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Application of Pendulum Theory to a Model of the Overturning of a Rectangular Hull in a Water Tank

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ABSTRACT

This paper reports on an experimental study of applying pendulum theory to determine whether a rectangular hull model would overturn. A pontoon used as a non-self-navigating work vessel was considered in this study. Its model begins to list when an unbalanced load is applied. It can then either stabilize in an inclined state or continue overturning, throwing the unbalanced load midway, and finally stabilizing in the inverted position. In a mid-air experiment, this correlation was made between two motion scenarios, pendular or rotational motion. The two motions were reproduced by changing the distance between the hull center and the rotary axis and the mass of the weight. The experimental results closely matched those of an analysis performed based on the Runge-Kutta method, which confirms the validity of the proposed analysis method.

1 INTRODUCTION

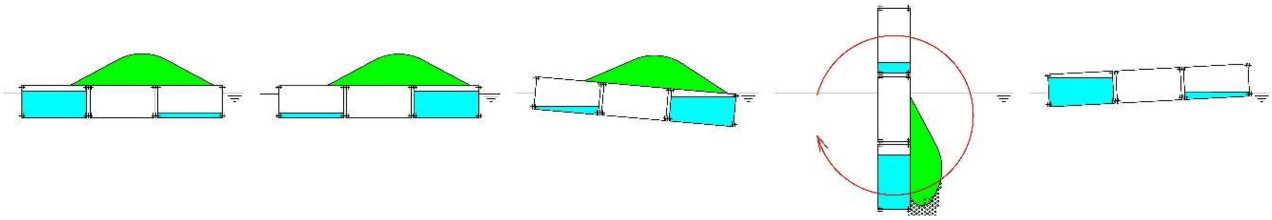
Connected rectangular knockdown pontoons are often used as hulls of various work vessels. The soil pontoon illustrated in Fig. 1 is one example of such work vessels. One study has introduced a self-dumping pontoon system with a hull overturning mechanism allowing to unload earth and sand from a work vessel [1]. According to this study, the hull initially keeps the horizontal position because the portside ballast tank is filled with water and the starboard deck is loaded with soil and sand as shown in Fig. 2(a). As the ballast water is moved to the starboard using compressed air (Fig. 2(b)), the hull overturns and drops the soil and sand (Figs. 2(c)-2(e)).

We have carried out some mid-air experiments to understand the full movement of a hull during the overturning process [2]. The rotational axis of the model used in the experiment is slightly offset from its center as shown in Fig. 3(a), and the mass of the weight changes. After the weight is dropped, the hull continues a rotary motion at a nonconstant speed or a pendulum motion as shown in Figs. 3(b)-3(d). We have correlated the rotary motion with the movement of overturning and the pendulum motion with the movement of not

Figure 1: Soil pontoon using knockdown pontoon

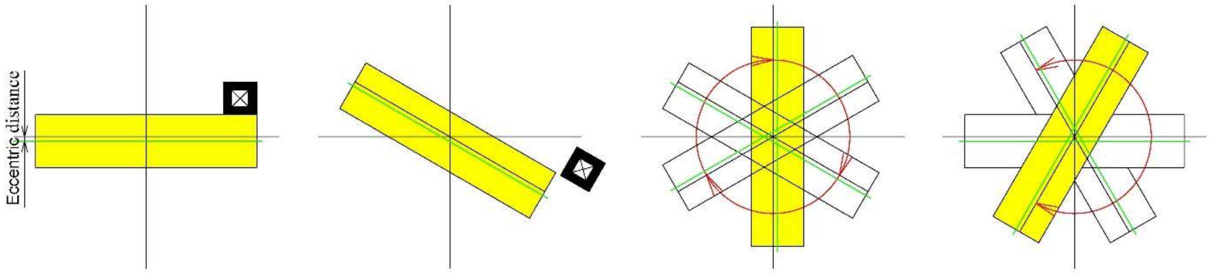
overturning [3-5]. The overturning hull has been considered to be a rigid pendulum with a large swing angle, and the Runge-Kutta method has been used to analyze its motion. For all considered cases, the experimental results have closely matched the analytical results.

In the study presented in this paper, we have conducted similar experiments in a water tank and compared the experimental and analytical results.



