Received: 11 June 2020 | Accepted: 24 November 2020

DOI: 10.1002/pamm.202000066

An Efficient Adjoint Sensitivity Analysis of Flexible Multibody Systems for a Level-set-based Topology Optimization

Ali Azari Nejat[1,*], **Alexander Held**[1], and **Robert Seifried**[1]

[1] Institute of Mechanics and Ocean Engineering, Hamburg University of Technology, Eißendorfer Straße 42, 21073 Hamburg, Germany

For large-scale topology optimization of flexible multibody systems, only little results exist. This is due to the complexity of the modeling of the flexible bodies and the big effort to provide exact gradients. The considered flexible multibody systems can undergo both large nonlinear motions as well as small elastic deformations. Here, the flexible components are modeled by the floating frame of reference approach. For gradient calculation, the fully coupled adjoint sensitivity analysis is used, which is a semi-analytical approach based on variational calculus. The computational effort strongly corresponds to the number of design variables. In this work, a design space reduction using radial basis functions is performed and the gradient of flexible components is constructed based on its exact value on a subset of selected design elements. In order to show the substantial gain in computation time, the exact and approximated gradient of a flexible crank in a slider-crank mechanism are computed and applied for a level-set-based topology optimization.

1 Floating frame of reference formulation

In dynamics, the method of flexible multibody systems is a well-known approach for modeling and analyzing dynamic systems, which are characterized by large nonlinear motions including vibrations and deformations. Provided that the deformations remain small and linear elastic, the overall motion of flexible bodies can be efficiently formulated using the floating frame of reference approach. Hereby, the body deformation is described in a body related reference frame undergoing large rigid body motions, see [2]. This allows an approximation of the elastic displacement $\boldsymbol{u}_\mathrm{P}$ and rotation $\boldsymbol{\nu}_\mathrm{P}$ for an arbitrary point P on an elastic body by the Ritz method as

$$\boldsymbol{u}_\mathrm{P} \approx \boldsymbol{\Phi}_\mathrm{P}\boldsymbol{q}_\mathrm{e}, \qquad \text{and} \qquad \boldsymbol{\nu}_\mathrm{P} \approx \boldsymbol{\Psi}_\mathrm{P}\boldsymbol{q}_\mathrm{e}. \tag{1}$$

Thereby, $\boldsymbol{\Phi}_\mathrm{P}$ and $\boldsymbol{\Psi}_\mathrm{P}$ are the matrices of the position-dependent global shape functions for the elastic displacements and rotations, and $\boldsymbol{q}_\mathrm{e}$ is the vector of time-dependent elastic coordinates. The global shape functions are often obtained by a model reduction from finely discretized finite-element models. The latter can be reused in the topology optimization. With an appropriate selection of the global shape functions, the flexible bodies can then be modeled and incorporated into multibody systems, so that a compact formulation of the equations of motion for the multibody systems results, which reveals a high modeling and simulation efficiency despite convenient accuracy.

2 Approximate adjoint sensitivity analysis

In this work, the objective function ψ is the integral compliance of flexible bodies over the simulation time interval $[t_0, t_1]$, and can be formulated with elastic coordinates $\boldsymbol{q}_\mathrm{e}$ and the reduced stiffness matrix $\boldsymbol{K}_\mathrm{e}$ as

$$\psi = \int_{t_0}^{t_1} \boldsymbol{q}_\mathrm{e}^\top \boldsymbol{K}_\mathrm{e}\boldsymbol{q}_\mathrm{e}\, \mathrm{d}t. \tag{2}$$

Utilizing the adjoint sensitivity analysis, the gradient $\nabla\psi$ of the objective function ψ with respect to a number n of selected design variables $\boldsymbol{x} \in \mathbb{R}^n$ can be deduced by variational calculus, see [1]. Here, the design variables are density-like parameters, which correspond to the filling amount of n design elements. The gradient results by the evaluation of the integral gradient function

$$\nabla\psi = \int_{t_0}^{t_1} \left(\boldsymbol{T}^x_\mathrm{OF} - \boldsymbol{\mu}^\top \boldsymbol{T}^x_\mathrm{KR} - \boldsymbol{\nu}^\top \boldsymbol{T}^x_\mathrm{EM}\right) \mathrm{d}t. \tag{3}$$

