

Bachelor Thesis

**Application of the Strain Energy Density
method to butt-welded joints**

**Bewertung der Schwingfestigkeit von
Stumpfstoßverbindungen mittels der
Formänderungsenergiedichte**



Atilla Acar
February 25, 2021

Institution:	M-10 - Institute for Ship Structural Design and Analysis
First supervisor:	Prof. D.Sc. (Tech.) Sören Ehlers
Second supervisor:	Moritz Braun M. Sc.
Institution:	TÜV Nord
Supervisor:	Dr.-Ing. Claas Fischer

Selbstständigkeitserklärung

Hiermit erkläre ich, Atilla Acar, Matrikelnummer 21596422, dass ich die vorliegende Arbeit selbstständig und ohne fremde Hilfe verfasst und keine anderen Hilfsmittel als angegeben verwendet habe. Insbesondere versichere ich, dass ich alle wörtlichen und sinngemäßen Übernahmen aus anderen Werken als solche kenntlich gemacht habe.

Hamburg, 25. Februar 2021

Atilla Acar

Contents

1	Introduction	1
2	Strain Energy Density	3
2.1	V-type notches	3
2.2	Half circle	4
2.3	Rounded notches	5
3	Meshing strategies for butt-welded joints	8
3.1	V-type notches	8
3.2	Half circle	10
3.3	Rounded notches	13
4	Fatigue assessment for butt-welded joints	15
4.1	V-type notches	15
4.1.1	Analysis for mirrored Case	15
4.1.2	Analysis for variable opening angles for only bottom side	16
4.1.3	Analysis between different distances and varying opening angle for bottom side	18
4.2	Half circle	19
4.2.1	Analysis for mirrored Case	19
4.2.2	Analysis for variable opening angles for only bottom side	21
4.2.3	Analysis between different distances and varying opening angle for bottom side	22
4.3	Rounded notches	24
4.3.1	Analysis for mirrored Case	24
4.3.2	Analysis for variable opening angles for only bottom side	26
4.3.3	Analysis between different distances and varying opening angle for bottom side	27
5	Comparison of the results	30
5.1	V-type notches comparison with Song's values	30
5.2	Comparison full surrounded with half circle surrounded	30
5.3	Rounded notches	31
5.3.1	Comparison small to large roundings	31
5.3.2	Analytical solution compared with numerical solution for roundings	34

6	Discussion	36
6.1	V-type notches	36
6.2	Half circle	37
6.3	Rounded notches	37
7	Conclusion	39
7.1	V-type notches	39
7.2	Half circle	39
7.3	Rounded notches	40
7.4	Correlation between the SED and the parameters in the model	41
8	References	42
9	Appendix	44

Nomenclature

2α	Opening angle of the notch
$2\alpha_b$	Bottom side opening angle of the notch
$2\alpha_t$	Top side opening angle of the notch
\bar{W}	Strain energy density
ad	Absolute difference between two SED values
d	Distance to the notch tip
d_b	x-Distance bottom side to the notch tip
d_t	x-Distance top side to the notch tip
f	Possible correction factor for the new suggested half circled control volume model
H^*	Factor to calculate the SED analytically
pd	Percentage difference between two SED values
R	Rounding radius
r_0	Correction parameter for the control Volume radius for rounded notches
R_2	New control volume radius for rounded notches
R_c	Control volume radius
R_c/R	Normalised radius
$R_{hc,new}$	new suggestion for the radius of the half circled control volume
R_{nZ}	Notch zone radius for refinement
r_{pos}	Positioning distance for the new suggested half circled control volume from notch tip to the center of the half circle
wss	welding seam superelevation

List of Tables

1	Analytical calculation for the SED of the rounded notches in a mirrored butt-welded joint	6
2	SED for V-Type notches with mirrored geometry	15
3	SED for V-Type notches with $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying	17
4	SED for V-Type notches with $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d	18
5	SED for half circled control volume notches with mirrored geometry	20
6	SED for half circled control volume notches with $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying	21
7	SED for half circled control volume notches with $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d	23
8	SED for rounded notches with mirrored geometry for different roundings R	24
9	SED for different rounded notches, $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying	26
10	SED for different rounded notches, $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d	28
11	Comparison of the SED for V-Type notches	30
12	Comparison of the SED for half circled control volume with the full surrounded control volume	31
13	Comparison of the SED for rounded notches for increasing rounding radii R for different opening angles 2α	32
14	Comparison of the SED between analytical and Ansys model solution	34

List of Figures

1	V-Type notches with different modelling strategy	9
2	V-Type notch for full surrounded free mesh	10
3	V-Type notches for subdivided control volume mesh	10
4	V-Type half circled control volume	11
5	V-Type notches for half circle control volume mesh	12
6	Rounded notch for $2\alpha = 135^\circ$, rounding radius R , new control volume radius $R_2 = R_c + r_0$ with $r_0 = R \frac{(\pi - 2\alpha)}{(2\pi - 2\alpha)}$	13
7	Meshing structure of a butt welded joint with rounded notches	14
8	SED for V-Type notches with mirrored geometry	16
9	SED for V-Type notches with $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying	17
10	SED for V-Type notches with $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d	19
11	SED for half circled control volume notches with mirrored geometry	20
12	SED for half circled control volume notches with $2\alpha_t = \text{con-}$ $\text{stant} = 135^\circ$ and $2\alpha_b$ varying	22
13	SED for half circled control volume notches with $2\alpha_t = \text{con-}$ $\text{stant} = 135^\circ$, $2\alpha_b$ varying and different distances d	23
14	SED for rounded notches with mirrored geometry for dif- ferent roundings R	25
15	SED for different rounded notches, $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying	27
16	SED for different rounded notches, $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d	28
17	Comparison of the SED for rounded notches for increasing rounding radii R for different opening angles 2α	33
18	Comparison analytical solution (A) with numerical solution (N)	35

1 Introduction

This research investigates whether and how the Strain Energy Density (SED) method can be applied for different models of butt-welded joints, especially for rounded V-type notches with bigger rounding radii R so that they can be called rounded notches now and for which the approximation as a V-type notch is no longer useful, but for which, despite all this, a notch effect is still noticeably large and this must therefore be taken into account for the fatigue assessment. Conversely, this also means that for really small rounding radii R an approximation as a V-type notch is possible. This research is important because the fatigue assessment for complex structures as ships and buildings is demanding. Therefore the entire structures of ships and buildings should be analyzed in it's substructures, which, among other things, consists of butt-welded joints, to simplify the fatigue assessment. Also it is tested out how to get accurate and fast numerical SED solutions with meaningful modelling and meshing strategies in Ansys Mechanical APDL.

The SED method will generally help to describe the fatigue assessment of notched (welded) joints and it's approach is based on the N-SIF (Notch Stress Intensity Factor) concept, which was introduced by Gross and Mendelson in 1972 [8]. For the SED calculation the local strain energy around the notch is averaged over a small size control volume with the radius R_c , which can be calculated for welded joints [7, 11] and therefore it is possible to automate the calculation for different opening angles 2α with numerical models as in Ansys Mechanical APDL. One of the main advantages of this SED approach compared to the N-SIF concept is that it just needs a high density of defined elements in it's control volume and can be coarse meshed in the remaining structure, which will lead to fast and accurate solutions [1, 12]. This is possible because the SED is calculated with the nodal displacements of the control volume and because it's center is being placed in the notch tip, which is also the point of singularity, it will lead to a fast solution in which every important node is involved[1, 6].

There are different possible variations to surround the V-type notch with the control volume and two of them are examined in this research. One of them is the fully surrounding control volume, which is a circle sector centered in the notch tip and the other one is a half circle surrounding control volume, which is placed in the bisecting line of the notch centered in his notch tip. The point here is that less nodes are involved in the calculation so that the calculation time is reduced. The main idea of

the hypothesis is that the SED's accuracy for the notch tip will increase, because the expected crack path could be around the bisecting line of the notch tip in which the half circle control volume will be placed and because of that the nodes along the notch tip have minor influence for the critical crack formation, which is the reason why they will be excluded[15, 16].

Two different models for the V-type notches with a fully surrounding control volume are analyzed here. The first one is the subdivided control volume and the second one is the freely meshed one. The results for the SED should be equal as it was already investigated by Fischer et. al [14]

Furthermore the SED calculations for rounded notches are also investigated. This roundings are made with adding a rounded area piece to the V-type notch. Also it shall be possible to edit the rounding radius R . There are already investigations for rounded notches in welded joints by Berto and Lazzarin [1], which will be considered here. The models should result for the SED that for rounded notches the SED is lower than for V-type notches and that for increasing rounding radii R the SED should decrease, because the point of singularity disappears for big rounding radii.

It is important to mention that just the right side of the butt-welded joint is modelled here because the left and right side is mirrored and it is simplified into a plane view to decrease the modelling effort. Also it is distinguished between top and bottom side of the butt-welded joint because the top and bottom side's weld parameters are each editable to generate different configurations of butt-welded joints. So that the SED is calculated separately for each side.

In total there are four different models to calculate the SED for the butt-welded joints, which are the V-type notches for full surrounded control volume with subdivided and freely meshed variation, the V-type notches with a half circle control volume variation and the rounded notches.

2 Strain Energy Density

2.1 V-type notches

In this section the analytical calculation of the SED for V-type notches is explained and for which areas of application the modelling can be already used.

The main idea of the SED method is to fully surround the notch's or crack's tip with a small size control volume to calculate the strain energy for each finite element in it's control volume to sum them up and then to divide them with the control volume sum to calculate the averaged total elastic SED [2, 3]. The SED has to been averaged in this way for the notch tip because the SED can not be calculated for a singularity like the V-type notch tip, because the stress and the SED itself would tend towards infinity [2].

So the next step for the SED calculation itself is to set an optimal control volume radius R_c which can change for different materials. There are already different investigations about the control volume radius R_c for V-type notches and that for steel welded joints $R_c = 0.28\text{mm}$ is a good approximation [7, 1, 18]. It is worth to mention that Livieri and Lazzarin found out for steel welded joints with failures formed from it's weld toe for V-type notches with opening angles 2α around 135° that only Mode I was significant for the N-SIF calculation, because the made up stress distributions are non singular for this case and with this researches it was possible to determine the mean N-SIF under Mode I value to calculate the control volume radius R_c [5].

It is important to mention that whenever using the SED method in Ansys it is important to refine the finite elements (FE) in the control volume to have enough FEs to approximate the actual value [7, 8, 14]. It is also tested out whether $R_c / 2$ is a good refinement value for the element size in the control volume of a butt-welded joint, because it was already fine for a transverse fillet-welded joint like in Fischer's et al model [14]. Also one important note is that it has been pointed out by Fischer et al. that generally misalignments effects for butt-welded joints, should be considered here when calculating the S-N Curves, because due to manufacturing issues and the welding process many axial and angular misalignments will lead to secondary bending stresses for axially loaded joints [4]. This misalignments will not be considered in this paper, because the S-N curves

won't be calculated.

Furthermore this SED method has been already applied for different cases, such as high-cycle fatigue under uniaxial and multiaxial loading, in which the result was that it doesn't matter which loading type it has because the critical SED value will be the same for both [18]. Also the method was proven for brittle fractures [15]. One other interesting investigation was the SED calculations for steel butt-welded [7] and fillet-welded joints [13] under sub-zero temperatures to simulate arctic conditions.

