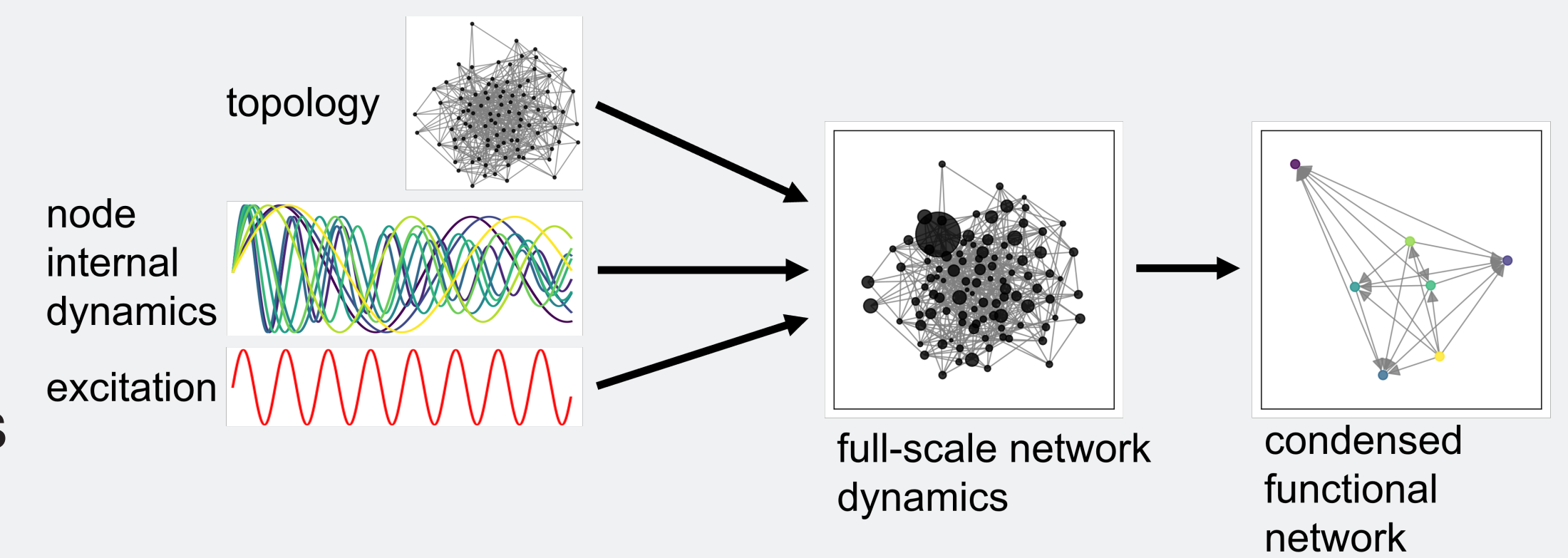


Embedding dynamics into network topology

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Bringing topology and dynamics together

- Background** information flow within network depends on topology, interaction mechanisms and (nonlinear) dynamics of each node
- Challenges** role of each factor difficult to assess → design of network for specific dynamics difficult
- Idea** generate a dynamics-based *functional network* to obtain a backbone-structure for dynamics and evaluate impact of different factors



Model

$$\ddot{x}_i + d\dot{x}_i = g_0(x_i) + c \sum_j^N A_{ij} g_1(x_i, x_j) + f_i$$

Network topology

A Erdős–Rényi–Network with $N = 100$ nodes and edge probability p

Nodal dynamics

nonlinear: $g_{0,nl}(x_i) = -kx_i - k_{nl}x_i^3$
linear: $g_{0,lin}(x_i) = -kx_i$