(a) Loading state (b) Ballast water: movement (c) Start of overturning (d) Dropping soil (e) Termination

Figure 2: Overturning mechanism for a self-dumping pontoon

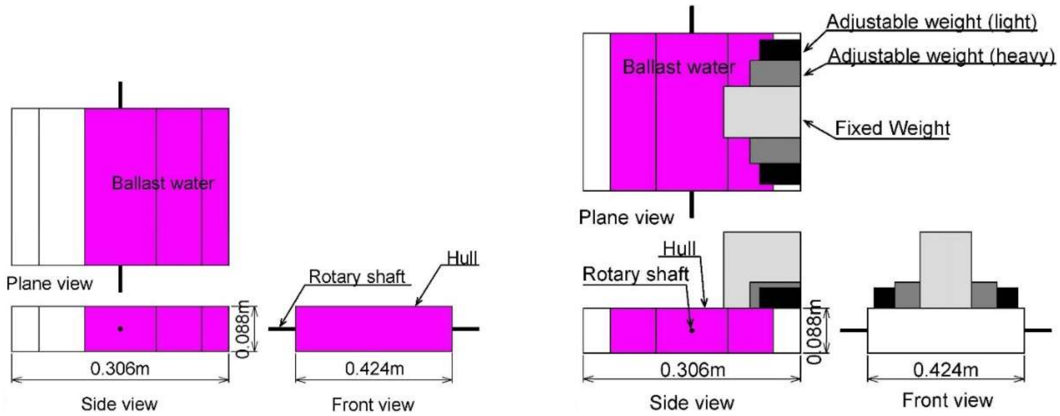


(a) Initial position (b) Overturning and fall of weight (c) Rotary motion at a nonconstant speed (d) Pendular motion

Figure 3: Stages of the motion of a model in a midair experiment

2 EXPERIMENTAL METHOD

Figure 4 illustrates two types of the considered rectangular hull model (hereafter, “model”). Mode A models the hull without soil and sand, and makes a typical pendular motion in water (Fig. 4(a)). Mode B models the hull with soil and sand, and makes pendular motion in the case when the buoyancy force of soil and sand changes (Fig. 4(b)). The rotary axis is located at the center of the model. The model is placed in a water tank as shown in Fig. 5. Ballast water or some weight is placed on the model to act as an eccentric load causing the model to attempt a rotation. To prevent the rotation, the model is supported with a rope. The rope is then cut to initiate the rotation, and the model starts the movement.



(a) Mode A (b) Mode B

Figure 4: Shapes of the hull models

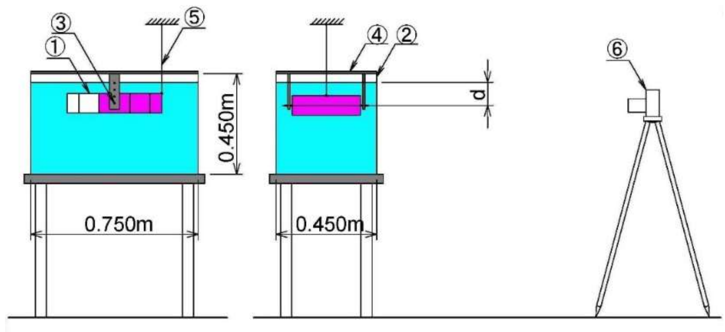


Figure 5: Experimental apparatus for simulating model motions

3 METHOD FOR EVALUATING THE MODEL OVERTURN

3.1 Equation of a simple pendular motion

Figure 6 illustrates a simple pendulum, where l denotes the length of the arm, θ_0 denotes the initial angle of the arm, θ denotes the angle of the arm from the vertical position, m denotes the mass, and v denotes the circumferential velocity. The equation of motion for the pendulum is shown in Eq. (1) [6].

