Modelling shear wave propagation in soft tissue surrogates using a finite element- and finite difference method

Jakob Ohlsen[1,*], **Maximilian Neidhardt**[2], **Alexander Schlaefer**[2], and **Norbert Hoffmann**[1]

[1] Dynamics Group, Hamburg University of Technology, Schloßmühlendamm 30, 21073 Hamburg

[2] Institute of Medical Technology and Intelligent Systems, Hamburg University of Technology, Am Schwarzenberg-Campus 3, 21073 Hamburg

Shear Wave Elasticity Imaging (SWEI) has become a popular medical imaging technique [1] in which soft tissue is excited by the acoustic radiation forces of a focused ultrasonic beam. Tissue stiffness can then be derived from measurements of shear wave propagation speeds [2]. The main objective of this work is a comparison of a finite element (FEM) and a finite difference method (FDM) in terms of their computational efficiency when modeling shear wave propagation in tissue phantoms. Moreover, the propagation of shear waves is examined in experiments with ballistic gelatin to assess the simulation results. In comparison to the FEM the investigated FDM proves to be significantly more performant for this computing task.

1 Preliminaries

The Navier-Cauchy equation, also known as the elastodynamic wave equation, describes the behavior of homogeneous isotropic linear-elastic materials for small deformations. It reads as follows:

$$\rho \ddot{\mathbf{u}} = \mu \Delta \mathbf{u} + (\lambda + \mu) \operatorname{grad} (\operatorname{div} (\mathbf{u})) + \rho \mathbf{k} \tag{1}$$

with ρ as the density, λ and μ as the Lamé parameters, $\mathbf{u}$ as the displacement vector and $\mathbf{k}$ as a source function (mass-distributed force) [3]. The computation of the shear wave velocity can be deduced from Eq. 1 as $c_s = \sqrt{\mu/\rho}$ [2].

2 Numerical simulations

Simulations were conducted with models representing cubical gelatin blocks of side $0.1 \, \mathrm{m}$ which are subject to a periodic excitation force ($0.001 \, \mathrm{N}$, $100 \, \mathrm{Hz}$) acting on the block-midpoint. The numerical experiments were performed for three linear elastic materials with parameters as listed in Table 1. The parameters were chosen based on previously performed mechanical compression tests on gelatin specimens. A spatial resolution of $7.8125 \times 10^{-4} \, \mathrm{m}$ was defined for the regular grid (FDM) as well as the corresponding mesh (FEM). In addition to this reference configuration, simulations were performed with a coarser ($1.1042 \times 10^{-3} \, \mathrm{m}$) and a finer ($6.25 \times 10^{-4} \, \mathrm{m}$) resolution to evaluate the computational efficiency of both methods. All computations were carried out on an Intel Core i7-8750H processor. Time periods of $8 \, \mathrm{ms}$ were simulated with fixed-size time steps Δt chosen to satisfy the Courant–Friedrichs–Lewy condition.

Finite element simulations were conducted using the commercial software Abaqus/CAE 2018 [4] and its explicit integration method. Hexahedral elements (type C3D8R) were used to create a regular mesh in order to comply with the Cartesian grid used in the FDM. Zero displacement boundary conditions were applied at all free surfaces of the cubic geometry.

The finite difference code used in this work was developed at the Dynamics Group of the Hamburg University of Technology. Data structures used in the C++ code are inspired by the open-source library OpenVDB [5] and allow for fast data access of high-resolution volumes. The underlying block structure enables cache efficient and vectorized operations on the Cartesian grid. A discrete form of Eqn. 1 is solved by using fourth order accurate differential operators [6] to compute spatial derivatives together with a backward Euler method of fourth order accuracy. Cross-correlation of node-displacements is used to compute shear wave velocities.

3 Experiment

All experiments described in the following were conducted using cuboid blocks of GELITA ballistic type 3 gelatin with dimensions of $100 \times 100 \times 60 \, \mathrm{mm}$. The setup is depicted in Fig. 1. We employ an ultrasound device (Cephasonics, 7.5 MHz linear array transducer) for excitation and imaging of propagating shear waves. Ultrasonic beams are focused at a depth of $30 \, \mathrm{mm}$ for shear wave excitation. Subsequently, high-frame-rate plane wave imaging is performed.

The experiment was carried out for gelatin concentrations 10%, 15% and 20% (mass gelatin/total mass). For each concentration two transducer positions were chosen. Shear wave propagation was captured with a minimum frame rate of 7000