Interaction

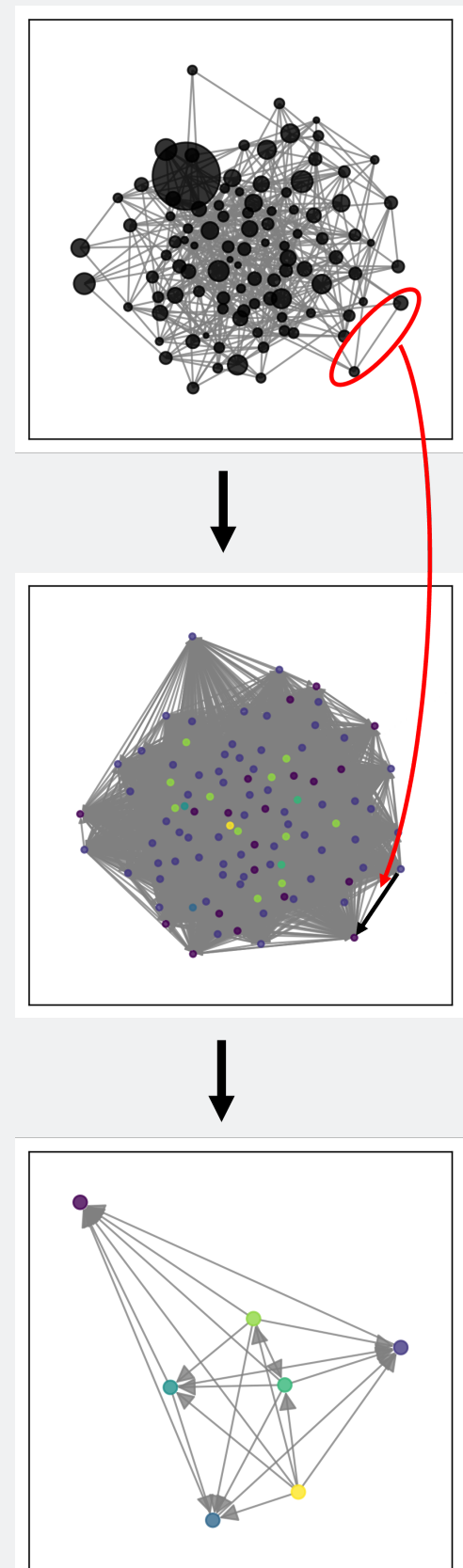
linear: $g_1(x_i, x_j) = x_i - x_j$
scaling factor c

Excitation

harmonic: $f_i = F \cos(\Omega t)$

Generating functional networks

workflow



- obtain dynamics for each node over time period (through excitation of one node)
- generate functional network [1]: each node in network = node in functional network for each node pair, obtain coupling direction [2] using [3]
 - recurrence-based high-dimensional embedding
 - interaction direction from cross-clustering and cross-transitivity
- condense using strongly connected components [4, 5]

