the TV-configuration, the wheel slip angles of the front and rear wheels are quite similar. From 11 s to 26 s, a slight understeering behaviour is observable because the front



(b) CRT - wheel slip angles - torque vectoring

Figure 7.16: CRT - wheel slip angles

slip angle  $\alpha_F$  is as higher than the rear slip angle  $\alpha_R$ . Until 10 s, the ET-configuration behaves similarly to the TV-configuration. From 10 s to 22 s, the rear slip angle  $\alpha_R$  of the ET increases constantly whereas the front slip angle  $\alpha_F$  of the ET increases strongly, especially when the driver starts to steer more strongly at 15 s. The ET-configuration has a strong understeering behaviour, which reduces the performance and safety of the vehicle. For the TV-configuration, the wheel slip angles  $\alpha_F$  and  $\alpha_R$  are closer together, so the vehicle behaves more neutrally but with slight understeering. The author comes to the conclusion that a small sideslip angle  $\beta$  is not desirable, but a sideslip angle  $\beta$ that leads to an equally distributed wheel slip angle  $\alpha$  is important.

#### **CRT** - understeer gradient

A number of researchers analyse the understeer gradients of vehicles ( [44], [49], [78], [80]). The understeer gradient is defined by the SAE norm [178] and is shown in Figure 7.17. Here, this analysis is not as good as it could be because steering angle and lateral acceleration values are corrupted with measurement noise. The driver performs longitudinal acceleration from  $-0.9 \text{ m/s}^2$  to  $1.2 \text{ m/s}^2$ , which further distorts the results. Nonetheless, the trend of the understeering gradient is visible for both, the ET-configuration and the TV-configuration. The ET-configuration operates in a linear understeer gradient strongly increases, peaking at  $8.4 \text{ m/s}^2$ . For the TV-configuration, the understeer gradient is lower, which means that the vehicle behaves more neutrally and its understeer gradient is linear



Figure 7.17: CRT - motor torque

until 7.6 m/s<sup>2</sup>. The nonlinear understeer regime rises until  $9.4 \text{ m/s}^2$ . Also, the understeergradient rises later, which means that the vehicle does not understeer as much as the ET-configuration does. Unfortunately, the constant radius turn was limited by the test facility to a radius of 15 m. This limits the comparison with other results [49], [78], [80] but the comparison between the ET and the TV-configuration is still possible.

# 7.3 Extreme Driving Manoeuvre: Double Lane Change

The DLC is defined by ISO norm 3888-2 [42] and is also known as the "elk-test" or "moose-test". This test simulates a strong steering manoeuvre to drive around an unexpectedly appearing obstacle. The basic test setup is shown in Figure 7.18, in which the driver follows a defined track setup. The DLC is a closed-loop test, which means that the



Figure 7.18: Double lane change - test layout

human driver reacts to the vehicle's behaviour and does not steer equally for every test. To keep the variance as small as possible, the same driver is performing all DLC tests. It is more difficult to compare different vehicle configurations with the driver in the loop, because the driver's requests vary. During the DLC, the driver performs a strong left turn, followed by a strong right turn and again a strong left turn. The initial velocity of the test is increased in 5 kph steps until the driver is unable to follow the track. In ISO norm 3888-2 [42], the driver is advised to keep the angular velocity of the internal combustion engine above 2000 rpm. This advice is not applicable to electric vehicles. Here, the driver switches to neutral gear when approaching the first cone setup. With the project's specific driver and the eFuture prototype [20], the test is successfully performed for the ET-configuration with an initial velocity of 60 kph. The maximum initial velocity of the TV-configuration is 65 kph. To analyse the difference between the ET- and TV-configurations, a successful TV-configuration test is compared with a failed ET-test at an initial velocity of 65 kph.

#### **DLC** - steering angle

Figure 7.19 compares the steering requests of the ET- and TV-configurations. The first



Figure 7.19: DLC - driver steering request

left turns from 0.8s to 1.2s are similar. However, the driver reacts differently for the right turns. With the ET-configuration, the right turn performs more strongly, with a maximum steering angle of  $-241^{\circ}$  at 2.4s. With the TV-configuration, the driver steers with a lower maximum steering angle of  $-218^{\circ}$  at 2.4s. From 1.6s to 2.3s, an oscillating steering behaviour is visible for the TV-configuration. Oscillations are again visible from 3.8s to 4.6s. The second left turn of the TV-configuration has a maximum of  $197^{\circ}$  at 3.35s. The second left turn for the ET-configuration is performed earlier and more strongly, with a maximum of  $240^{\circ}$  at 3.32s.