The analytical calculation for the total elastic SED \bar{W} averaged over the control volume area (it is an area because it was simplified into a plane view) is based on the leading order terms of William's solution and is calculated as shown in the following equation [7, 10]:

$$\bar{W} = \frac{e_1}{E} \left[\frac{K_1}{R_c^{1-\lambda_1}} \right]^2 + \frac{e_2}{E} \left[\frac{K_2}{R_c^{1-\lambda_2}} \right]^2 \quad (1)$$

E is the Young's modulus which is given for different materials and e_1 and e_2 are correction factors, which depend on the stress- strain field, notch opening angle 2α and the Poisson's ratio [7]. The values for this are already listed in the literature for different important 2α . λ_1 and λ_2 are the eigenvalues of the Williams' stress field solution for the N-SIF [7] and also for this literature values are given like for the correction factors. For V-type notches in butt-welded joints the analytical calculation of K_1 and K_2 which are the N-SIF under loading modes I and II is considered a demanding ask to solve because they have to be integrated [11] or solved numerically with the computer. The easier way is to get the numerical solution. But if the computer is calculating the numerical solution, it could also directly determine the SED. Therefore the SED for V-type notches is not calculated analytically. But the Ansys SED values are compared with a different model which is by S. Song for butt-welded joints.

2.2 Half circle

In this section the half circle surrounded V-type notches are considered.

As already explained the half circle control volume is centered in the bisecting line of the notch tip and surrounding the notch tip just as a half circle so that the expected less important nodes for the crack formation,

which should be along the notch tip aren't considered in this variation's SED calculation [15, 16]. In former investigations by Radaj et al. this variation has been already analyzed for placing the half circle control volume into the expected direction of the crack propagation, which can be determined with the Erdogan-Sih maximum tangential stress criterion and they have found out that in most cases the full surrounding circle should be used to determine the SED and just for exceptional cases when the total SED's angular distribution is one sided the half circle control volume can be more advantageous than the full surrounding control volume [15, 16].

Reminding that the half circle control volume is just useful in exceptional cases it has to been shown whether the model is applicable for the V-type notches when considering opening angles $90^\circ \leq 2\alpha \leq 165^\circ$. And also the analytical SED calculation for this half circle control volume is analogue to the full surrounding control volume and therefore just the numerical solutions for the SED values of the variations are compared to each other.

2.3 Rounded notches

In this section the rounded notches with big enough rounding radii R , whose SED calculations can not be approximated with the calculation for V-type notches anymore, are considered here.

The investigations for the rounded notches, which are also called blunted V-shaped notches, are mainly based on the researches by Berto and Lazzarin [1] and there are not many researches about the rounded notches modelled with Ansys Mechanical APDL. But the state of the art for the rounded notches in general is that it is possible to calculate accurate SED values for rounded notches [15]. Also it is important to mention that the rounding will be realized with a circular arc. But other rounding strategies are also possible as a hyperbolic arc, which is generally useful to round other types of notches like sharp opening angle V-type notches or undercuts [17].

The main point in modelling the rounded notches is that the control volume has to been placed in a new position with an adapted new control volume radius of $R_2 = R_c + r_0$ [1]. This is to make sure to not underestimate the SED. Also there is already a way to calculate the SED for rounded notches under mixed mode loading with the following equation which

was defined by Berto and Lazzarin [1]:

$$\bar{W} = H^* \left(2\alpha, \frac{R_c}{R} \right) \times \frac{\pi \sigma_{max}^2}{4E} \quad (2)$$

The H^* function is depending on the notch's opening angle 2α , the poisson ratio ν , which is set as $\nu=0.3$ in this research and the normalised radius R_c/R between the former control volume radius $R_c=0.28\text{mm}$ and the rounding radius R . The values for the H^* function are already listed by Berto and Lazzarin [1] and will be used for the calculations. σ_{max} is the maximum value for the principal stress along the rounded notch, which has to be output with Ansys for every calculation. When outputting the principal stress it is important to get the principal stress for nodes and not for elements. E is again the Young's modulus, which is set as $E=206000\text{MPa}$ in this research.

The possible calculations for the SED for rounded notches for $2\alpha \geq 90$ are calculated with the equation (2) and are compared to the numerical SED solutions in the later section. The geometry of the butt-welded joint is mirrored and an initial press of 100MPa is set on the right side of the butt-welded joint, so that the SED results for top and bottom side are equal.

2α [°]	2α [rad]	R_c/R	H^*	σ_{max} [MPa]	\bar{W} [MJ/m ³]
90	$\pi/2$	0.3	0.3296	228.119	0.06539
		0.5	0.2361	267.448	0.06439
120	$2\pi/3$	0.1	0.392	166.450	0.04141
		0.3	0.2578	227.557	0.05090
		0.5	0.1851	264.788	0.04948
135	$3\pi/4$	0.1	0.3206	166.588	0.03392
		0.3	0.2082	225.277	0.04028
		0.5	0.1572	261.476	0.04098

Table 1: Analytical calculation for the SED of the rounded notches in a mirrored butt-welded joint

In the table (1) it can be seen for the analytical solution of the SED that for an increasing opening angle 2α under a given normalised radius R_c/R the values are decreasing and for a constant opening angle but increasing normalised radius R_c/R , which does also mean that the rounding radius R is decreasing for a given control volume radius R_c , the SED values are increasing.

This analytical solution will be even further explained in the section 5.3.2 when comparing to the Ansys model results.

3 Meshing strategies for butt-welded joints

3.1 V-type notches

In this part of the research C. Fischer's and M. Braun's Ansys Mechanical APDL codes for transverse fillet-welded joints were analyzed to write a new one for butt-welded joints with a subdivided control volume. Also Fischer's et al. recommendations for the numerical analyses of the SED was considered here. [14]

The first variation of this model is the full surrounding subdivided control volume. It's segmentation of the subdivided control volume is in a ratio of 15° . This variation is compared to an equal model without a subdivided control volume, so that it is freely meshed and also full surrounding the control volume. As already explained this models are compared with the butt-welded joint model by S. Song.

Now the modelling is explained.

First of all the keypoints for the mirrored half of a butt-welded joint's structure with an opening angle $2\alpha = 90^\circ$ for both notch sides is figured out. After that the code's parameter can be manipulated in that way that the angles, plate thicknesses and distances for top and bottom side are individually editable to calculate the SED for different cases. After the general structure is created, a control volume area with the radius R_c is created at the notch tip. This control volume's area is a circular section that is fully surrounding the notch with the area of $A_c = R_c^2(\pi - \alpha)$. The angle should be input as rad. Also there is an outer circular section with the radius R_{nZ} to refine the FEs around the control volume area as recommended by Fischer et al. [14]. The calculation of it's area is analogue to the control volume area, but for the calculation it has no relevance. Besides the SED calculation are done with the recommended $R_c = 0.28\text{mm}$ for welded joints as explained in 2.1.

This modelling strategy is working for both variations. But in the variation with the subdivided control volume there is one bigger circle section to connect the control volume's FEs with the remaining model. The idea behind of this is to avoid black spots in the structure and to connect the nodes flawless. And the the main difference in generating the subdivided control volume is that it is done with a do loop in a range of 15° . But all in all the geometry for both versions is the same.

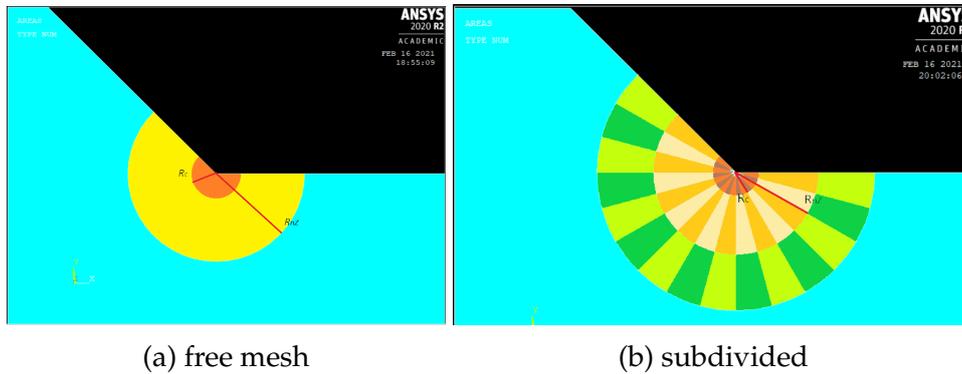


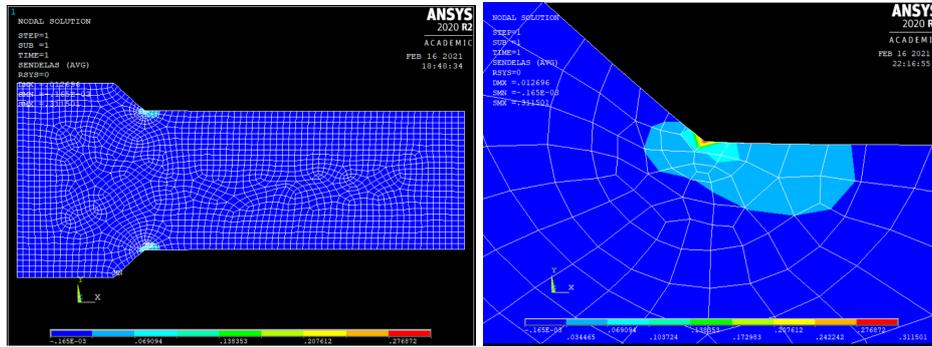
Figure 1: V-Type notches with different modelling strategy

After this the area is ready to be meshed and the result of that is a flawless mesh without any black spots. Ideally the control volume should be meshed with a higher density to generate more FEs as mentioned like before [7, 8, 14] and the remaining part can be meshed in a less dense way to minimize the computational effort.

Now the boundary conditions are written down. In this research a press of 100 MPA on the butt-welded joint's right side ending has been set and the left side of the butt-welded joint is cut off. Therefore the right side is fixed on that cut off.

The next step is to solve and evaluate the SED from the notch tip to R_c . Therefore the meshed nodes in that area are selected by a command. The ideal way is to use a local coordinate system to make the code as easy as possible. After that the SED and the volume (even if it is an area in the plane view the command is still the volume) for each FE in the control volume area is calculated and added up until that area is completed. To finish this calculation and to get a mean SED value the SED sum has to be divided by the volume sum. The top and bottom SED calculations work the same way.

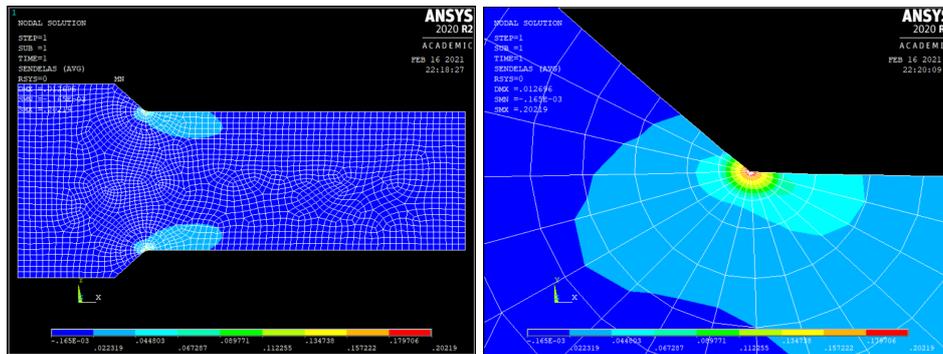
The SED values are calculated for different cases for top and bottom side notch, which will be shown in 4.1. Also the values are compared to Shi Song's model's SED values. The idea here is to check up whether the values are nearly equal or differ from each other. Also it shall be figured out whether it is needed to subdivide the control volume in pieces.



(a) butt-welded joint

(b) top notch close up

Figure 2: V-Type notch for full surrounded free mesh



(a) butt-welded joint

(b) top notch close up

Figure 3: V-Type notches for subdivided control volume mesh

In the following part for the fatigue assessment different cases for the SED calculations are analyzed. There the opening angles and the distances will vary. The figures (2) and (3) are meshed for $2\alpha = 135^\circ$ and for top and bottom's notch tip distance $d = 8.66025\text{mm}$.