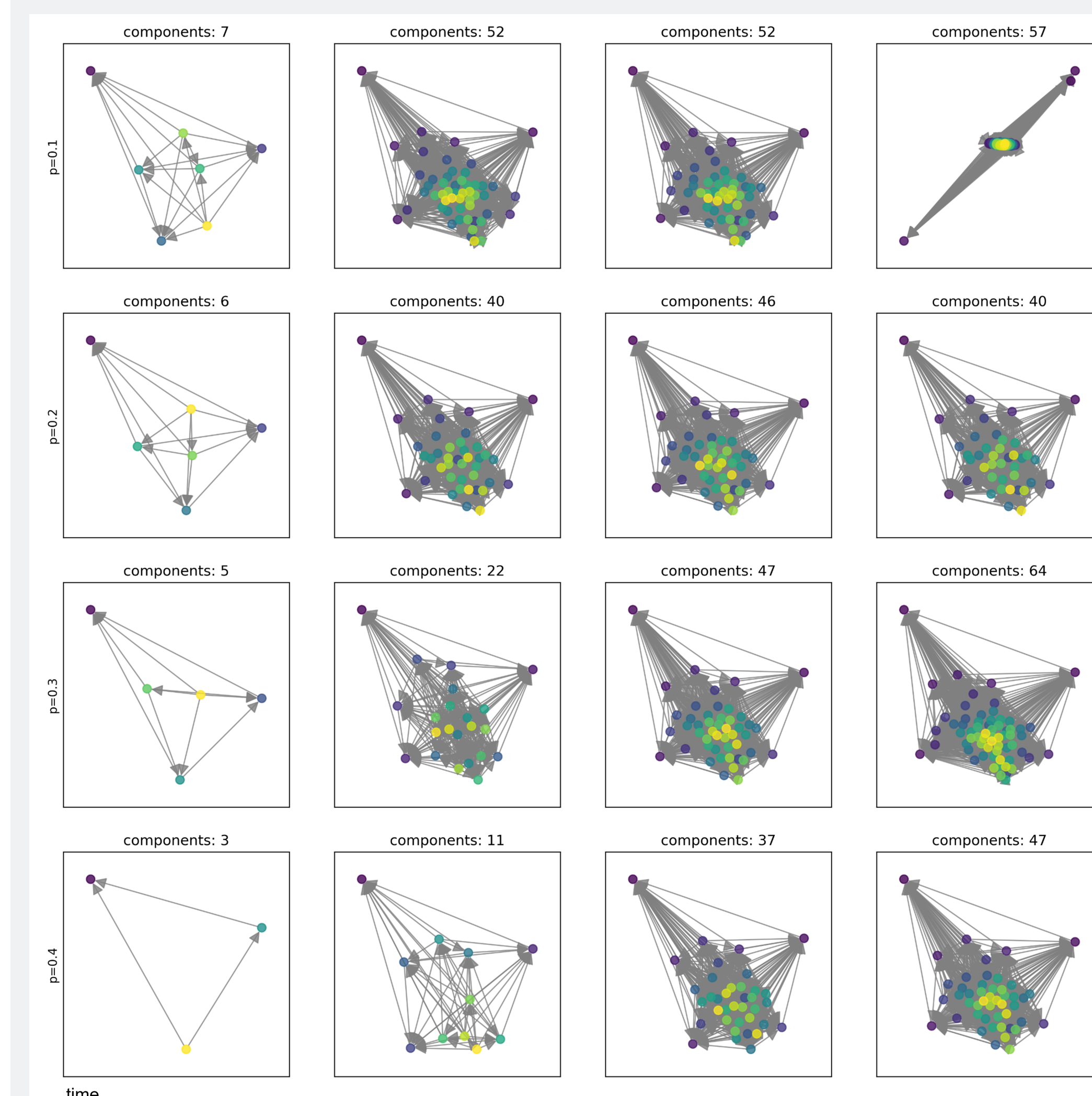
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Impact of different elements on network dynamics