### **DLC** - longitudinal velocity

An earlier and stronger steering angle for the ET-configuration suggests that the vehicle drives faster during the test. Examining Figure 7.20, the opposite appears true. For the ET-configuration, the vehicle starts at 63 kph. During the TV-configuration test, the vehicle starts at 62.5 kph. After 2.1 s, the ET-configurations slows down more strongly than the TV-configuration. Also, the final velocity of the TV-configuration is 51.5 kph, whereas the ET-configuration drives at 46 kph at the end of the test. The lower velocity



Figure 7.20: DLC - longitudinal vehicle velocity

implies that the ET-configuration performs higher lateral work, which slows down the vehicle.

## **DLC** - lateral acceleration

If higher lateral work is performed, the measured lateral acceleration should be higher. Comparing lateral accelerations (Figure 7.21) does not prove this inference. The mea-



Figure 7.21: DLC - lateral acceleration

sured lateral acceleration suggests that the vehicles move in nearly the same way. The steering measurements in Figure 7.19 show that the driver steers earlier and more strongly for the ET-configuration, but the lateral accelerations are fairly similar.

## DLC - yaw rate

The lateral vehicle motion is also described by the yaw rate r of the vehicle as shown in Figure 7.22. The desired yaw rate (calculated from the velocity and steering angle)



Figure 7.22: DLC - yaw rates

is compared with the measured yaw rate of the equal torque configuration in Figure 7.22a. With the ET-configuration, the tracking error is 0.14 rad/s between 1.25s and 1.5s. Between 2.7s and 3.4s, the measured yaw rate is 0.2s delayed from the desired yaw rate. The desired and measured yaw rates for the TV-configuration are shown in Figure 7.22b. The yaw rate tracking is closer for the TV-configuration, as compared to the ET-configuration. The maximum yaw rate error for the first left steering is 0.09 rad/s and the delay for the second left steering is 0.04 s. The maximum yaw rate error for the second left steering is, at 0.24 rad/s, higher than for the ET-configuration at 0.08 rad/s. One interesting fact about the maximum yaw rate of the second left turn is that the measured vehicle yaw rates are similar, with 0.87 rad/s for the ET-configuration and 0.86 rad/s for the TV-configuration. However, the desired yaw rates differ at this point, with 0.8 rad/s for ET and 0.62 rad/s for the TV-configuration. The desired yaw rate for TV is lower because the vehicle moves faster at this point, and the driver steers less, compared to ET-configuration.

#### **DLC** - motor torque

After reviewing the vehicle states, the control inputs are reviewed. Figure 7.23 shows the motor torques for the ET- and TV-configurations. For the ET-configuration, the motor torques are constantly 0 Nm because the neutral gear is engaged before reaching the first track setup. No longitudinal torque request is generated. The torque requests



Figure 7.23: DLC - motor torque

of the TV-configuration are symmetrical about the 0 torque axis because of the neutral gear selection. An interesting fact about the TV-configuration is that the maximum motor torque and the power limit of the battery are no major limitations for the DLC test. However, the motor torque slew-rate limitation has a major effect. This effect can be seen in the limited gradient of the motor torque, which is observable from 3.09 s to 3.27 s, for example.

### DLC - wheel slip

Figure 7.24 shows the wheel slip values for the driven wheels of the TV-configuration. The ET-configuration is omitted here because it applies no torques and therefore gen-



Figure 7.24: DLC - wheel slip

erates no longitudinal wheel slips. The wheel slip values of the TV-configuration are acceptable at all times. However, from 2.4s to 2.6s the TSL is active to suppress ex-

cessive brake slip at the front right wheel. The excessive slip of the wheel starts at a negative torque of about -400 Nm and is reduced to about -150 Nm from 2.6s to 3 s.

From 1.6 s to  $1.7 \,\mathrm{s}$ , -400 Nm is also applied and no excessive slip is visible. In one time point, the wheel starts to lock; in at the other, it does not lock at -400 Nm. This behaviour is related to the roll motion of the vehicle. Between 2 s and 3 s the vehicle performs a strong right turn. The vehicle rolls to the left side, which increases the vertical tyre forces on the left (outer) side and reduces the vertical tyre forces on the right (inner) side. With the reduced vertical tyre forces, the right wheel starts to lock, with lower propulsion moment. So the roll motion of the vehicle causes different wheel slip behaviour even if the propulsion forces are similar. The same effect is visible in the spinning tendency of the front left wheel from 3.6 s to 3.74 s. The vehicle rolls to the right side and reduces the vertical load of the front left tyre. The reduced vertical force leads to a faster spinning tendency. The TSL suppresses the spinning of the front left wheel.