3.2 Half circle

This model works like the free meshed notch model just with the difference that the control volume is not surrounding the whole notch. The half circle control volume is being placed into the opening angle's bisector. The calculation difference is that the selected area is that half circle instead of the whole surrounding.

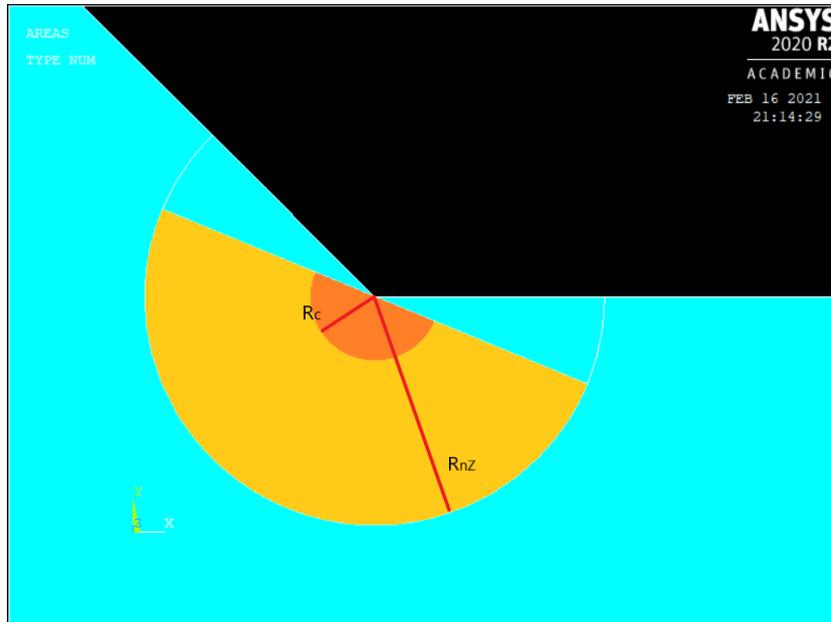
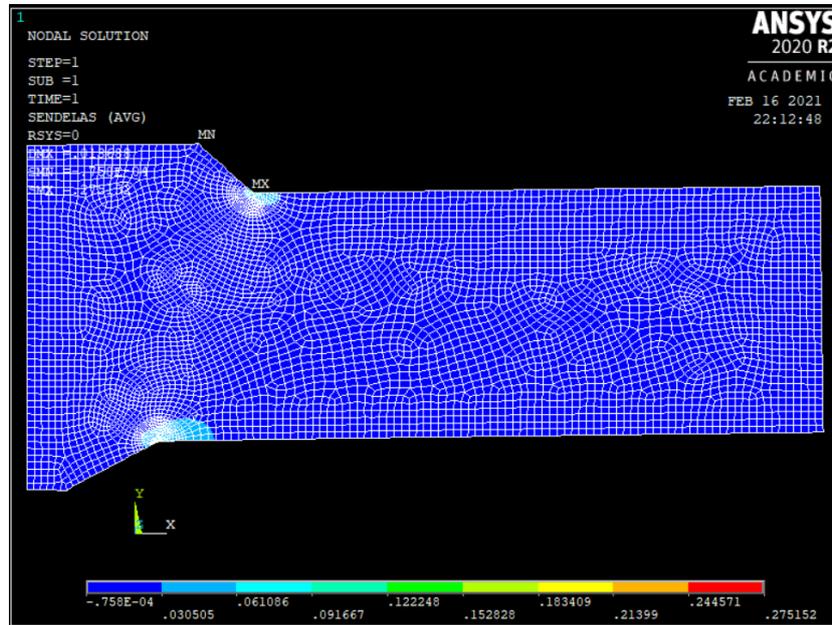
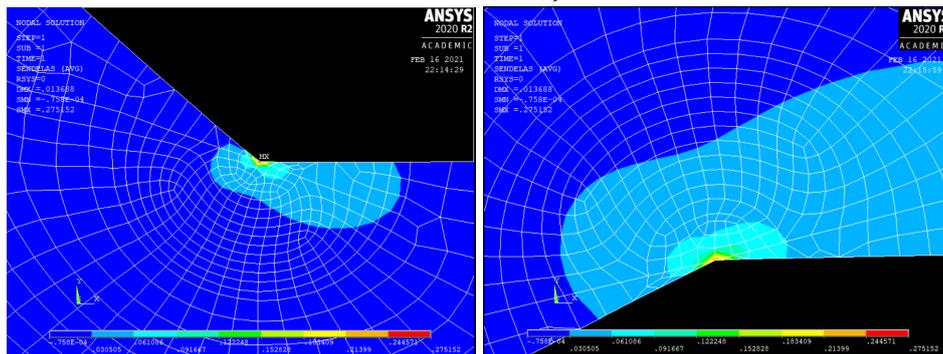


Figure 4: V-Type half circled control volume



(a) butt-welded joint



(b) top notch close up

(c) bottom notch close up

Figure 5: V-Type notches for half circle control volume mesh

In the figures (5) the mesh structure for the half circle control volume can be seen. For the top side the parameters are $2\alpha_t = 135^\circ$ with a notch tip distance $d_t = 8.66025\text{mm}$ and for the bottom side the parameters are $2\alpha_b = 150^\circ$ with a notch tip distance of $d_b = 5\text{mm}$.

The SED values between the full surrounded control volume and the half circle values are compared to each other in 5.2. The values for full surrounded control volume are the same as for the free mesh variation. Therefore the values are just copied down for each case.

3.3 Rounded notches

The outline for a butt-welded joint with a rounded notch and the V-type notch are the same. The difference is that an area is added to the notch tip on top and bottom side of the butt-welded joint to generate a rounding. To make this possible the contact points are calculated, which are in dependence of the opening angle 2α . Then the circle segment of the radius is added. The angle of the circle segment is $180^\circ - 2\alpha$. The circle segment are fully surround the notch, because it's tangentials are intersecting the notch tip and are overlapping with the lines along the notch edge. After that the area between notch and circle segment can be generated with selecting that keypoints and line which were defined before and to finish this the circle segment area has to be deleted, so that only the rounding is left. The control volume area has to be moved outside of the rounding and the control volume radius is adopted R_2 in the same way as Berto and Lazzarin have presented it [1]. The following figure presents all the rounding parameters.

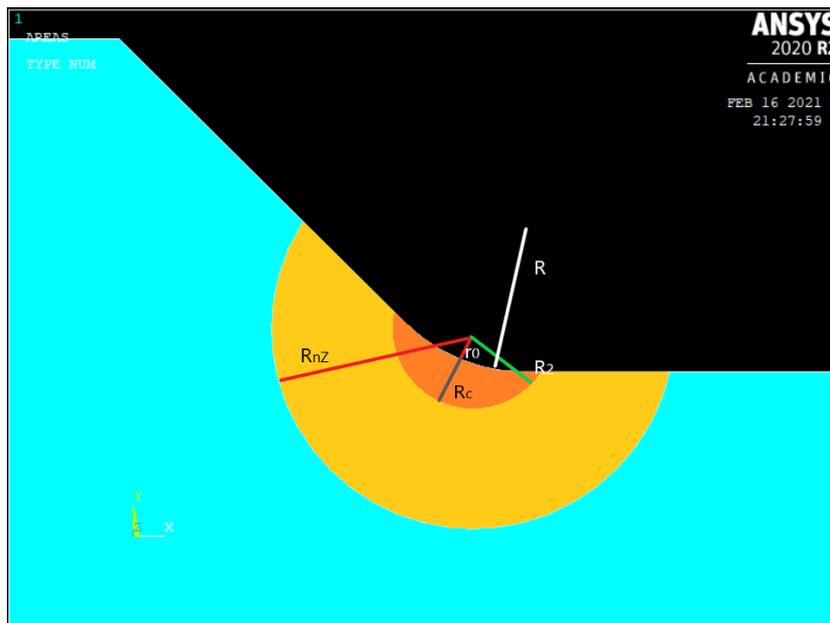
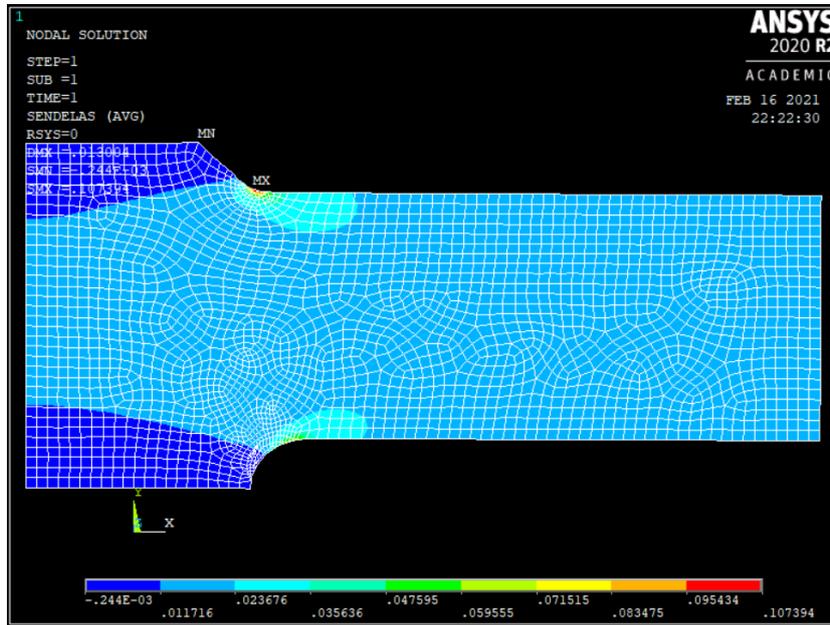
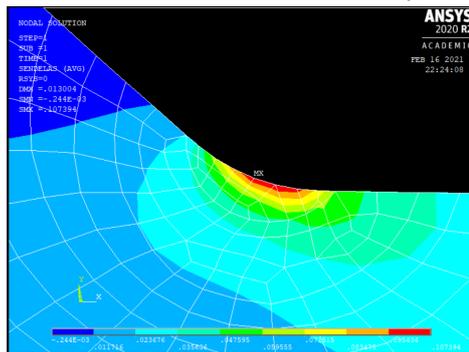


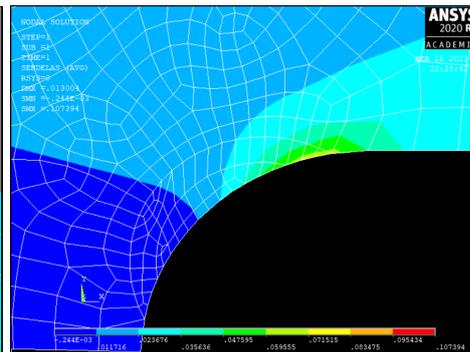
Figure 6: Rounded notch for $2\alpha = 135^\circ$, rounding radius R , new control volume radius $R_2 = R_c + r_0$ with $r_0 = R \frac{(\pi - 2\alpha)}{(2\pi - 2\alpha)}$ adopted from Berto and Lazzarin ¹.



(a) butt-welded joint with rounded notches



(b) top rounding close up



(c) bottom rounding close up

Figure 7: Meshing structure of a butt welded joint with rounded notches

The figure (7) shows how the meshing structure of the butt welded joint with rounded notches and the closed up control volumes of it looks like. In this case the distances to the former notch tip for top and bottom are equal $d = 8.66025\text{mm}$ and the parameters for top side rounding is $R = 1\text{mm}$ and $2\alpha_t = 135^\circ$ and for bottom side $R = 2\text{mm}$ and $2\alpha_b = 90^\circ$. Here it can also be seen, cause of the rounding, that the welding seam superlevation $w_{ss} = 2\text{mm}$. Therefore there will be no results for $R > 2\text{mm}$ when having this opening angle. This parameter is editable but it makes no sense in general to have larger values for the welding seam superlevation w_{ss} when the weld seam itself is only 5mm high without the superlevation.