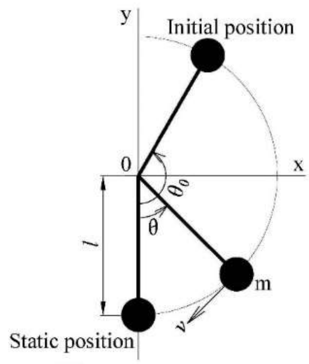


Figure 6: Diagram of a simple pendulum

$$\frac{d^2\theta}{dt^2} = -\frac{g}{l}\sin\theta \tag{1}$$

Figure 7 illustrates the relationship of the gravitational and buoyancy forces acting on the model, where l_{Wx} denotes the distance between the rotary axis and the gravity center, W_o denotes the gravitational force acting on the horizontal component of the model, l_{Bx} denotes the distance between the rotary axis and the buoyancy center, B_o denotes the buoyancy force acting on the horizontal component of the model, l_x denotes the pendulum arm length, W denotes the gravity force acting on the pendulum rigid body, B denotes the buoyancy force acting on the pendulum rigid body, and T denotes the apparent specific gravitational force acting on the pendulum rigid body. Equations (2)-(4) indicate the relationship among these valuables. The gravitational acceleration, g, in Eq. (1) corresponds to the apparent gravitational acceleration, g', in Eq. (5), where M denotes the total mass of the model, i.e., the sum of the masses of the hull, ballast water, and weight. The impact of the buoyancy on the pendular motion was evaluated using the apparent gravitational acceleration g'.

$$W = \frac{W_0 * l_{Wx}}{l_x} \tag{2}$$

$$B = \frac{B_0 * l_{Bx}}{l_x} \tag{3}$$

$$T = W - B \tag{4}$$

$$g' = \frac{T}{M} \tag{5}$$

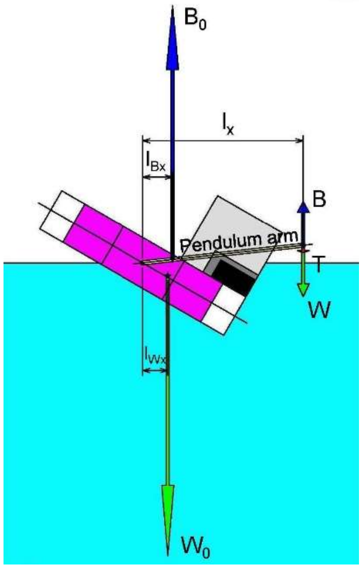


Figure 7: Relationship of gravitational and buoyancy forces acting on the model

Equation (1) of the simple pendular motion can be rewritten as Eqs. (6) and (7) using the angular velocity ω. Quantitative analysis of an ordinary differential equation using the Runge-Kutta method is applied as follows [7]:

$$\frac{d\theta}{dt} = \omega \tag{6}$$

$$\frac{d\omega}{dt} = -\frac{g}{l}\sin\theta \tag{7}$$

where l is defined according to the following equation using the total inertia moment of the hull I, i.e., the sum of the inertia moments of the hull, the ballast water, and the weight, and using the distance, h, between the center of gravity and the rotational axis.

$$l = \frac{I}{Mh} \tag{8}$$

3.2 Validation method

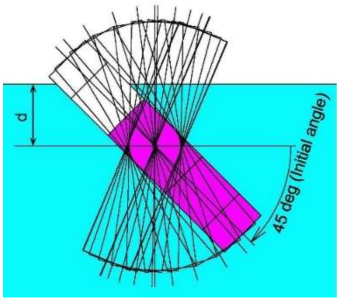
Experiments were repeated three times under the same conditions. The experimental results of these replications were plotted on the graph representing the analytical results to see if they matched, which would mean that the analytical results can be used to determine whether a model would overturn. The progress of the overturning experiment was captured every 0.2 s using continuous photography.

4 EXPERIMENT OF THE RIGID PENDULUM IN MODE A

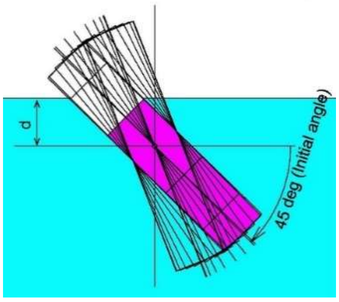
In Mode A, the apparent gravitational acceleration was changed by changing the water level without the weight. The experimental conditions are listed in Table 1, where d denotes the distance between the rotary axis and the water surface and g'_i denotes the apparent gravitational acceleration at the initial point. Figure 8 illustrates the turnover situations in representative experimental Cases 1-1 and 1-7. The turnover angle in the initial point is 45°, and d in Fig. 8(a) is larger than that in Fig. 8(b). The buoyancy caused by this water level acts as the gravity on the pendulum. Therefore, the apparent gravitational acceleration in Case 1-1 increases more than that in Case 1-7 (Table 1).