#### **DLC** - sideslip angle

The major difference in vehicle behaviours can be seen again in the sideslip angles  $\beta$  of the vehicles which are shown in Figure 7.25. For the first left turn, the sideslip angle of



Figure 7.25: DLC - lateral velocity

the ET-configuration has a small delay of around 0.08 s between 1 s and 2 s. The main difference appears in the right turn between 2 s and 3.3 s. The maximum sideslip angle of the ET-configuration rises to 0.255 rad, whereas the maximum sideslip angle of the TV-configuration is limited to 0.126 rad. During the second left turn between 3.3 s and 4.5 s, the maximum sideslip angle of the ET-configuration is 0.023 rad stronger and 0.2 s delayed, as compared to the TV-configuration.

#### DLC - wheel slip angle

As with the CRT in Section 7.2, the wheel slip angles  $\alpha_F$  and  $\alpha_R$  are used to classify the vehicle's behaviour. Figure 7.26 shows the steering angle  $\delta$  of the front wheels, the



type slip angle of the front wheels  $\alpha_F$  and the slip angle of the rear wheels  $\alpha_R$ . In Figure

(b) DLC - wheel slip angles - torque vectoring

Figure 7.26: DLC - wheel slip angles

7.26a the ET-configuration is displayed; the TV-configuration is shown in Figure 7.26b. At the first left turn, between 1s and 1.4s, both configurations behave similarly, with a comparable steering trajectory and a similar wheel slip angle  $\alpha_F$  and  $\alpha_R$ . After 1.5 s the two configurations begin to differ. For the ET-configuration, the front type slip angle declines to a value of -0.266 rad at 2.63 s. The maximum negative slip angle of the rear tyres reaches -0.266 rad at 2.94 s. For the TV-configuration, these maximum values are lower, at -0.177 rad at 2.84s at the front axle and -0.145 rad at 3.02s at the rear tyres. For the second left turn, both configurations behave similarly. For the ET-configuration, the maximum type slip angle of 0.17 rad is reached at 3.97 s. For the TV-configuration, the maximum wheel angle of 0.153 rad is reached at 3.65 s. Besides the maximum tyre slip angles, the reaction times between the steering angle and the tyre slip angles are intersting. With the ET-configuration, the front steering angle reaches a value of 0.05 rad at 2.89 s. The front slip angle reaches this value at 3.41 s, and the rear tyre slip angle reaches 0.05 rad at 3.45 s. The time difference from the steering angle to the front tyre slip angle is 0.52 s, and from the steering angle to the rear type slip angle 0.56 s. For the TV-configuration, the front wheel steering angle reaches 0.05 rad at 3.17 s. The front type slip angle reaches the same value at 3.45 s and the rear type slip angle at 3.45 s. The time difference between the steering angle and the front type slip angle is 0.28 s, and between the steering angle and the rear steering angle  $0.28 \,\mathrm{s}$ .

The arrows and the black line at the bottom are introduced to visualise the differences between Figure 7.26a and Figure 7.26b. The lengths of the vertical arrows indicate the different vehicle behaviours. The bottom line indicates the reduced maximum wheel slip angle  $\alpha$  of the TV-configuration. The horizontal arrows indicate the reduced time delay of the TV-configuration, compared to the ET-configuration.

#### DLC - vehicle behaviour

Using the definitions from Figure 7.15, the vehicle behaviour can be described by the terms understeer, oversteer and neutral operation. During the DLC, the ET-configuration has an understeering behaviour between 1s and 1.5s. Between 1.5s and 1.9s, the vehicle has an oversteering behaviour. Between 1.9s and 2.7s, the ET-configuration slightly understeers. Between 2.7s and 3.4s, the vehicle has a strong oversteering behaviour, which is followed by a neutral phase between 3.4s and 3.8s. Between 3.8s and 4.3s, the vehicle slightly oversteers. Afterwards, the vehicle behaves neutrally.

The TV-configuration generally behaves like the ET-configuration with phases of understeering (1 - 1.4 s), oversteering (1.4 - 1.9 s), understeering (1.9 - 2.9 s), oversteering (2.9 - 3.4 s), neutral (3.4 - 3.7 s), and oversteering (3.7 - 4.2 s). The understeering phases are similar. However, the oversteering phases are weaker with the TV-configuration. The most extreme example of oversteering is the second oversteering phase, where the maximum angle between the front and rear tyre slip angles is 0.191 rad at 3.06 s for the ET-configuration and 0.115 rad at 3.22 s for the TV-configuration. As a result the TV-configuration.