4 Fatigue assessment for butt-welded joints

In this section it shall be investigated how the SED on the notch is interacting for different opening angles, distances and roundings for the different models. Also there are analyzed three different cases for all of them.

4.1 V-type notches

4.1.1 Analysis for mirrored Case

This case is that top and bottom is mirrored to check up whether the calculation is working equally. This is calculated for opening angles $90^\circ \leq 2\alpha \leq 165^\circ$. For smaller angles it makes in general no sense because the welding seam superelevation w_{ss} should not protrude out of the notch tip's vertical line. The x-distance from the middle of the half of a butt-welded joint to the notch tip is $d = 8.66025\text{mm}$. This distance is for a half welding seam of 60° and a height of 5mm. The plate thickness is 5mm for top and bottom side and the welding seam superelevation $w_{ss} = 2\text{mm}$ for both sides. The control volume radius and notch zone radius is set as $R_c = 0.28\text{mm}$ and $R_{nZ} = 1\text{mm}$.

2α [°]	Song's \bar{W} [MJ/m ³]		\bar{W} for subdivided [MJ/m ³]		\bar{W} for free mesh [MJ/m ³]	
	top	bottom	top	bottom	top	bottom
-						
90	0.06642	0.06642	0.05776	0.05776	0.05774	0.05775
105	0.07013	0.07013	0.06101	0.06101	0.06098	0.06099
120	0.07314	0.07310	0.06389	0.06388	0.06385	0.06385
135	0.07268	0.07279	0.06401	0.06402	0.06399	0.06399
150	0.06450	0.06449	0.05740	0.05740	0.05737	0.05738
165	0.04572	0.04553	0.04107	0.04107	0.04106	0.04106
∅	0.06543	0.06541	0.05752	0.05752	0.05750	0.05750

Table 2: SED for V-Type notches with mirrored geometry

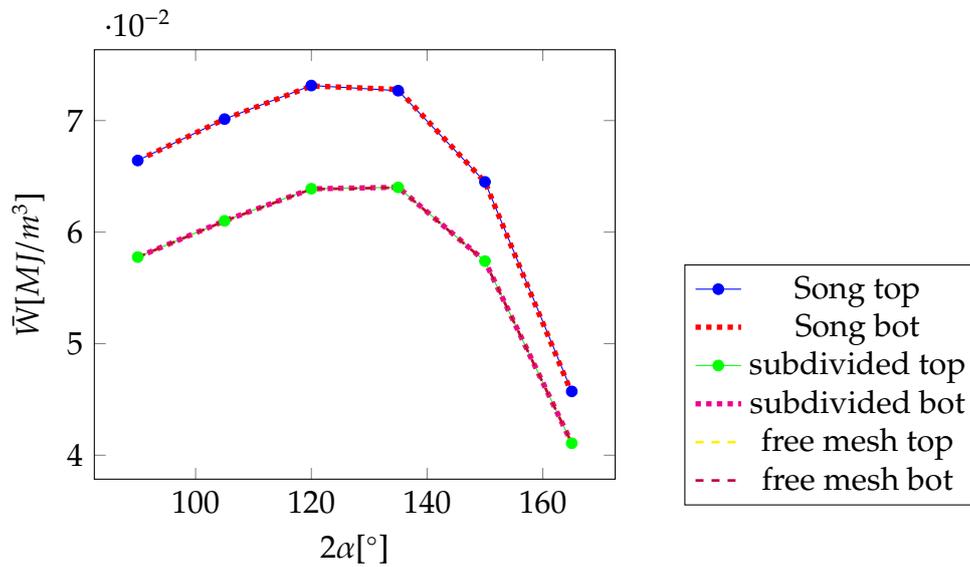


Figure 8: SED for V-Type notches with mirrored geometry

There are differences between Song's SED values and the new models' SED values. It can be seen that Song's SED values are a bit bigger but kind of parallel to the new ones' values. Also it can be seen that there is like no difference between subdivided control volume and a control volume with free mesh. Furthermore the top and bottom SED values for a given angle is nearly equal, so that for a parallel outline the SED for top and bottom should be the same. This can be seen also in the figure (8) because the different points are overlaying on each other and therefore there are only two visible plots.

The interesting part for the SED here is that it is increasing from $2\alpha = 90^\circ$ until $2\alpha = 135^\circ$ even though it's angle is getting blunter.

4.1.2 Analysis for variable opening angles for only bottom side

This case is nearly similar to the mirrored one. The difference is that the top side notch's opening angle is constant $2\alpha_t = 135^\circ$, but the opening angle on bottom side is variable.

$2\alpha_b$ [°]	Song's \bar{W} [MJ/m ³]		\bar{W} for subdivided [MJ/m ³]		\bar{W} for free mesh [MJ/m ³]	
	top	bottom	top	bottom	top	bottom
-						
90	0.07294	0.06617	0.06436	0.05743	0.06435	0.05741
105	0.07307	0.06975	0.06434	0.06069	0.06432	0.06066
120	0.07298	0.07281	0.06427	0.06362	0.06424	0.06359
135	0.07268	0.07279	0.06401	0.06402	0.06399	0.06399
150	0.07166	0.06536	0.06311	0.05816	0.06309	0.05815
165	0.06753	0.04826	0.05948	0.04344	0.05946	0.04343
∅	0.07181	0.06586	0.06326	0.05789	0.06324	0.05787

Table 3: SED for V-Type notches with $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying

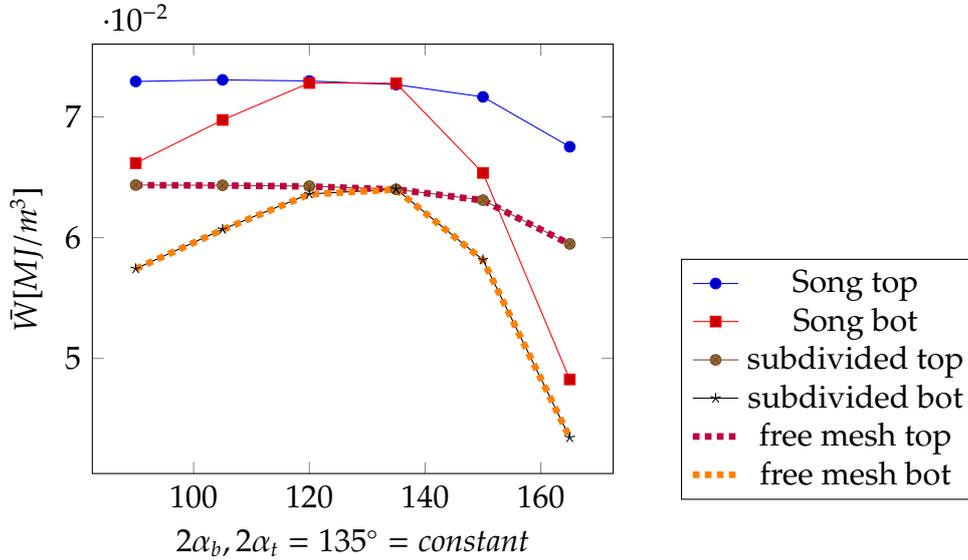


Figure 9: SED for V-Type notches with $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying

It can be seen that having a constant angle on one side (top) and one variable angle on the other side (bottom) will lead to small differences for the constant angle which is shown in figure (9). This small differences can be explained by having different mesh structures for different angles for the bottom side. It is also possible that the bottom side is causing this small differences. The only bigger change for all 3 models is for an opening angle $2\alpha_b = 165^\circ$. The SED for the top side plots is slightly decreasing even the opening angle is constant. Also it can be seen again that the points of free mesh and subdivided control volume mesh are overlaying. Also it is

important to say that the SED values for bottom are nearly similar in mirrored and in this case. This can be seen also when comparing the bottom side plots in figure (8) and (9). For more detailed analyses the values can also be compared to each other from table (2) and (3). Furthermore it is to mention that Song's SED values are again bigger and parallel to the new models' SED values.

The interesting part here is again that the SED is increasing from $2\alpha_b = 90^\circ$ until $2\alpha_b = 135^\circ$ even though it's angle is getting blunter.

4.1.3 Analysis between different distances and varying opening angle for bottom side

This case is nearly similar to the case with the varying bottom side opening angle and constant top side opening angle, but with the difference that the distance to the bottom side notch tip is $d_b = 5\text{mm}$ instead of 8.66025mm . In this case $2\alpha_b = 165^\circ$ is not calculated because when having a welding seam superelevation $w_{ss} = 2\text{mm}$ like here the model will not be generated correctly.

$2\alpha_b$ [°]	Song's \bar{W} [MJ/m ³]		\bar{W} for subdivided [MJ/m ³]		\bar{W} for free mesh [MJ/m ³]	
	top	bottom	top	bottom	top	bottom
-						
90	0.06004	0.07436	0.05296	0.06477	0.05295	0.06475
105	0.06019	0.07861	0.05295	0.06843	0.05294	0.06839
120	0.06007	0.08208	0.05292	0.07174	0.05290	0.07171
135	0.05995	0.08205	0.05282	0.07226	0.05280	0.07223
150	0.05953	0.07388	0.05245	0.06576	0.05243	0.06574
Ø	0.05996	0.07820	0.05282	0.06859	0.05280	0.06857

Table 4: SED for V-Type notches with $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d

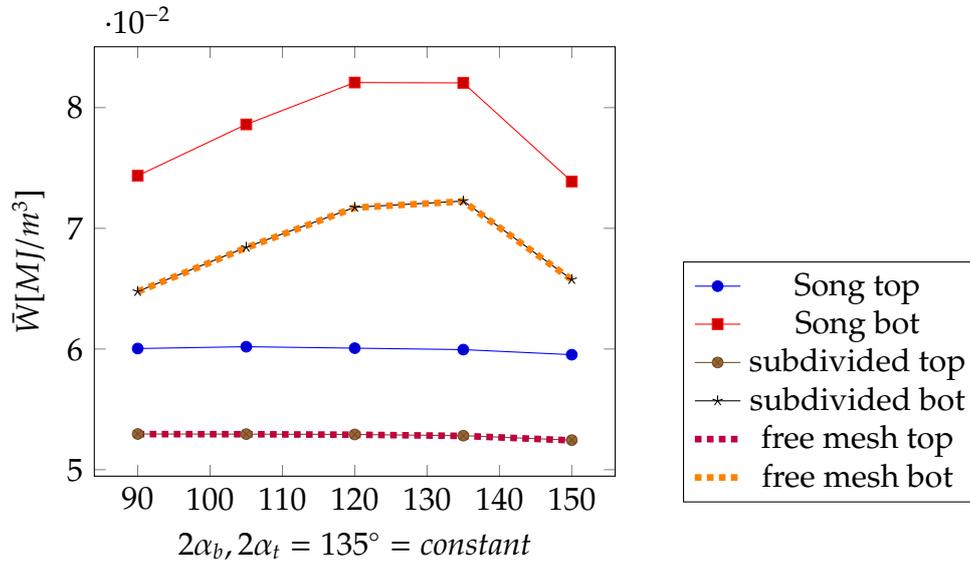


Figure 10: SED for V-Type notches with $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d

Like in the cases before it can be seen in figure (10) that Song's values are again bigger and parallel to the new models' SED values and again the SED values for subdivided and free meshed control volume are overlaying. Also it can be seen that the SEDs are increasing for having a shorter distance on bottom side, but also the SED is decreasing for the top side. But the form of the plots for top and bottom are again comparable with the ones in figure (9).

Also for this case the interesting part is that the SED is increasing from $2\alpha = 90^\circ$ until $2\alpha = 135^\circ$ even though it's angle is getting blunter.

4.2 Half circle

Now the SED values of the half circle control volume model are listed and compared with the values for the full surrounded control volume model.