Table 1: Experimental conditions of Mode A

Experimental case	Experimental condition	Hull model		Rigid pendulum	
	d m	M kg	I kg·m^2	l m	g'_i m/s^2
1-1	0.106	8.660	0.1148	0.2561	3.758
1-2	0.102	8.660	0.1148	0.2561	3.268
1-3	0.098	8.660	0.1148	0.2561	2.796
1-4	0.094	8.660	0.1148	0.2561	2.344
1-5	0.090	8.660	0.1148	0.2561	1.910
1-6	0.086	8.660	0.1148	0.2561	1.496
1-7	0.082	8.660	0.1148	0.2561	1.100



(a) High water surface (Case 1-1)



(b) Low water surface (Case 1-7)

Figure 8: Diagram showing the overturning procedure for Mode A

The experimental and analysis results are shown in Figs. 9 and 10, respectively. The model swung in experimental Case 1-1–1-4 (Fig. 9), whereas it stood still at around 90° and did not swing in Case 1-5. This tendency was similar according to both the analysis and experimental results. The model swung as illustrated in Fig. 10 in the cases with a large water surface (up to Case 1-6), whereas it did not swing in the cases with a small water surface (Case 1-7 and above). According to both the experimental and analysis results, the turnover speed became low as the apparent gravitational acceleration decreased (Table 1). Because of these results, it can be concluded that it is possible to analyze the hull motion using the pendulum theory.

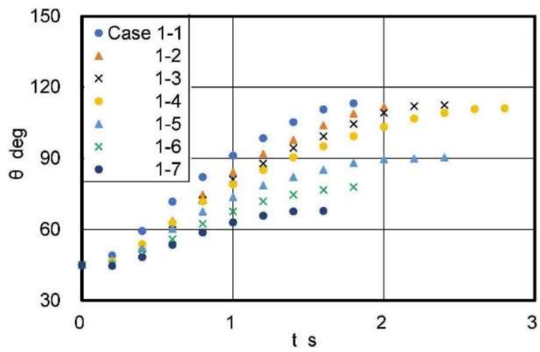


Figure 9: Experimental results for Mode A

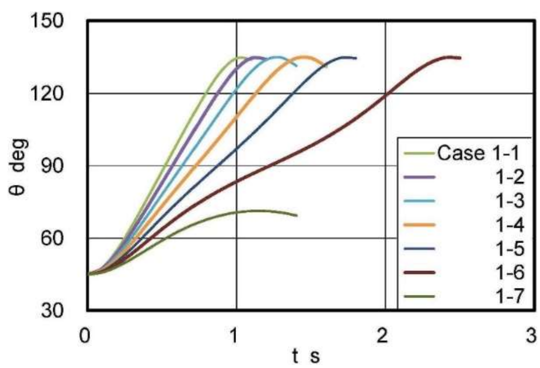


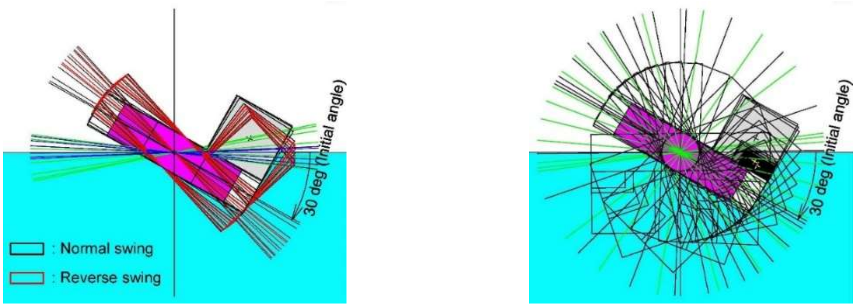
Figure 10: Analytical results for Mode A

5 EXPERIMENT OF THE RIGID PENDULUM IN MODE B

In Mode B, the mass of the weight was changed to assume the change of the load capacity of a self-dumping pontoon. The experimental conditions are listed in Table 2, where mw denotes the mass of the weight. Figure 11 illustrates the turnover situations in the representative experimental Cases 2-1 and 2-7. The turnover angle in the initial point is 30°. In all cases, the water surface is as high as the rotary axis of the model. The ballast tank at both ends of the model is empty. Therefore, the weight has to be placed on the model to initiate the overturning. At the initial point, the gravitational force acting on the model is larger than the buoyancy force because the weight is exposed from the surface of the water. As the model overturns, the buoyancy force gradually increases. Therefore, the model does not overturn when the weight is light (Fig. 11(a)), whereas it overturns when the weigh is heavy (Fig. 11(b)).