# 7.4 Test Driving: Conclusion

To analyse the lateral vehicle behaviour, several tests are performed. A general driving scenario, the constant radius turn and the double lane change are analysed. For the general driving scenario, only the function of torque vectoring is tested. A comparison with the equal torque distribution was not possible because no similar test drives were available. The constant radius turn is a static lateral test, and allows a torque vectoring vehicle to be compared with an equal torque distribution vehicle. With torque vectoring, the driver has to steer less to follow the desired circle and the velocity of the vehicle is about 1.5 kph higher. More important, if torque vectoring is activated, the inner wheel does not spin. It is possible to steer the vehicle because the TSL suppresses excessive wheel slip and type abrasion is reduced. Additionally, this test shows the reduced understeering behaviour of the vehicle with torque vectoring. The double lane change is a dynamic lateral test, which forces vehicles to oversteer. With activated torque vectoring, the driver successfully performs the DLC with an initial velocity of 65 kph. With equal torque, the driver successfully performs this test only at 60 kph. Hence, the performance and safety of the vehicle with TV are superior. Objective criteria, reflected in the yaw rate tracking and wheel slip angle analyses, show a faster vehicle response to steering requests. With torque vectoring, the vehicle behaves more neutrally. The delay time between steering wheel movement and vehicle response is reduced.

Several tests were performed with the eFuture prototype [20]. Drivers liked the quicker response of the vehicle and the smoother steering wheel feeling during curves. Unfortunately, drivers reported an undesirable oscillation of the steering wheel when they drove straight. Apart from that problem, the vehicle reacted quicker and accurately to the requests given. Straight line braking and accelerating tests were performed as longitudinal tests. No striking results were obtained there, only confirmation that the vehicle behaves as expected by the driver.

**Remark 7.1** The steering wheel oscillations of the eFuture prototype [20] are related to distributed front wheel forces and the "direct" steering column. The eFuture prototype is equipped with mechanical front wheel steering with no power-assisted steering. Vehicles with servo-power-assisted steering would create smaller feedback forces at the steering wheel. If the vehicle is equipped with active steering or steer by wire devices, this effect can be suppressed completely.

# 8 General Conclusions and Future Work

This chapter provides a conclusion to the thesis and an outlook for interesting topics that arose during this study and will be addressed in the future.

# 8.1 General Conclusions

The dynamics of an automotive vehicle are explained in Section 2. The basic equations of motion are introduced, and a three-dimensional vehicle model is generated. This model has 14 degrees of freedom and the vehicle model can move in all three axes in space, rotate around all three axes, and its suspension movement is governed. This model is used for the development of different vehicle functions in the eFuture project [20]. Additionally, the model is integrated into the driving simulator from WIVW [150]. It is possible to operate this simulator on a computer with standard computer components, or inside a real hydraulic test environment that simulates the forces acting on a driver in a real vehicle. The main challenges in these simulations are calculating type force exactly. Different tyre models from the literature are reviewed, and the Dugoff tyre model [38] and the Paceika type model [36] are calibrated following real test drives. However, the three-dimensional model is too complex to implement a torque vectoring controller based on this model. A simplified single-track model is extracted from the three-dimensional model, and this simple model is used for the controller design. After the controller synthesis, the controller is validated with the 14 degrees of freedom model for proper operation.

Before designing the torque vectoring controller, a review of actual torque vectoring controller designs is given in Section 3. Various control strategies are extracted, and the major inputs and outputs are described. Commonly used controller synthesis procedures are summarised. Additionally, the non-linearity of the single-track vehicle model is analysed, and requirements for torque vectoring are explained. Finally, an LPV based control concept is chosen because it combines the well known linear control strategies for optimal and robust control with the non-linear nature of the single-track model.

The LFT-LPV and the polytopic LPV controller designs are described in Chapter 4. Both designs use parameter-dependent plants, and the controllers calculated guarantee stability and performance. The controllers achieve similar results, and the main differences are related to different tunings of the controller. However, the LFT-LPV controller cannot be implemented here because the design uses a parameter-dependent Lyapunov function, which requires a matrix-inversion inside the microcontroller. The polytopic LPV controller obtains similar simulation results and seems preferable in terms of the numerical robustness of the solution and computation effort, and is chosen as the vehicle dynamics controller. An anti-windup scheme is developed for the LPV controller because the controller must be tuned aggressively to achieve good performance. However, an aggressive controller might drive the actuators to physical limitations. These limitations are not included in the controller design and degrade the performance of the vehicle and may even lead to instability. Additionally, the actuator limitations are not fixed and depend on many parameters, such as the velocity of the vehicle, the temperature, the power consumption, the status of the battery and efficiency requests from the driver. The proposed antiwindup scheme improves the performance of the closed-loop system as far as possible. Besides addressing the limitations of the electric machines, the anti-windup scheme is extended to the TSL, where wheel slip limitations for safe vehicle operation are included as actuator constraints. With this extension, the propulsion wheels do not spin or lock. The combination of LPV controller and TSL works nicely and improves vehicle performance. However, natural physical limits cannot be exceeded, and the vehicle's performance can only be improved within these boundaries.