4.2.1 Analysis for mirrored Case

It is the same case as in 4.1.1, so that it is a mirrored structure.

2α [°]	\bar{W} [MJ/m ³] for full surrounded		\bar{W} [MJ/m ³] for half circle	
	top	bottom	top	bottom
90	0.05774	0.05775	0.05879	0.05956
105	0.06098	0.06099	0.05813	0.05843
120	0.06385	0.06385	0.05879	0.05956
135	0.06399	0.06399	0.05887	0.05887
150	0.05737	0.05738	0.05466	0.05498
165	0.04106	0.04106	0.04038	0.04034
∅	0.05750	0.05750	0.05493	0.05529

Table 5: SED for half circled control volume notches with mirrored geometry

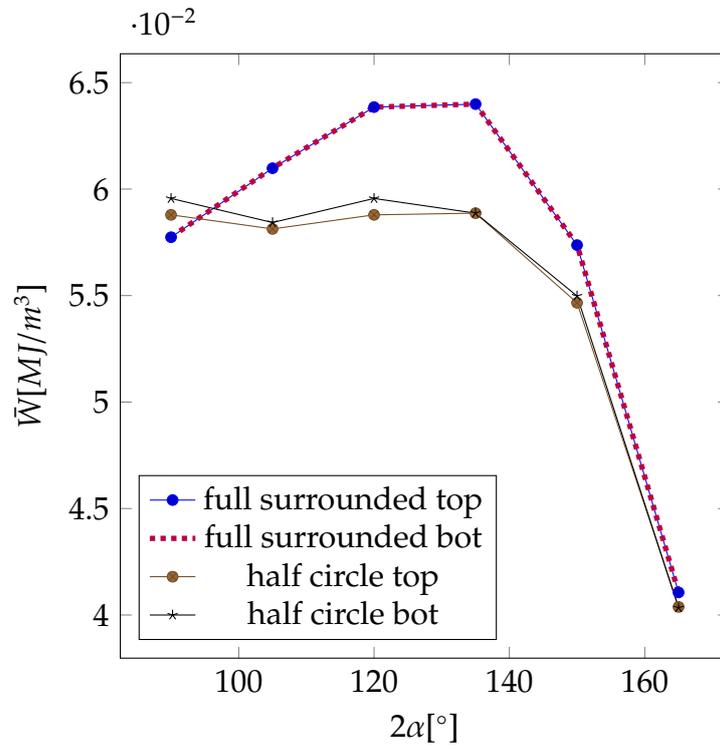


Figure 11: SED for half circled control volume notches with mirrored geometry

In figure (11) it can be seen that the SED values for different angles for half circle are varying a bit even though the outline is parallel in this case so the SED for top and bottom should be equal like the SED values

for full surrounded. This could be problematic, but it is explained in the discussion.

4.2.2 Analysis for variable opening angles for only bottom side

It is the same case as in 4.1.2, so that only bottom side's opening angle is varying.

$2\alpha_b$ [°]	\bar{W} [MJ/m ³] for full surrounded		\bar{W} [MJ/m ³] for half circle	
	top	bottom	top	bottom
90	0.06435	0.05741	0.05910	0.05648
105	0.06432	0.06066	0.05908	0.05736
120	0.06424	0.06359	0.05902	0.05820
135	0.06399	0.06399	0.05887	0.05887
150	0.06309	0.05815	0.05797	0.05571
165	0.05946	0.04343	0.05468	0.04266
Ø	0.06324	0.05787	0.05812	0.05488

Table 6: SED for half circled control volume notches with $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying

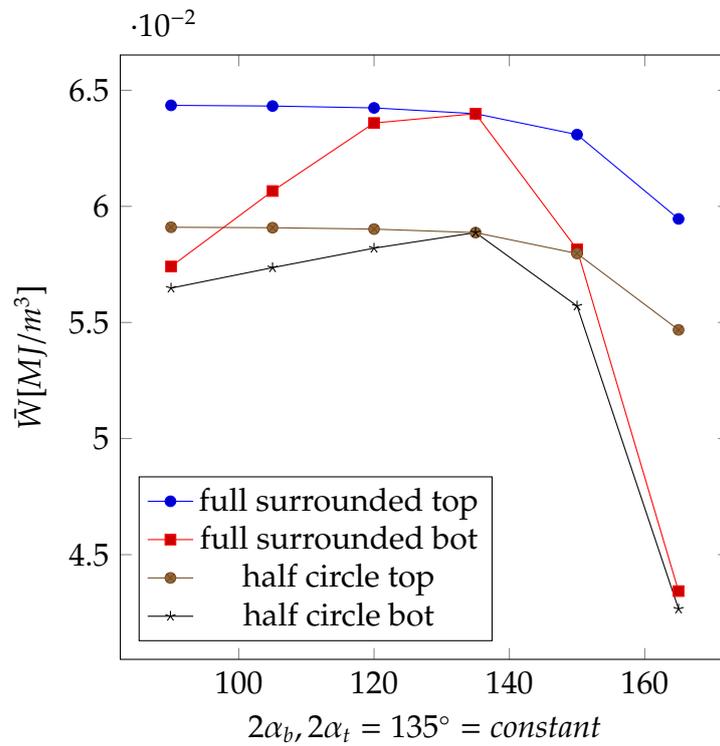


Figure 12: SED for half circled control volume notches with $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying

In the figure (12) it can be seen that the SED values for top side are nearly parallel and that the full surrounded one has bigger SED values. The half circle bottom side's plot's form is also varying a bit so that the SED is not parallel to the top full surrounded one's plot.

Now also the SED values for the half circle bottom side are increasing strictly for opening angles between $90^\circ \leq 2\alpha_b \leq 135^\circ$.

4.2.3 Analysis between different distances and varying opening angle for bottom side

It is the same case as in 4.1.3, so that only the bottom side opening angle is varying and the distance for the notch tip is shorter than for the top side notch.

$2\alpha_b$ [°]	\bar{W} [MJ/m ³] for full surrounded		\bar{W} [MJ/m ³] for half circle	
	top	bottom	top	bottom
90	0.05295	0.06475	0.04876	0.06268
105	0.05294	0.06839	0.04875	0.06409
120	0.05290	0.07171	0.04872	0.06604
135	0.05280	0.07223	0.04863	0.06720
150	0.05243	0.06574	0.04829	0.06243
∅	0.05280	0.06857	0.04863	0.06449

Table 7: SED for half circled control volume notches with $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d

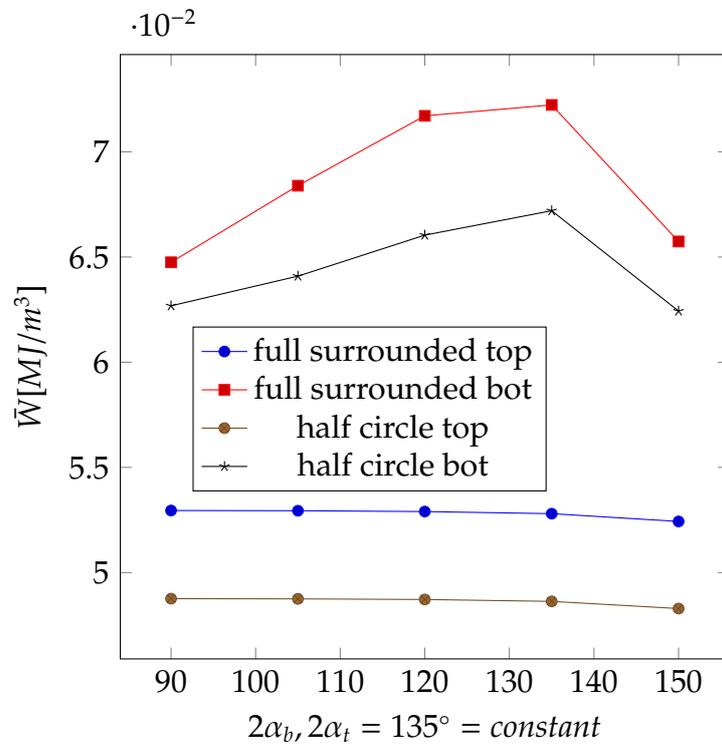


Figure 13: SED for half circled control volume notches with $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d

In the figure (13) it can be seen again that the SED values for top side are parallel for the different models and the SED values for bottom side seems to be parallel. Also the values are bigger for the bottom side as already explained in 4.1.3.

And also here the SED's are increasing until $2\alpha_b = 135^\circ$.

4.3 Rounded notches

Now the rounded notches will be modelled and the SED values will be listed for different cases.

4.3.1 Analysis for mirrored Case

It is the same case as in 4.1.1, so a mirrored geometry but with the change that there is a rounding now. The rounding radius is R . Also it has been analyzed one set of specimen with equal roundings for top and bottom side and one with different.

2α [°]	\bar{W} [MJ/m ³]		\bar{W} [MJ/m ³]	
	top $R = 1\text{mm}$	bottom $R = 1\text{mm}$	top $R = 2\text{mm}$	bottom $R = 0.5\text{mm}$
-	0.03543	0.03543	0.01802	0.05019
90	0.04352	0.04352	0.02714	0.05489
105	0.05098	0.05099	0.03717	0.05908
120	0.05563	0.05563	0.04592	0.06069
135	0.05346	0.05346	0.04530	0.06143
150	0.04025	0.04025	0.04876	0.05574
165				

Table 8: SED for rounded notches with mirrored geometry for different roundings R

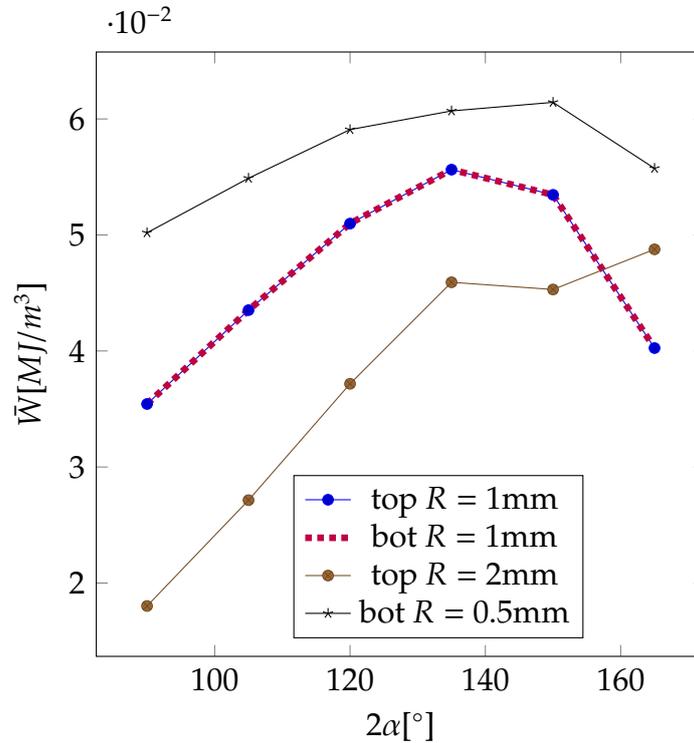


Figure 14: SED for rounded notches with mirrored geometry for different roundings R

In the figure (14) it can be seen that whenever R is equal for both sides the SED is the same and the form of the plot is kind of similar to the V-type notches plot's form. The SED for a combined plotting with bigger R on top side and smaller R on the other side bottom results into smaller SED values for the side with the bigger rounding and for bigger SED values for the top side as it should be because the SED values for rounded notches approximate to the SED values of V-type notches, which are bigger for an equal set up. Also the top side's plot for $R = 2\text{mm}$ looks different than the other ones, because it's SED value is increasing for an opening angle $2\alpha_t = 165^\circ$. It can already be seen for $2\alpha = 165^\circ$ that the SED will converge maybe to the same SED value for larger opening angles even when having different radii for top and bottom side. This makes also absolutely sense because the rounding area is pretty small for larger opening angles and then the V-type notch form has the bigger influence.