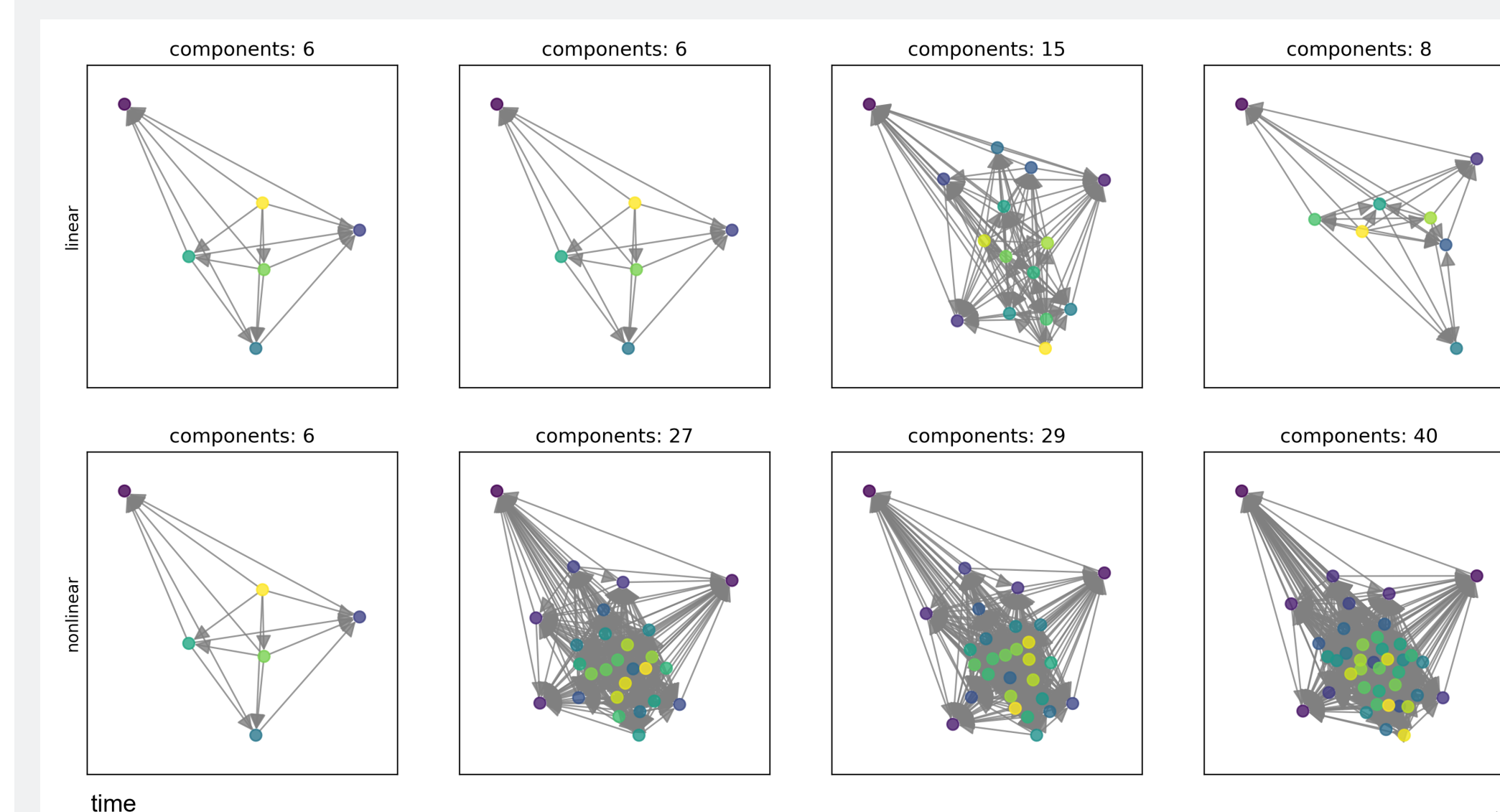
Topology: connection probability p

increased connectivity – more/faster information flow – more uniform dynamics – less components
nonlinear nodal dynamics, $c = 0.1$, $\Omega = 2$