Chapter 6 gives an overview of the necessary steps for implementation of the controller into an automotive microcontroller. These steps could have been directly included in Chapters 4 and 5. However, it seemed better to show the general controller development in the familiar continuous-time representation, and to use Chapter 6 for the required discretisation and fixed-point conversion. Other drivetrain functions interacting with torque vectoring are briefly described, and the range of torque vectoring activation is discussed. Finally, the implementation of a polytopic LPV controller for vehicle dynamics in a standard automotive microcontroller, satisfying AUTOSAR standards, is described.

Following validation of the virtual controller, the software is implemented in the microcontroller and tested in the vehicle. Various tests have been performed, and three relevant tests for torque vectoring are described. A general driving scenario is analysed in order to clarify torque vectoring in an average, daily drive and to illustrate proper control of the vehicle. Afterwards, a constant radius turn is reviewed to examine vehicle movement in a constant, lateral behaviour. The difference between an equal torque distribution and the torque vectoring controller is analysed. The understeering of the vehicle is improved with torque vectoring, and the driver can follow the requested trajectory more easily. The maximum possible velocity is higher with the torque vectoring controller. Excessive wheel slip is suppressed, which improves the handling of the vehicle. A double lane change is also reviewed. This test analyses the dynamic, lateral performance of the vehicle. The test driver was able to perform the test with an 8%higher initial velocity. However, the major result from this test is the faster reaction of the vehicle with torque vectoring to given steering inputs. The faster response improves the agility and "fun to drive" aspect of the vehicle. Additionally, the oversteering tendency of the vehicle is reduced, which improves the safety of the vehicle in critical manoeuvres. These three tests showed that calibrating the desired vehicle states, tuning the LPV controller, and adjusting the torque slip limiter are complex and challenging tasks.

Nonetheless, vehicle performance is positively improved with the torque vectoring controller. The wheels do not spin or lock because of the slip limitation characteristics of the TSL component. The TSL prevents motor saturation and avoids overshoots and oscillations that are related to motor saturations. The LPV controller improves the performance and safety of the vehicle by controlling the yaw rate and the longitudinal velocity. The understeering of the vehicle is reduced, which is demonstrated by the constant radius turn. The double lane change naturally forces the vehicle to oversteer but the torque vectoring controller reduced the oversteering behaviour and improved the handling of the vehicle.

The main achievements of this thesis are the development and implementation of an LPV controller for an electric drivetrain with two independent front motors. The controller can be activated during normal driving and improves the vehicle's behaviour in safety critical driving conditions. When the torque vectoring controller was activated the vehicle never skidded during several tests, whereas the vehicle skidded at least four times when using the equal torque distribution configuration, according to the author's knowledge. Additionally, the combination of an anti-windup scheme with the desired wheel slip limitation is a major result of this work. The concept of implementing limitations of model properties (wheel slip) as actuator limits rather than as model states could be interesting for other applications that are under-actuated.

## 8.2 Future Work

The main disadvantage of the proposed torque vectoring controller is the feedback of the front wheel torque difference to the steering wheel. For the constant radius turn, this feeling is helpful to the driver because it reduces the arm force needed to steer the vehicle. For the DLC or straight line driving, the feedback leads to an oscillating steering behaviour. This problem is noticeable in the prototype because no steering support is integrated, and the steering wheel is directly linked to the front wheels. With servo-assisted steering, this feedback is reduced. Active front steering or steer by wire technology would suppress this feedback successfully. This effect also vanishes completely if torque vectoring is applied to the rear wheels.

The understeering behaviour cannot be corrected for a front-wheel-drive vehicle as well as it can for a four-wheel-drive vehicle. It is beneficial to apply longitudinal wheel forces at the rear wheel if the vehicle understeers. During understeering, lateral forces of the front wheels are saturated, and additional longitudinal forces do not solve this problem and may even worsen it. Here the understeering of the vehicle is improved because the torque applied to the inside front tyre is reduced, and the outside front wheel force is increased. The outside wheel is not as crucial because the vertical load of the outside wheel is higher and the outside wheel can generate higher lateral forces. However, the understeering could be corrected more efficiently by applying longitudinal forces to the rear wheels. With oversteering, it is the other way around. Applying longitudinal forces to the front wheels. With four independent electric machines, oversteering and understeering can be corrected efficiently. Furthermore, the force difference at the front wheels can be used to improve the steering wheel feeling for the driver.