Also here it can be seen that the SED is increasing between $90^\circ \leq 2\alpha_b \leq 135^\circ$.

4.3.2 Analysis for variable opening angles for only bottom side

It is the same case as in 4.1.2, so that the opening angle for bottom side is changing while top side's opening angle is constant. Also the rounding is added here. The rounding radius is R and again there will be a set of two specimens one with equal roundings on each side and one with different roundings.

$2\alpha_b$ [°]	\bar{W} [MJ/m ³]		\bar{W} [MJ/m ³]	
	top $R = 1\text{mm}$	bottom $R = 1\text{mm}$	top $R = 2\text{mm}$	bottom $R = 0.5\text{mm}$
-				
90	0.05770	0.03403	0.04688	0.04672
105	0.05691	0.04248	0.04655	0.05255
120	0.05626	0.05040	0.04626	0.05790
135	0.05563	0.05563	0.04592	0.06069
150	0.05463	0.05437	0.04519	0.05700
165	0.05140	0.04277	0.04251	0.04356

Table 9: SED for different rounded notches, $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying

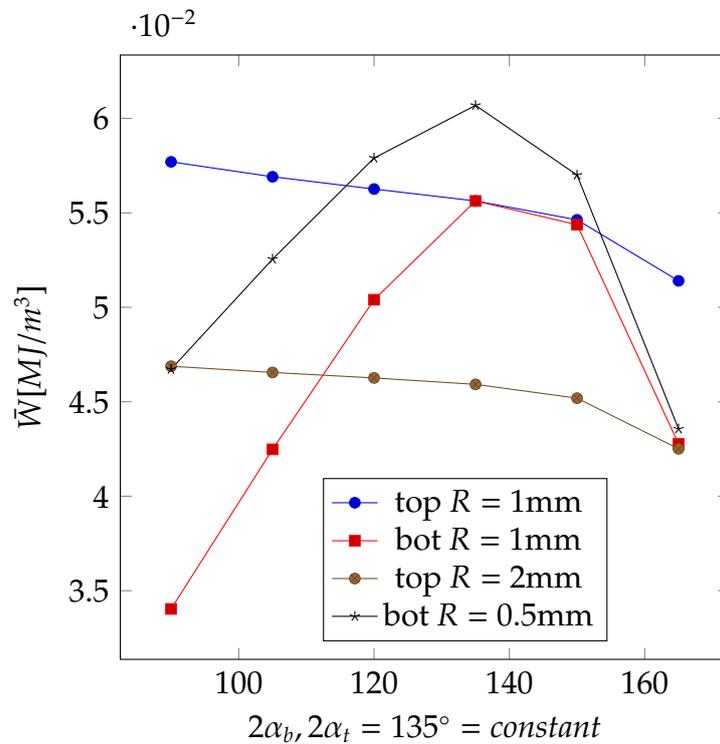


Figure 15: SED for different rounded notches, $2\alpha_t = \text{constant} = 135^\circ$ and $2\alpha_b$ varying

In the figure (15) it can be seen that the form of the plots for equal roundings for both side are nearly similar to the V-type notches case. When having different roundings for top and bottom side the form of the plots are varying, but there can be seen a parity. The plots are parallel for the top side as it should be. Also the SED values for the bottom side with different roundings are again approximating to the same value for large opening angles.

Also here it can be seen that the SED is increasing between $90^\circ \leq 2\alpha_b \leq 135^\circ$.

4.3.3 Analysis between different distances and varying opening angle for bottom side

It is the same case as in 4.1.3, so that the opening angle for bottom side is varying while for the top side it is constant and that the bottom side's notch has shorter distance. Also here the specimens are analyzed for one

set with equal rounding radii R on both sides and one with different.

$2\alpha_b$ [°]	\bar{W} [MJ/m ³]		\bar{W} [MJ/m ³]	
	top $R = 1\text{mm}$	bottom $R = 1\text{mm}$	top $R = 2\text{mm}$	bottom $R = 0.5\text{mm}$
90	0.04695	0.03960	0.03818	0.05325
105	0.04647	0.04875	0.03799	0.05945
120	0.04609	0.05729	0.03784	0.06523
135	0.04575	0.06293	0.03767	0.06826
150	0.04530	0.06143	0.03736	0.06416

Table 10: SED for different rounded notches, $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d

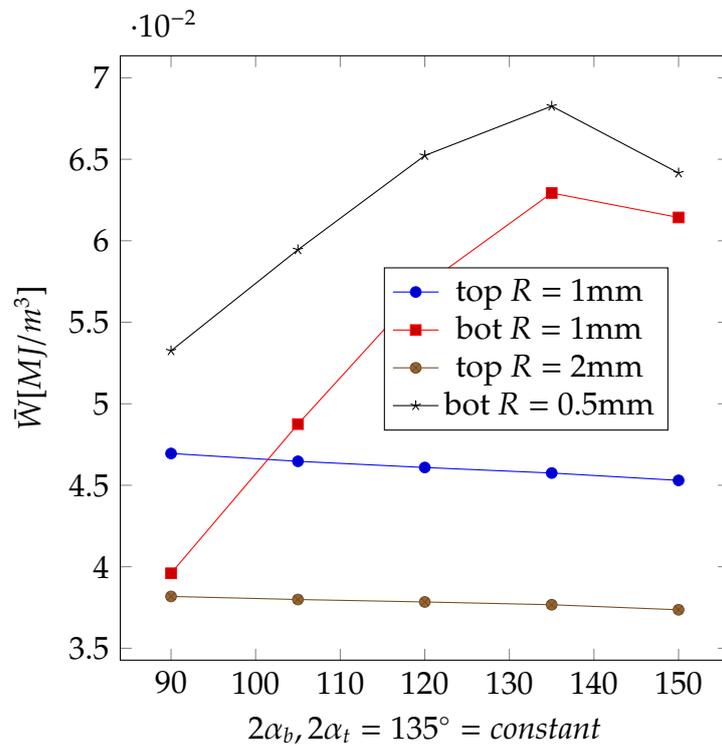


Figure 16: SED for different rounded notches, $2\alpha_t = \text{constant} = 135^\circ$, $2\alpha_b$ varying and different distances d

In the figure (16) it can be seen that top side's and bottom side's SED values are nearly parallel for different radii. And again the smaller the rounding is the bigger the SED values become until to the point in it is

approximating the V-type notches SED values for large opening angles 2α .

Also here it can be seen that the SED is increasing between $90^\circ \leq 2\alpha_b \leq 135^\circ$.

5 Comparison of the results

5.1 V-type notches comparison with Song's values

In this section the absolute difference ad and the percentage difference pd averaged SED values between Song's model and the results in this research are compared to each other. It can be seen in figures (8,9,10) that the variation between free meshing and meshing with a subdivided control volume resulting into nearly the same SED values. So it doesn't matter whether it is freely meshed or subdivided and then meshed when having a high degree of refinement for the FE as already Fischer et al. [14] and Foti et al. [9] has been mentioned this. Therefore in the following only the SED values between Song's model and the SED values with a free mesh are considered.

-		\bar{W} [MJ/m ³]		difference	
Case		Song	free mesh	absolute ad	percentage pd [%]
1	Top	0.0654	0.0575	0.0079	13.79
	Bot	0.0654	0.0575	0.0079	13.76
2	Top	0.0718	0.0632	0.0086	13.55
	Bot	0.0659	0.0579	0.0080	13.80
3	Top	0.0600	0.0528	0.0072	13.55
	Bot	0.0782	0.0686	0.0096	14.05

Table 11: Comparison of the SED for V-Type notches

The possible reason for this difference between this two Ansys APDL models might be that Song is lining up the control volume's keypoints with the general structure's keypoints whereas in this research the notch tip area is deleted and then reconstructed with the control volume circle. Therefore this can result to this percentage differences which is ranging between $13.55\% \leq pd \leq 14.05\%$ as shown in table (11).

5.2 Comparison full surrounded with half circle surrounded

In this section the absolute difference ad and the percentage difference pd for the averaged SED values between full surrounded notch (equal to the free mesh) and the half circle surrounded notch results are compared to each other.

-		$\bar{W}(MJ/m^3)$		difference	
		full surrounded	half surrounded	absolute ad	percentage pd [%]
1	Top	0.05750	0.05493	0.00257	4.67
	Bot	0.05750	0.05529	0.00221	3.99
2	Top	0.06324	0.05812	0.00513	8.82
	Bot	0.05787	0.05488	0.00299	5.45
3	Top	0.05280	0.04863	0.00417	8.58
	Bot	0.06857	0.06449	0.00408	6.33

Table 12: Comparison of the SED for half circled control volume with the full surrounded control volume

The percentage differences for different cases is kind of inconsistent and the absolute values are also ranging too much from each other. Also the averaged SED for the half circle should be higher than for the full surrounded because only the point of singularity is considered. Due to this problems there is suggested a new variation of this model in the conclusion 6.2.

5.3 Rounded notches

5.3.1 Comparison small to large roundings

In this comparison the values are plotted in a different way to see how important the rounding for a given angle is. The no rounding SED results are the same results as the V-type notches for free mesh in 4.1.1 from table (2). The SED results for the rounded notches are plotted with the mirrored geometry case from 4.3.1.

top	\bar{W} [MJ/m ³] for different $2\alpha_t$					
R	90°	105°	120°	135°	150°	165°
0.0mm	0.05774	0.06098	0.06385	0.06399	0.05737	0.04106
0.5mm	0.04717	0.05265	0.05766	0.06002	0.05568	0.04069
1.0mm	0.03543	0.04352	0.05098	0.05563	0.05346	0.04025
1.5mm	0.02590	0.03492	0.04402	0.05097	0.05123	0.03978
2.0mm	0.01939	0.02841	0.03814	0.04645	0.04891	0.03928
2.5mm	—	0.02349	0.03342	0.04251	0.04660	0.03877
3.0mm	—	—	0.02961	0.03916	0.04442	0.03825
bot	\bar{W} [MJ/m ³] for different $2\alpha_b$					
R	90°	105°	120°	135°	150°	165°
0.0mm	0.05775	0.06099	0.06385	0.06399	0.05738	0.04106
0.5mm	0.04717	0.05267	0.05766	0.06002	0.05556	0.04069
1.0mm	0.03543	0.04352	0.05099	0.05563	0.05346	0.04025
1.5mm	0.02590	0.03492	0.04402	0.05098	0.05122	0.03978
2.0mm	0.01939	0.02840	0.03813	0.04640	0.04892	0.03928
2.5mm	—	0.02349	0.03342	0.04251	0.04659	0.03877
3.0mm	—	—	0.02961	0.03916	0.04442	0.03824

Table 13: Comparison of the SED for rounded notches for increasing rounding radii R for different opening angles 2α

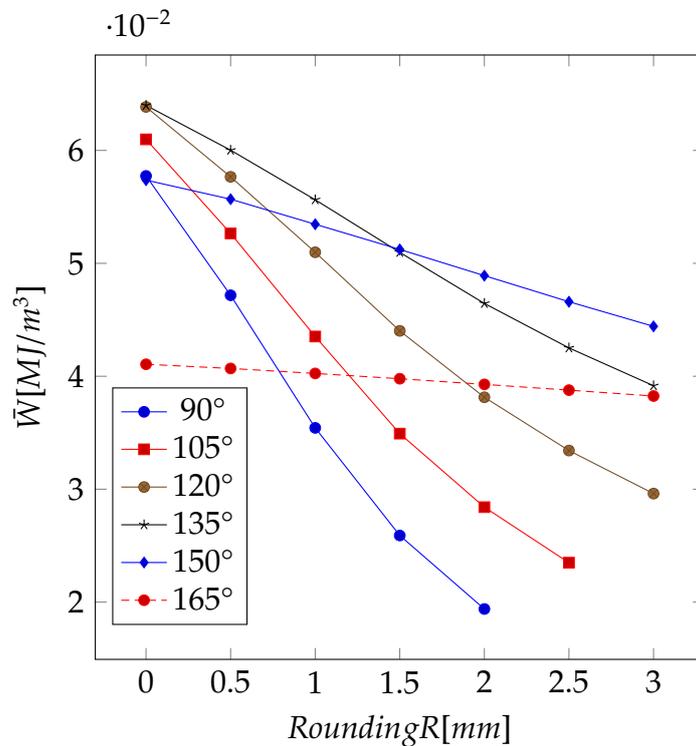


Figure 17: Comparison of the SED for rounded notches for increasing rounding radii R for different opening angles 2α

In the figure (17) only the top side values have been added because the bottom side is nearly equal, also it is to mention that there are no values for some combination as in table (13) shown. It can be seen that the SED values are decreasing the larger the rounding radius is. Also it can be seen that the maximum value for the SED is at $2\alpha = 135^\circ$ when having no rounding. But the larger the rounding radius R becomes the more the maximum SED value shifts to the opening angle $2\alpha = 150^\circ$. For even bigger rounding radii the maximum SED number could be for an opening angle $2\alpha = 165^\circ$.