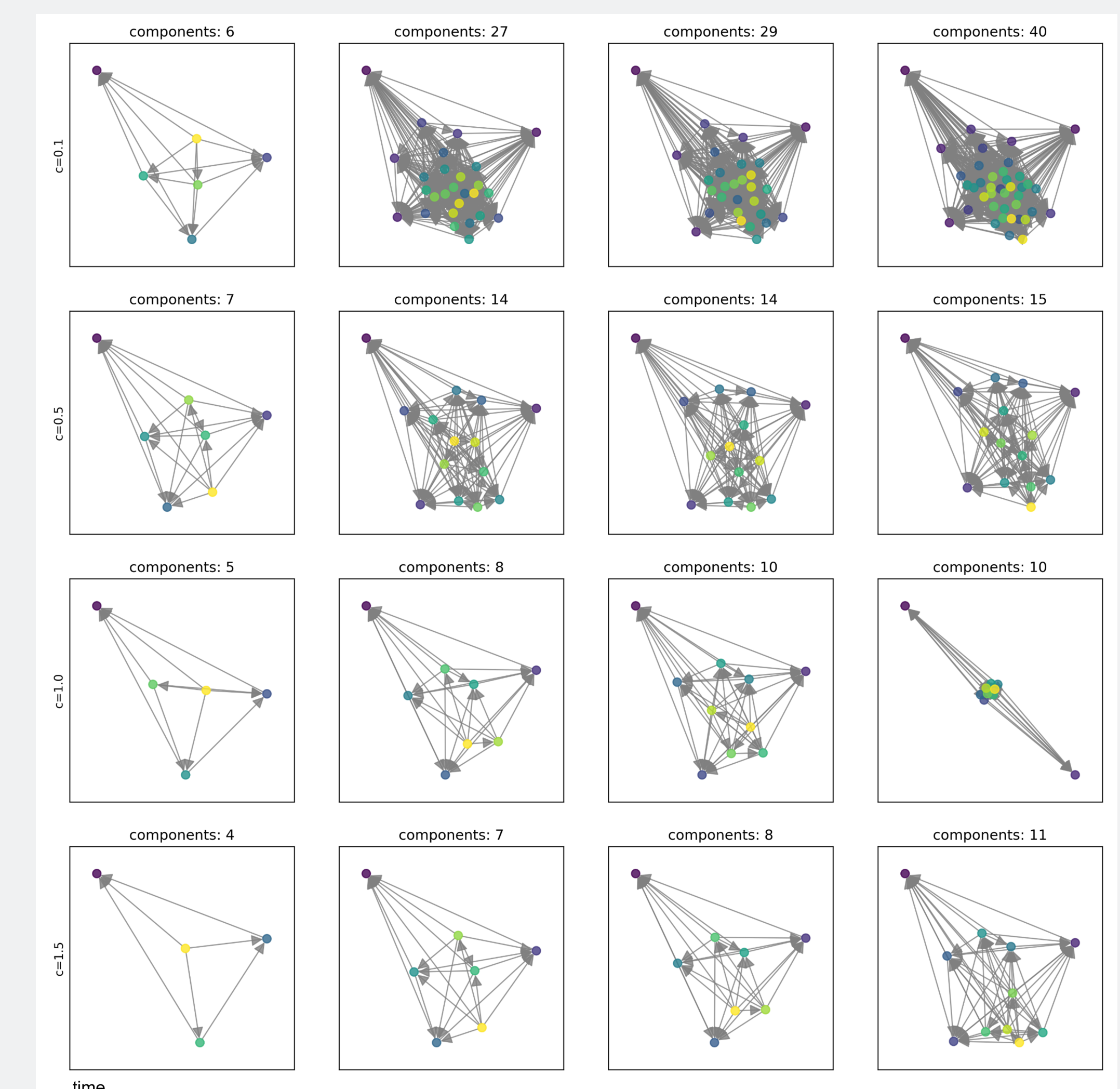
Nodal dynamics: linear or nonlinear

linear nodal dynamics less complex
 $p = 0.2$, $c = 0.1$, $\Omega = 2$



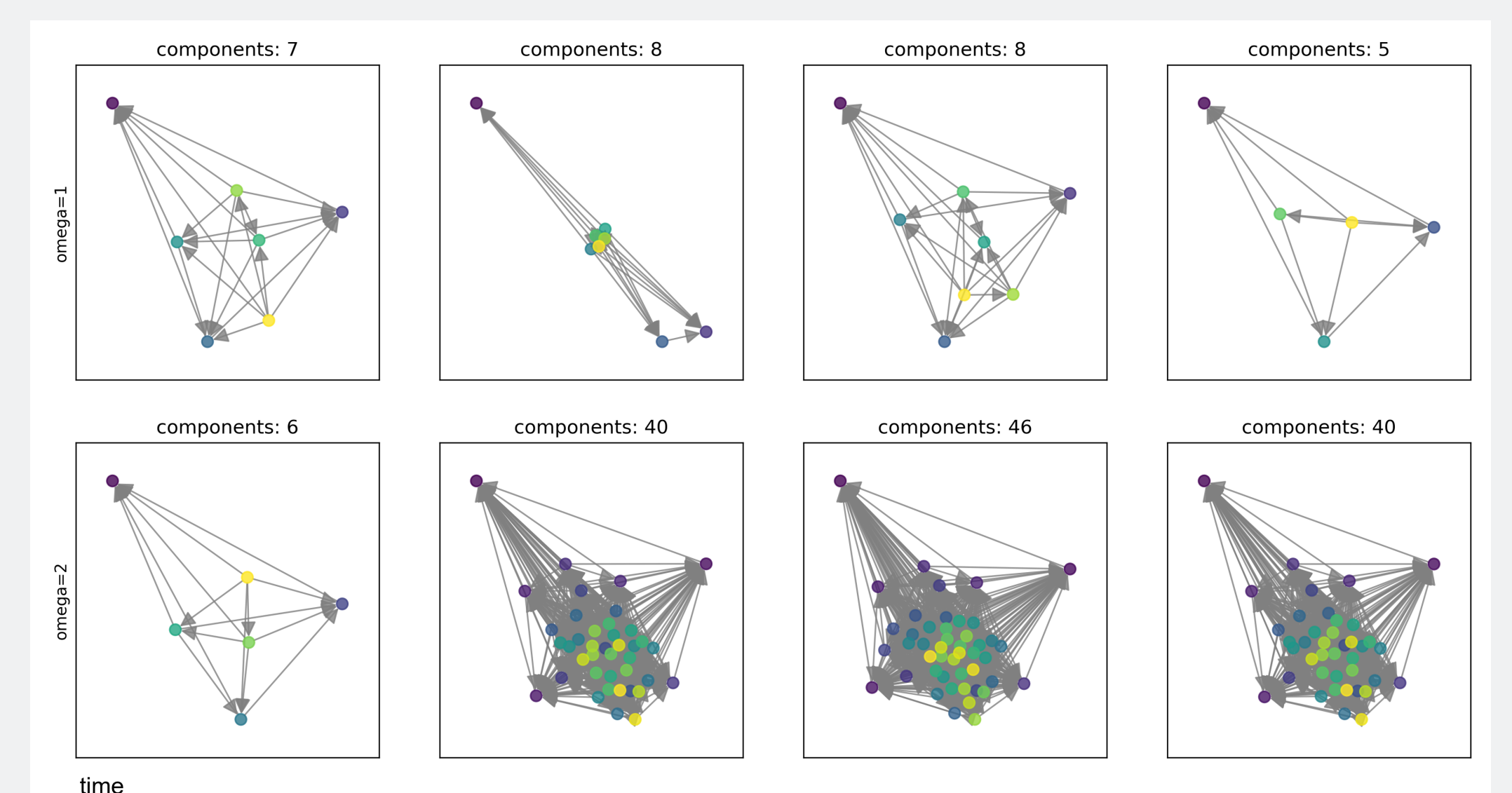
Interaction: interaction factor c

larger coupling weights – more equalizing movements – more uniform motion – less components
nonlinear nodal dynamics, $p = 0.2$, $\Omega = 2$



Excitation: frequency Ω

lower excitation frequency – less cluster formation
nonlinear nodal dynamics, $p = 0.2$, $c = 0.1$



Conclusion and outlook

- condensed functional networks reveal backbone structure of network dynamics, reveals function-based clusters and main paths of information flow
- method enables study of impact of different elements on network dynamics

Future studies: quantify influence of different factors, exploit findings for network design

<https://doi.org/10.1016/j.jsv.2024.118544>