Besides the general drivetrain layout of the prototype, the tuning of the vehicle dy-

namics simulation model should be improved in the future. At present, model tuning is performed heuristically, which requires time and knowledge. Especially the tuning of the tyre forces is complicated. An appropriate identification algorithm that uses the existing sensor signals and vehicle parameters should solve this problem.

For controller synthesis, an affine LFT-LPV controller should be tested in the future. The polytopic controller achieved positive results and seemed to be the safest solution for implementing the controller because no major implementation drawbacks were visible. However, the demands on the structure of the model imply undesirable design limitations. For example, it would be beneficial to consider the air-drag of the vehicle in the controller design, without increasing the number of scheduling parameters. In this way, the LPV controller could deal directly with the disturbance. Including air drag and using a less demanding controller implementation [140] should further improve the LPV controller in terms of performance and implementation complexity. Designing more sophisticated parameter-dependent shaping filters should also lead to an optimised controller performance.

The gain-scheduled approach to TSL deals very well with motor saturations for varying driving conditions. However, stability guarantees can be improved with an LPV based TSL algorithm that is tested in simulations but so far not implemented in the real vehicle. The drawback of the TSL is the challenging calibration process. If the TSL gains are tuned "smoothly" the wheel torques do not oscillate during the constant radius turn. However, the TSL is too weak to counteract fast controller reactions in the double lane change that lead to locking wheels. Alternatively, tuning the TSL "aggressively" leads to a fast reaction in the DLC but to oscillating wheel torque in the constant radius turn. This problem might be solved if the rate of saturations is included in the anti-windup design. Hence, the calculation of anti-windup gains might be different in the future, but the combination of motor saturation and wheel slip seems to be very beneficial for torque vectoring design and will continue to be used in the future.

# **Appendix**

# A.1 Linearised Vehicle Model

The vehicle model is linearised around operating points  $v_{x_0}$  and  $\delta_0$  where  $v_{x_0}$  represents a constant longitudinal velocity and  $\delta_0$  represents a constant steering angle. These two parameters are used to calculate the steady-state lateral velocity  $v_{y_0}$  and yaw rate  $r_0$ 

$$0 = -\frac{C_{y,F} + C_{y,R}}{mv_{x_0}}v_{y_0} + \left(\frac{l_R C_{y,R} - l_F C_{y,F}}{mv_{x_0}} - v_{x_0}\right)r_0 + \frac{C_{y,F}}{m}\delta_0 \tag{1}$$

$$0 = \frac{l_R C_{y,R} - l_F C_{y,F}}{I_z v_{x_0}} v_{y_0} - \frac{l_F^2 C_{y,F} + l_R^2 C_{y,R}}{I_z v_{x_0}} r_0 + \frac{a_1 C_{y,F}}{I_z} \delta_0.$$
(2)

After defining the steady-state operating points  $\boldsymbol{x}_0 = [v_{x_0}, v_{y_0}, r_0]^T$ , the nonlinear single track model (2.13 - 2.15) is linearised around  $\boldsymbol{x}_0$  using

$$\begin{bmatrix} \Delta \dot{v}_{x} \\ \Delta \dot{v}_{y} \\ \Delta \dot{r} \end{bmatrix} = \begin{bmatrix} -c_{w}A\rho v_{x_{0}} & r_{0} & v_{y_{0}} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} \Delta v_{x} \\ \Delta v_{y} \\ \Delta r \end{bmatrix} + \begin{bmatrix} 0 & \frac{1}{m} & 0 \\ \frac{C_{y,F}}{m} & 0 & 0 \\ \frac{l_{F}C_{y,F}}{m} & 0 & \frac{1}{l_{z}} \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta F_{x} \\ \Delta M_{z} \end{bmatrix}$$
(3)  
$$a_{21} = \frac{(C_{y,F} + C_{y,R})v_{y_{0}} + (l_{F}C_{y,F} - l_{R}C_{y,R})r_{0}}{mv_{x_{0}}^{2}} - r_{0}$$
$$a_{31} = \frac{(l_{F}C_{y,F} - l_{R}C_{y,R})v_{y_{0}} + (l_{F}^{2}C_{y,F} + l_{R}^{2}C_{y,R})r_{0}}{I_{z}v_{x_{0}}^{2}}$$
$$a_{22} = -\frac{C_{y,F} + C_{y,R}}{mv_{x_{0}}} \qquad a_{23} = \frac{l_{R}C_{y,R} - l_{F}C_{y,F}}{mv_{x_{0}}} - v_{x_{0}}$$
$$a_{32} = \frac{l_{R}C_{y,R} - l_{F}C_{y,F}}{I_{z}v_{x_{0}}} \qquad a_{33} = -\frac{l_{F}^{2}C_{y,F} + l_{R}^{2}C_{y,R}}{I_{z}v_{x_{0}}}$$