Table 2: Experimental conditions of Mode B

Experimental case	Experimental condition	Hull model		Rigid pendulum	
	m_w kg	M kg	I kg·m^2	l m	g'_i m/s^2
2-1	3.587	13.132	0.2332	0.3602	-1.1957
2-2	3.775	13.340	0.2396	0.3525	-0.6380
2-3	3.956	13.501	0.2451	0.3453	-0.1794
2-4	4.144	13.689	0.2515	0.3387	0.2822
2-5	4.332	13.877	0.2579	0.3327	0.7030
2-6	4.514	14.059	0.2634	0.3269	1.0533
2-7	4.702	14.247	0.2698	0.3217	1.4101



(a) Light weight (Case 2-1) (b) Heavy weight (Case2-7)

Figure 11: Diagram showing the overturning procedure for Mode B

The experimental and analysis results are shown in Figs. 12 and 13, respectively. The model did not overturn in experimental Case 2-1–2-3 (Fig. 12), whereas it overturned in all cases from Case 2-4. This tendency was similar in both the analysis and experimental results (Fig.13). The model did not overturn when the weight was light in all cases up to Case 2-3, whereas it overturned as the weight became heavy in all cases from Case 2-4. According to both the experimental and analysis results, the turnover speed became high as the weight increased (Table 2).

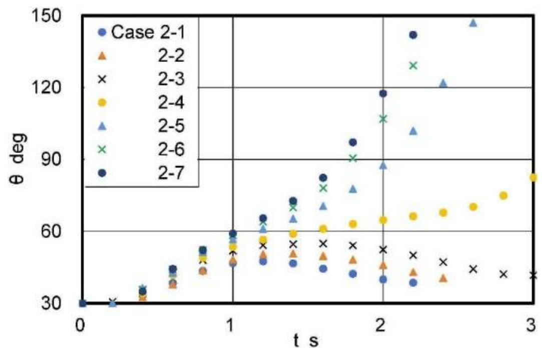


Figure 12: Experimental results for Mode B

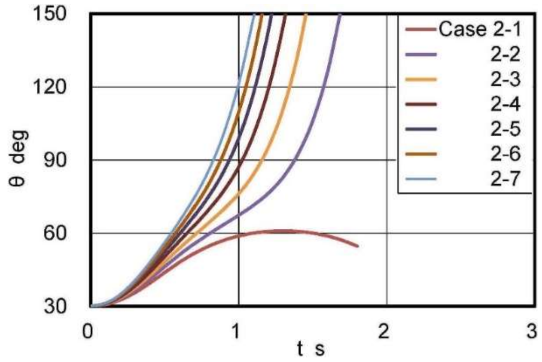


Figure 13: Analytical results for Mode B

6 CONCLUSION

In this study, pendulum theory was applied to the rotating model of a hull in a water tank and several experiments were conducted with the model. The experimental results closely matched the results of the analysis using the Runge-Kutta method confirming its validity. Hence, it can be concluded that it is possible to evaluate whether the vessel model would overturn using the proposed analytical analysis. In the future, we plan to extend the proposed analysis by considering the viscous force.

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REFERENCES

[1] Doya, Y. and Tsutsumi, S. “Self-dumping pontoon system utilizing the versatile unit construct flotation”. In: *International Journal of the Society of Materials Engineering for Resources*, 20 (2014), 118-125.

[2] Doya, Y. and Sawada, T. “Application of Pendulum Theory to Overturning Model of the Rectangular Hull”. In: The 10th International Workshop on Ship and Marine Hydrodynamics, Keelung, Taiwan, (2017).

[3] Clement,A. et al. “The SEAREV wave energy converter”. In: *6th European Wave and Tidal Energy Conference*, Glasgow, UK, (2005), 81-89.

[4] Zelei, A. and Stepain, G. “Stabilizing effect of vertical periodic motion on athletes-paddle systems”. In: *Proceedings of the 3rd Hungarian Conference on Biomechanics*, Budapest, Hungary, (2008), 415-422.

[5] Arief, I. S. et al. “Response to pontoon and pendulum motion at wave energy converter based on pendulum system” In: *6th International Energy Conference, E3S Web of Conference*, 43, 01022, (2018).

[6] Ference, M. J. R., Lemon, H. B. and Stephenson, R. J. *Period of a simple pendulum, Analytical experimental* physics. University of Chicago Press, IL, (1966), 132-134.

[7] Stanton, R. G. The Runge-Kutta method, *Numerical methods for science and engineering*. Prentice Hall, NJ, (1961), 151-154.