On the one hand it can be seen that the values for the $2\alpha = 165^\circ$ are decreasing really slow and they could also be called constant. The reason for this might be that the rounded area for $2\alpha = 165^\circ$ is really small and the rounded area's growth from $R = 0\text{mm}$ to $R = 3\text{mm}$ is also really small. So a rounding for $165^\circ \leq 2\alpha < 180^\circ$ might be unnecessary and they could be also called V-type notches instead of rounded. But on the other hand the biggest changes for the SED values are for $2\alpha = 90^\circ$. For this angle the most rounded area has been added to the notch. Also it can be seen that the less

rounded area is added to the notch the less are the SED values decreasing and the more rounded area is added the more are the values decreasing which is making absolutely sense. This might be the reason why the SED for larger roundings for $2\alpha = 150^\circ$ is higher than for $2\alpha = 135^\circ$.

All in all there has to be a parameter for the function SED $\bar{W}(2\alpha, R)$ that is explaining the correlation of this. So whenever a SED value is given for a combination of an opening angle and a rounding radius R it should be possible to derive the other combinations with that function and also it is important to mention that for decreasing opening angles the best effects can be achieved with the roundings, which will lead to a better toughness.

5.3.2 Analytical solution compared with numerical solution for roundings

In this section the analytical results of the SED for rounded notches from table (1) is compared with the numerical solutions from the Ansys model. For this the percentage difference p will be calculated.

$2\alpha(^{\circ})$	R_c/R	\bar{W} [MJ/m ³] Analytical	\bar{W} [MJ/m ³] Model	percentage pd
90	0.3	0.06539323	0.036969	43.466624
	0.5	0.06438688	0.045846	28.796049
120	0.1	0.04140724	0.031036	25.0469317
	0.3	0.05089627	0.051912	-1.99568239
135	0.5	0.04947954	0.056885	-14.9667049
	0.1	0.03392139	0.040434	-19.1991337
	0.3	0.04028442	0.056235	-39.5949171
	0.5	0.04097688	0.059509	-45.2258023

Table 14: Comparison of the SED between analytical and Ansys model solution

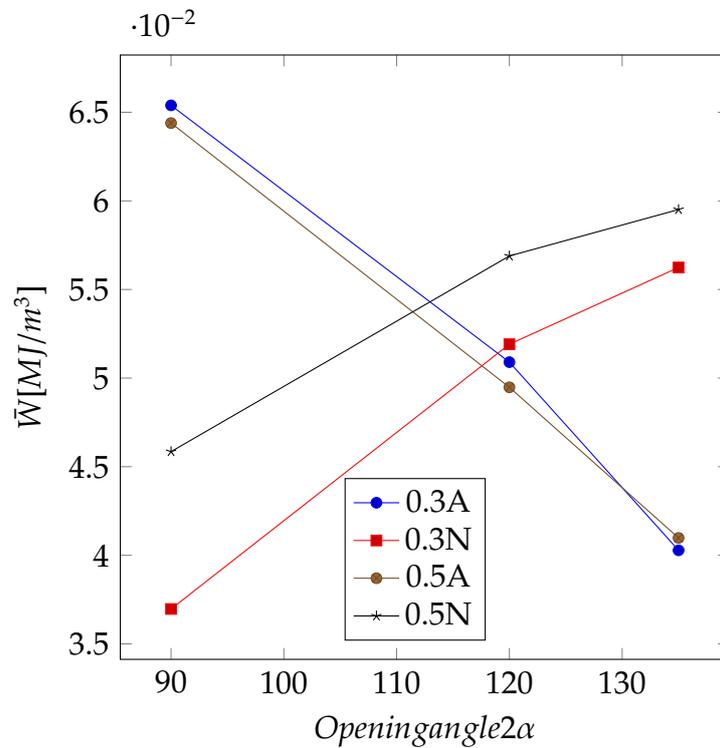


Figure 18: Comparison analytical solution (A) with numerical solution (N)

The percentage differences are inconsistent, because one plot is growing and one decreasing. And in the figure (18) it can be seen for the $R_c/R = 0.3$ and $R_c/R = 0.5$ that the SED values for the analytical solutions are decreasing whereas the model SED solutions are increasing for a growing opening angle. The point is that the SED should increase from opening angles $90^\circ \leq 2\alpha_b \leq 135^\circ$, because the roundings behave similar like V-type notches and for them the SED is increasing between the opening angle interval as it can be seen in the researches by Gaiotti et al. [8]. Also it is important to mention that for the numerical solution it was shown that the SED values will decrease for opening angles $2\alpha > 135$, which can not be compare with the analytical solutions because the H^* function table by Berto and Lazzarin is limited [1]. Also the analytical and numerical solution should be calculated for more specimens to generate a larger more meaningful pool of analysis.

6 Discussion

In this section the different investigations are explained and analyzed.

6.1 V-type notches

The important point here is that the SED values for V-type notches for opening angles $90^\circ \leq 2\alpha \leq 135^\circ$ are increasing, as it can be seen in all plotted figures, which can also be seen in the results by Gaiotti et. al [8]. In their researches they also modelled a butt-welded joint without the bottom side, but with the left and right side of the butt-welded model. Their SED values can not directly compared with the SED values here in this research, because in their research different parameters and a bit different model design were selected, but it can be said that the SED for V-type notches should increase for this $90^\circ \leq 2\alpha \leq 135^\circ$. Furthermore the increase is like a parabolic function.

A possible explanation for the increase of the SED for the analyzed opening angles is that the combination of the easier force deflection due to the the flattening of the opening angles and that the notch effect is still noticeably large for opening angles until $2\alpha = 135^\circ$, will lead to larger nodal displacement's in the control volume area, which can be directly translated into an increase of the SED value. Also the force deflection will result into new combinations of the stresses for the notch, which should be analyzed in the next investigations.

The decrease of the SED values for opening angles $2\alpha > 135^\circ$ can be explained, because of the fact that the V-type notch is flattening until $2\alpha = 180^\circ$ and that the notch effect is decreasing until it is disappeared. Also here the decrease could be illustrated with a parabolic function, so that the increasing and the decreasing parts could be combined into one function. This function should involve in dependence of the opening angle the general material parameters as Young's modulus, Poisson ratio, also the stresses in a parabolic relation and a correction factor.

An explanation for the point that the SED is increasing when having a shorter distance to the notch tip for one side and a longer distance to the notch tip on the other side could be that the oblique alignment result into different combinations of compressive and shear tension, which will then cause the different SED values. So in general constructions this case should be avoided, when the geomery allows it. But in special cases the different distances could help to deal with mixed mode loadings.

One problem for the SED calculation in Ansys was that S. Song's SED values and the SED values in this research are not approximately equal whereas the models had the same geometry and the same conditions, just the control volume was generated in an other way. Despite all this, there were average percentage differences $13.55\% \leq pd \leq 14.05\%$. So the modelling in Ansys should be done carefully in a standardised way so that the different results could be compared to each other.

6.2 Half circle

The modelled half circle surrounding variation seems not to be expressive, because for the analyzed specimens the SED values are behaving kind of inconsistent, which is shown in table 12.

There are possible answers for this. The easiest answer for this is that the modelling for this was done incorrectly and that there is a better strategy to modelling a butt-welded joint with a half circle control volume.

One other possible answer for this is that the crack path of the notch is not always around the bisecting line, which will lead to that the SED is underestimated and that the nodes along the notch edge have a bigger influence in many cases. The SED for $2\alpha = 90^\circ$, which can be seen in figure (11) was a bit higher, so that this is possibly one exceptional case for which the half circle surrounding control volume can be modelled. But for many cases the half circle model should be avoided when modelling with this variation. So this results are corresponding to the results of Radaj et al. that this half circle surrounding control volume should just be used for exceptional cases [16, 15]. So this solutions are underlining the former results.

6.3 Rounded notches

There is a problem between the results for the numerical and the analytical solution of the SED for the rounded V-type notches, because the numerical results which are presented in figure (18) show that the SED is increasing between $90^\circ \leq 2\alpha \leq 135^\circ$ and the analytical solutions, which is calculated with the equation (2) by Berto and Lazzarin [1] are showing the opposite. The first possible explanation for this is that the SED for this butt-welded joint model can not be calculated with the equation (2), which could make sense, because there has to been modelling differences between this rounded notches model and Berto's and Lazzarin's model, which can not be verified, because their model is not available.

One other explanation is that this equation (2) is not recommended for butt-welded joints because it was designed while modelling transverse fillet-welded joints, but that the SED calculation for transverse fillet-welded and butt-welded joints is behaving in an opposite way, is also questionable. It could also be possible that modelling mistakes are causing this results, but in figure (17) it can be seen that the model seems to be self-consistent, because the correlation between a growing rounding radius R and a decreasing SED is given. Also the plots for the different cases of rounded notches have the same structure as the plots for V-type notches just with decreased SED values which are proportional to the rounding radius R as mentioned like before, so this is also underlining the self-consistent. Also the new control volume radius R_2 seems also to be a good approximation when considering the results in this paper, so there has to be one other problem.

The problem could be that the normalised radius for the H^* from the equation (2) is not R_c/R but R/R_c , because Berto and Lazzarin are listing values between $0.01 \leq R_c/R \leq 1$ [1]. For $0.01 \leq R_c/R \leq 1$ and $R_c = 0.28\text{mm}$ rounding radii between $28\text{mm} \geq R \geq 0.28\text{mm}$ would result which would be really questionable because a rounding radius of $R = 28\text{mm}$ could be too large to be considered as a rounded notch and then other approaches would be more sensible. But for the opposite ratio $0.01 \leq R/R_c \leq 1$ and $R_c = 0.28\text{mm}$ rounding radii between $0.03572\text{mm} \geq R \geq 1\text{mm}$ would result, which would be more understandable and more useful for the further calculation and with the really small rounding radius it could be demonstrated that the SED for rounded notches is approximating to to the same value as the V-type notches. Therefore it should be made clear which variation of the normalised radius is the correct one.

7 Conclusion

7.1 V-type notches

For the numerical SED calculation it will make no difference whether the control volume is subdivided or meshed freely because only the number of the FEs in the control volume area matter and after a certain point of refinement the SED changes are really small and then higher refinements will be just causing into more computational effort which should be avoided. In all Ansys models a refinement $0.5 \times R_c$ was enough to get good and fast results.