Here, the air resistance force

$$F_{x,airDrag} = \frac{1}{2} c_w A \rho v_x^2 \tag{4}$$

is included for a more accurate vehicle model and to achieve a stable system. The air drag coefficient is defined as  $c_w$ , the cross-section area of the vehicle as A and the air

density as  $\rho$ . The linearised vehicle model is defined as

$$\boldsymbol{x} = \boldsymbol{x}_0 + \Delta \boldsymbol{x} \tag{5}$$

$$\Delta \dot{\boldsymbol{x}} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{x}} \Big|_{0} \partial \boldsymbol{x} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{u}} \Big|_{0} \partial \boldsymbol{u}$$
(6)

which results in the linearised state space model

$$\Delta \dot{\boldsymbol{x}} = A \Delta \boldsymbol{x} + B \Delta \boldsymbol{u} \tag{7}$$

$$\boldsymbol{y} = C\left(\boldsymbol{x}_0 + \Delta \boldsymbol{x}\right) + D\left(\boldsymbol{u}_0 + \Delta \boldsymbol{u}\right).$$
(8)

# A.2 Practical Stability

The described stability norms are important for the controller design. However, it is important to keep in mind that all these stability constraints rely on an accurate plant model. Here, a single track model is used as plant model and this model is valid for the most conditions. However, there are cases where the model is not valid any more and all stability guarantees are lost. These conditions are reverse driving or very bad road conditions like icy roads. More complicated vehicle models are developed for these cases but these complex models can not be used for controller design because they lead to too complex controllers. For a practical solution, the controller is based on a simple model and various simulations are used to validate the controller performance for different driving scenarios. Tests are listed in Table A.1. Additionally, the vehicle stability was

	-
Name	Norm
Straight line acceleration	related to ISO 21994
Straight line braking	ISO 21994
Constant radius turn	ISO 4138
Lift off oversteer	ISO 9816
Brake in Bend	ISO 7975
Accelerate in Bend	related to ISO 7975
Sinus with Dwell	NHTSA recommendation
Double lane change	ISO 3888-2
Simulator driving	Random
Failure: straight line	
Failure: constant radius turn	

Table A.1: List of simulation manoeuvres for torque vectoring

analysed in the side slip - yaw rate plain, as recommended by [102]. Therefore, a singletrack model and a Pacejka tyre model are used with various torque vectoring and steering wheel interactions.

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## Acronyms

- $\mathbf{CO}_2$  carbon dioxide
- **ABS** anti-lock braking system
- $\boldsymbol{\mathsf{ACC}}$  adaptive cruise control

 $\ensuremath{\mathsf{ADAS}}$  advanced driver assistance systems

 $\ensuremath{\mathsf{ARS}}$  active roll stabilisation

 $\ensuremath{\mathsf{ASS}}$  active suspension system

 $\boldsymbol{\mathsf{AWC}}$  anti-windup compensator

**BEV** battery electric vehicle

 ${\sf BEVs}$  battery electric vehicles

 ${\sf BRL}\,$  bounded real lemma

**CoG** centre of gravity

- $\ensuremath{\mathsf{CRT}}$  constant radius turn
- **DLC** double lane change
- $\ensuremath{\mathsf{DTM}}$  dual-track model
- $\ensuremath{\text{DU1}}$  command decision unit
- $\ensuremath{\text{DU2}}$  drive-train decision unit
- ${\sf DVG}$  desired value generator
- **EBA** emergency brake assistant
- ${\sf EM}$  electric machine
- $\ensuremath{\mathsf{ESC}}$  electronic stability control
- $\ensuremath{\mathsf{ESS}}$  electric storage system