---

* Corresponding author: e-mail jakob.ohlsen@tuhh.de, phone +49 40 428 782 993

PAMM · Proc. Appl. Math. Mech. 2020;**20**:1 e202000148. www.gamm-proceedings.com **1 of 2**

https://doi.org/10.1002/pamm.202000148 © 2021 The Authors *Proceedings in Applied Mathematics & Mechanics* published by Wiley-VCH GmbH
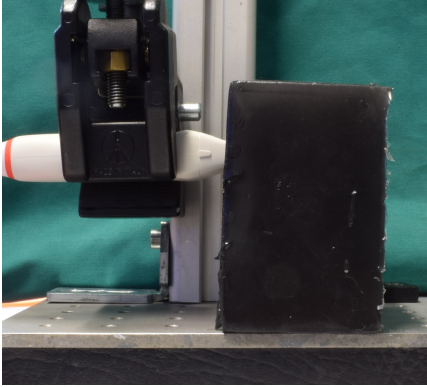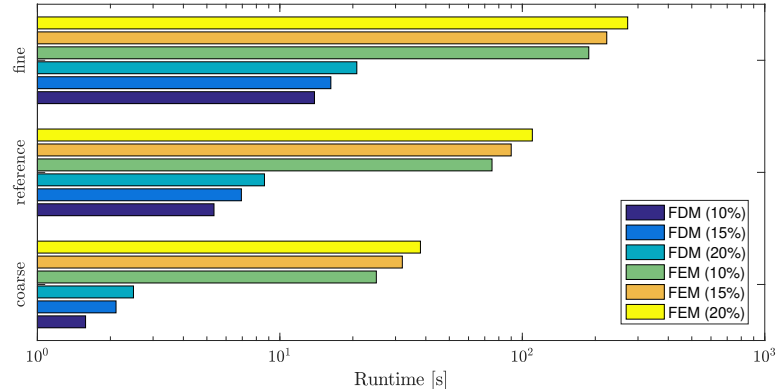
Fig. 1: Experimental setup

Fig. 2: Runtimes of numerical simulations for coarse to fine spatial resolutions

Table 1: Examined gelatin concentrations and shear wave speeds c_s [m s^{-1}] (theoretical, modelled, measured) together with the Young's Moduli E [kPa], Poisson ratios ν, densities ρ [kg m^{-3}] and time increments Δt_c, Δt_r, Δt_f [µs] (coarse, reference, fine).

Material	E	ν	ρ	Δt_c	Δt_r	Δt_f	$c_{s,\text{theo}}$	$c_{s,\text{FEM}}$	$c_{s,\text{FDM}}$	$c_{s,\text{exp}}$
10% Gelatin	49.77	0.49	1017	27	14	7	4.05	3.86 ± 0.03	4.10 ± 0.04	4.23 ± 0.10
15% Gelatin	80.27	0.49	1065	22	11	5	5.03	4.81 ± 0.05	5.09 ± 0.04	6.32 ± 0.19
20% Gelatin	119.32	0.49	1090	18	9	4	6.06	5.80 ± 0.10	6.15 ± 0.06	7.68 ± 0.23

frames per second. To reduce noise and artifacts the ultrasound data was filtered according to [7] and processed using a Loupas' algorithm to compute displacement fields [8,9]. Shear wave velocities were then estimated by using cross-correlation of displacement signals and averaged over the data points of the computed displacement fields.

4 Results and Discussion

The simulated shear wave velocities show a good agreement to the theoretical values for all three linear elastic materials as shown in Table 1. The average shear wave velocities simulated by using the FEM and FDM differ by 0.21 to 0.26 m s^{-1} and respectively 0.05 to 0.09 m s^{-1} from the theoretical shear wave velocity. Depending on the problem size the computational costs of the finite difference simulations were 13 to 15 times lower than the respective finite element simulations as shown in Figure 2 while being more accurate at the same time. The experimentally determined wave velocities increase with the gelatin concentration. However, a discrepancy to the theoretical wave velocities can be observed in particular for higher gelatin concentrations. The discrepancy of simulated and experimentally determined wave speeds appears to be due to the fact that the approximation of tissue-like materials by a linear elastic material model is not sufficiently accurate. Efficient memory management and therefore fast grid operations make the finite difference code used in this work preferable over the finite element method within the Abaqus framework for this computing task. It should be noted that the investigated FDM in its current state is limited to periodic boundary conditions and allows only simple geometries. The treatment of free boundaries and the implementation of more complex material models is subject of ongoing work and shall be investigated in further research activities.

References

[1] C. H. Suh, H. M. Yoon, S. C. Jung, and Y. J. Choi, PLOS ONE **14**(7), e0219621 (2019).
[2] A. P. Sarvazyan, O. V. Rudenko, S. D. Swanson, J. Fowlkes, and S. Y. Emelianov, Ultrasound in Medicine & Biology **24**(9), 1419–1435 (1998).
[3] H. Altenbach, Kontinuumsmechanik (Springer Berlin Heidelberg, 2015).
[4] M. Smith, ABAQUS User's Manual, Version 2018 (Dassault Systèmes Simulia Corp, United States, 2018).
[5] K. Museth, ACM Transactions on Graphics **32**(3), 1–22 (2013).
[6] V. N. Kukudzhanov, Numerical Continuum Mechanics (DE GRUYTER, January 2012).
[7] T. Nordenfur, Comparison of pushing sequences for shear wave elastography, Master's thesis, KTH Royal Institute of Technology, 2013.
[8] T. Loupas, J. Powers, and R. Gill, IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control **42**(4), 672–688 (1995).
[9] G. Pinton, J. Dahl, and G. Trahey, IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control **53**(6), 1103–1117 (2006).