When comparing Song's model's SED values with the free mesh model's SED values there was a nearly constant percentage difference, which was eventually caused by a bit different modelling strategy because everything else was the same. So there should be a standardised modelling strategy. One recommendation for further researches is that it should be investigated how to make a model that is combining V-type notches and rounded notches to better understand their interactions whenever having one side V-type notched and the other side rounded. Also the SED for oblique aligned (different distances for top and bottom side notch) notches should be analyzed much more to investigate the cases for which combinations of stresses and forces this can be a better solution to implement this for butt-welded joint constructions.

7.2 Half circle

The problem is that the SED values for the half circle model wasn't as intended. They should be higher than the full surrounded model's SED values because the control volume area is just surrounding the point of singularity in the notch tip and not the nodes along the notch edge at all. The only exceptional case for the half circle was when having an opening angle $2\alpha = 90^\circ$ for the mirrored case.

The conclusion here is that this single point of singularity is not enough to describe the SED correctly for many cases and therefore a suggestion for further researches the half circle's center should be positioned a bit outside of the notch tip in its bisecting line so that there are included more nodes along the notch edge than just the point of singularity, also the new half circle radius for the new control volume should be $R_{hc,new} = R_c + f \times r_{pos}$. $R_c = 0.28\text{mm}$ could be still reasonable for steel welded joints and r_{pos} would be the positioning distance for the new half circle's center to the notch tip, which can be multiplied with correction factor f , but it could be also f

=1.

The hypothesis here is that the accuracy of the SED values will increase compared to the full surrounding one's and that it could be used in a more generalised way, which would be a great progress and this should also be possible to implement in Ansys.

Alternatively the crack path for each opening angle for real constructed notched butt-welded joints could be analyzed and then the half circle could be positioned into that crack patch in dependence of the opening angle. But this would be a huge effort. So for most cases the full surrounding control volume is the better option.

7.3 Rounded notches

The positive thing here is that the Ansys model is working as intended and that all the correlations between different roundings and opening angles are self-consistent. But it couldn't be find out whether the values for the SED in the model are in a correct range because the analytical solution with equation (2) is differing as explained and that maybe first of all it should be clarified by Berto and Lazzarin whether their notation in their paper is correct [1].

So for further investigations there should be modelled more rounded notches as butt-welded joints which should be compared to this results especially to develop a function that is making it possible to compare the results with real solutions. An other interesting investigation could be to expand this SED approach for rounded notched butt-welded joint's model for sub-zero temperatures as it was already investigated for for butt-welded joints by Braun et al. Braun and fillet welded joints by Braun et al. [13] have analyzed this already for V-type notches, to simulate arctic conditions. The investigations about the fillet-welded joints could be a good supplement to improve for this suggestion that have more specimens for this special area of application.

Also it should be considered to include the investigations of Savruk and Kazberuk in further modelling of rounded welded joints, because the rounding has not to be always a circular arc, it could also be a hyperbolic arc, which could be plausible for sharp rounded notches [17].

7.4 Correlation between the SED and the parameters in the model

In the last point of the conclusion it shall be explained and summarised in a short way how the SED was changing when modifying the parameters of the butt-welded joint's structure.

- Whenever the top and bottom side of the structure was mirrored the SED values were the same which makes absolutely sense. But it was not tested out for smaller plate thicknesses. The guess is that the SED should be bigger but equal for both sides
- When having a constant opening angle for the notch on one side and a varying opening angle on the other side the SEDs for the constant opening angle were not differing too much but they were a bit influenced by the varying side.
- When changing the distances for one side the SED increased the further the distances to the pressure side was.
- For the rounded notches the important points is that the SED was decreasing the larger the rounding is and the larger the opening angle is the less important is a rounding and the more it could be seen as a V-type notch.
- For really small roundings the influence of it was infinitesimal and therefore the rounded notch could be approximated as a V-type notch.
- For different rounding radii R on top and bottom side the SED values were also approximating to the same value for larger opening angles because the effect of the rounding was decreasing.
- Some parameters were not changed for the SED calculation in the model, which among us could explain much better the correlations between the parameters. The unchanged relevant parameters were: the control volume radius R_c , the welding seam superelevation w_{ss} , the plate thickness, the total length of the butt-welded joint and the type of loading and their values

8 References

References

- [1] F. Berto, P. Lazzarin. *A review of the volume-based strain energy density approach applied V-notches and welded structures*. In: Theoretical and Applied Fracture Mechanics 52 (2009), pp.183-194.
- [2] P. Lazzarin, R. Zambardi. *A finite-volume-energy based approach to predict the static and fatigue behavior of components with sharp V-shaped notches*. In: International Journal of Fracture 112 (2001), pp.275-298.
- [3] P. Lazzarin, F. Berto. *Some expressions for the strain energy in a finite volume surrounding the root of blunt V-notches*. In: International Journal of Fracture 135 (2005), pp.161-185.
- [4] C. Fischer, W. Fricke, C.M. Rizzo. *Review of the fatigue strength of welded joints based on the notch stress intensity factor and SED approaches*. In: International Journal of Fracture 84 (2016), pp.59-66.
- [5] P. Livieri ,P. Lazzarin. *Fatigue strength of steel and aluminium welded joints based on generalised stress intensity factors and local strain energy values*. In: International Journal of Fracture 133 (2005), pp.247-276.
- [6] P. Lazzarin, F. Berto, M. Zappalorto. *Rapid calculations of notch stress intensity factors based on averaged strain energy density from coarse meshes: Theoretical bases and applications*. In: International Journal of Fracture 32 (2010), pp.1559-1567.
- [7] Braun M, Fischer C, Fricke W, Ehlers S. *Extension of the strain energy density method for fatigue assessment of welded joints to sub-zero temperatures*. In: Fatigue Fract Eng Mater Struct. 2020;1–16. <https://doi.org/10.1111/ffe.13308>.
- [8] M. Gaiotti, C.M. Rizzo, F. Berto. *Assessment of welded joints by strain energy density approach accounting for misalignments and geometrical imperfections*. In: Proceedings of the ASME 2017 36th International Conference on Ocean, Offshore and Arctic Engineering OMAE2017, 2017, Trondheim, Norway.
- [9] P. Foti, S.M.J. Razavi, M.R. Ayatollahi, L. Marsavina, F. Berto. *On the application of the volume free strain energy density method to blunt V-notches*

- under mixed mode condition*. In: Engineering Structures 230 (2021), pp.1-9.
- [10] P. Lazzarin, F. Berto, M. Zappalorto. *Rapid calculations of notch stress intensity factors based on averaged strain energy density from coarse meshes: Theoretical bases and applications*. In: International Journal of Fatigue 32 (2010), pp.1559-1567.
- [11] A. Düster, C. Fischer, W. Fricke. *Fatigue strength assessment of welded joints based on the local strain energy density*. Technische Universität Hamburg-Harburg, Hamburg.
- [12] C. Fischer, A. Düster, W. Fricke. *Different finite element refinement strategies for the computation of the strain energy density in a welded joint*. In: Advances in Marine Structures - Guedes Soares & Fricke (eds)(2011), pp.289-294.
- [13] M. Braun, A.-S. Milaković, F. Renken, W. Fricke, S. Ehlers. *Application of local approaches to the assessment of fatigue test results obtained for welded joints at sub-zero temperatures*. In: International Journal of Fatigue 138 (2020) 105672, pp.1-11.
- [14] C. Fischer, W. Fricke, C.M. Rizzo. *Experiences and recommendations for numerical analyses of notch stress intensity factor and averaged strain energy density*. In: Engineering Fracture Mechanics 165 (2016), pp.98-113.
- [15] D. Radaj. *State-of-the-art review on the local strain energy density concept and its relation to the J-integral and peak stress method*. In: Fatigue & Fracture of Engineering Materials & Structures 38 (2015), pp.2-28.
- [16] D. Radaj, F. Berto, P. Lazzarin. *Local fatigue strength parameters for welded joints based on strain energy density with inclusion of small-size notches*. In: Engineering Fracture Mechanics 76 (2009), pp.1109-1130.
- [17] M.P. Savruk, A. Kazberuk. *RELATIONSHIP BETWEEN THE STRESS INTENSITY AND STRESS CONCENTRATION FACTORS FOR SHARP AND ROUNDED NOTCHES*. In: Material Science, Vol. 42, No. 6 (2006), pp.725-738.
- [18] F. Berto, J. Torgersen, A. Campagnolo. *A review of the fatigue strength of structural materials under multiaxial loading in terms of the local energy density*. In: Engineering Solid Mechanics (2017), pp.245-270.

9 Appendix

BACHELOR THESIS 2020

for

Atilla Acar

Bewertung der Schwingfestigkeit von Stumpfstoßverbindungen mittels der Formänderungsenergiedichte

(engl.: Application of the strain energy density method to butt-welded joints)

To this day many different local fatigue assessment methods have been proposed and are still being developed. Each of them has benefits and downsides for particular problems.

The most common local fatigue assessment method in ship and offshore industry is the structural hot-spot stress approach. One of the more sophisticated methods is based on the integration of the averaged strain energy density (SED) in a small volume surrounding for example the notch at a weld toe. In different studies the accuracy of the method has proven to be eminent; however, the computational effort is still high, due to particular requirements on the finite element mesh used for calculation of the SED.

In order to improve the applicability of the method for fatigue assessment of larger structures like ships or offshore structures it is necessary to decrease the computational effort of this method and to verify its accuracy based on different scenarios. Thus, this thesis aims at comparing different approaches for SED-based assessment of butt-welded joints. The project contains the following tasks:

- 1) Development of different meshing strategies for butt-welded joints (V-type notch, half circle, and rounded notch)
- 2) Fatigue assessment of various butt-welded joints based on the developed meshing strategies
- 3) The results should be compared with literature results

Literature studies of specific topics relevant to the thesis work shall be included.

The work scope may prove to be larger than initially anticipated. Subject to approval from the supervisors, topics may be deleted from the list above or reduced in extent.

In the thesis the candidate shall present his personal contribution to the resolution of problems within the scope of the thesis work.

Theories and conclusions should be based on mathematical derivations and/or logic reasoning identifying the various steps in the deduction. The candidate should utilise the existing possibilities for obtaining relevant literature.

Thesis format

The thesis should be organised in a rational manner to give a clear exposition of results, assessments, and conclusions. The text should be brief and to the point, with a clear language and the objective to be published in a conference article and/or scientific journal. It is thus desirable that the thesis is written in English. Telegraphic language should be avoided.

The thesis shall contain the following elements: An executive summary, list of symbols and acronyms, followed by the main body of the thesis consisting of a brief background introduction, a state of the art defining the knowledge gaps defining the scope or work and limitations, the actual contribution chapters, conclusions with recommendations for further work, references and (optional) appendices. All figures, tables and equations shall be numerated.

The supervisors require that the candidate, in an early stage of the work, presents a written plan for the completion of the work. The plan may include a budget for the use of computer and laboratory resources if applicable, which will be charged to the department. Overruns shall be reported to the supervisors.

The original contribution of the candidate and material taken from other sources shall be clearly defined following basic academic principles and an acknowledged referencing system, which includes the name of the referred authors followed by the publication year in the text. The subsequent reference list can thus be alphabetical.

The report shall be submitted in two copies:

- Signed by the candidate
- The text defining the scope included
- In bound volume(s)
- Drawings and/or computer prints, which cannot be bound should be organised in a separate folder.
- The report shall also be submitted in PDF along with essential input files for computer analysis, spread sheets, MATLAB files etc. in digital format.

Ownership

According to the current rules, the candidate has the ownership of the thesis. Any use of the thesis has to be approved by TUHH M-10 and Vattenfall AB. TUHH M-10 has the right to use the thesis as if a TUHH M-10 employee carried out the work, if nothing else has been agreed in advance.

Thesis supervisors

TUHH: Prof. D.Sc. (Tech.) Sören Ehlers, Moritz Braun

TÜV Nord: Dr.-Ing. Claas Fischer

Deadline: 24.01.2021

Hamburg, 23.11.2020