<b>ET</b> equal torque
<b>EV</b> electric vehicles
EVs electric vehicless
<b>FCEVs</b> hydrogen fuel cell electric vehicles
<b>FL</b> front left
<b>FR</b> front right
${\it gridding-based}\ LPV$ gridding-based linear parameter-varying
<b>HEV</b> hybrid electric vehicle
<b>HEVs</b> hybrid electric vehicles
<b>ICE</b> internal combustion engine
<b>ICEs</b> internal combustion engines
<b>LDW</b> lane departure warning
$\label{eq:linear} \textbf{LFT-LPV} \ \text{linear fractional transformation linear parameter-varying}$
LHP left half plane
<b>LKAS</b> lane keeping assistance system
<b>LMI</b> linear matrix inequality
<b>LPV</b> linear parameter-varying
<b>LQG</b> linear-quadratic-Gaussian
<b>LQR</b> linear-quadratic regulator
<b>LTI</b> linear time-invariant
MIMO multi input multi output
<b>NODS</b> near object detection system
<b>P</b> proportional
<b>PI</b> proportional-integral
<b>PID</b> proportional-integral-derivative

 $polytopic \ LPV$  polytopic linear parameter-varying

RL rear left

 $\boldsymbol{\mathsf{RR}}\xspace$  rear right

**SISO** single input single output

 $\textbf{STM} \ {\rm single-track} \ {\rm model}$ 

 $\textbf{T1} \ \mathrm{turn} \ \mathrm{one}$ 

 $\textbf{T2} \ \mathrm{turn} \ \mathrm{two}$ 

**TCS** traction control system

 $\ensuremath{\mathsf{TSL}}$  motor torque and wheel slip limiter

 $\boldsymbol{\mathsf{TV}}$  torque vectoring

VehObs vehicle observer

 ${\bf ZOH}$  zero-order-hold

## List of symbols

- $A(\theta)$  Parameter dependent A-matrix
- $K(\theta)$  Parameter dependent controller
- $a_y$  Lateral acceleration of the vehicle
- $\alpha~$  Tyre slip angle
- $\alpha_F$  Front type slip angle
- $\alpha_R$  Rear tyre slip angle
- B Input matrix of the system
- $\beta$  Sideslip angle of the vehicle
- $C_x$  Longitudinal type stiffness
- $C_{y,F}$  Cornering stiffness of the front axle
- $C_{y,R}$  Cornering stiffness of the rear axle
- $\dot{\omega}$  Angular acceleration
- $\delta$  Steering angle of the front wheel
- $F_{\sf lin}$  Linear, longitudinal type force acting onto the vehicle
- $F_{\text{sample}}$  sampling factor for system discretisation
- $F_x$  Longitudinal force acting onto the vehicle
- $F_x^w$  Longitudinal force in the wheel coordinate frame
- $F_{y}^{w}$  Lateral force in the wheel coordinate frame
- $F_z$  Vertical force
- $F_z^w$  Vertical force in the wheel coordinate frame
- $G(\theta)$  Parameter-dependent state space system
- $\mathbf{H}_2$  H two norm
- $\boldsymbol{\mathsf{H}}_\infty~\operatorname{H}$  infinity norm

- $\gamma$  Limit of the H<sub> $\infty$ </sub> performance index
- $I_w$  Moment of inertia of the wheel around the turning axis
- $I_z$  Moment of inertia around the vertical vehicle axis
- $K(\theta)S(\theta)$  Parameter-dependent, closed-loop, output control sensitivity
- $l_F$  Distance between the CoG and the front axle
- $l_R\,$  Distance between the CoG and the rear axle
- $\lambda$  Longitudinal wheel slip
- $\lambda_{FL}$  Longitudinal wheel slip of the front left wheel
- $\lambda_{FR}$  Longitudinal wheel slip of the front right wheel
- m Mass of the vehicle
- $M_z$  Yaw moment around the vertical axis of the vehicle
- $M_z^w$  Moment around the vertical axis of the wheel coordinate frame
- $\mu$  Road surface adhesion coefficient
- $\omega$  Angular velocity
- $\mathcal{P}$  convex polytope of the parameter space
- R Effective type radius
- r Yaw rate of the vehicle
- $r_{des}$  Desired yaw rate of the vehicle
- $\mathcal{L}_2$  induced  $L_2$  norm of the nonlinera system
- $S(\theta)$  Parameter-dependent, closed-loop output sensitivity
- $T_{cl}$  Closed loop system, including plant and controller
- $T_s$  Discrete sample time
- $\theta\,$  Scheduling signal of an LPV system
- $v_x$  Longitudinal velocity of the vehicle
- $v_x^w$  Longitudinal velocity in the wheel coordinate frame
- $v_y$  Lateral velocity of the vehicle
- $v_u^w$  Lateral velocity in the wheel coordinate frame

- $W^{C}\,$  Closed-loop control sensitivity shaping filter
- $W^C(\theta)$ Parameter-dependent shaping filter for control sensitivity
- $W^{S\,}$  Closed-loop sensitivity shaping filter
- $W^S(\theta)$ Parameter-dependent shaping filter for sensitivity
- x State of the control system