* Corresponding author: e-mail ali.azari@tuhh.de, phone +49 40 428 782 742, fax +49 40 427 314 601

Thereby, $\boldsymbol{\mu}$ and $\boldsymbol{\nu}$ are adjoint variables, which are the solution of an adjoint system, see [1]. Furthermore, the auxiliary vector $\boldsymbol{T}^x_{\mathrm{OF}}$ and the auxiliary matrices $\boldsymbol{T}^x_{\mathrm{KR}}$ and $\boldsymbol{T}^x_{\mathrm{EM}}$ include, one after the other, the derivatives of the objective function, kinematic relation and the equations of motion with respect to the design variables $\boldsymbol{x}$. For the computation of these terms, among others, the derivatives of the shape functions and of the volume integrals of system matrices with respect to the design variables are required. The computational effort of these two steps and gradient evaluation (3) depends directly on the number n of design variables. Hence, an appropriate reduction of the design variables helps to limit the required computation time. For this purpose, radial basis functions are used here, see for instance Fig. 1. Thereby, the unknown gradient at an arbitrary point P on the flexible body with the position vector $\boldsymbol{r}_{\mathrm{P}}$ is approximated by a linear combination of a number m of gradient parameters α_j and radial basis functions g_j as

$$\nabla\psi(\boldsymbol{r}_{\mathrm{P}}) \approx \sum_{j=1}^{m} \alpha_j g_j(\boldsymbol{r}_{\mathrm{P}}), \tag{4}$$

see also [3]. The gradient parameters α_j result from a parametrization of the known gradient values $\nabla\psi_j$ for the chosen design variables $\bar{\boldsymbol{x}} \in \mathbb{R}^m$, which are selected independently of the underlying FE-mesh from the entire set of design variables, i. e. $\bar{\boldsymbol{x}} \subseteq \boldsymbol{x}$. In other words, here, the sensitivity analysis is performed for a reduced design space.

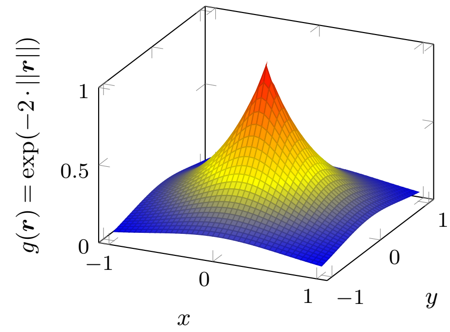

Fig. 1: Exponential spline g

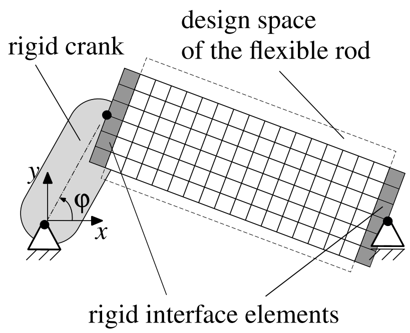

Fig. 2: Application example

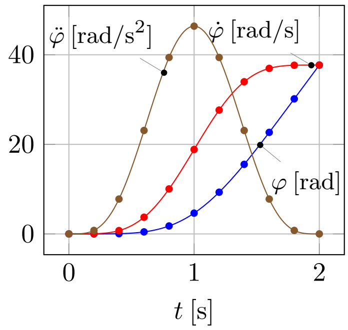

Fig. 3: Crank motion

3 Application example and conclusion

In this work, the compliance minimization problem of the flexible rod in a slider-crank mechanism is considered, see Fig. 2. The flexible rod is discretized by 200×20 elements. Thereby, the SIMP-parametrized elements compose the design space, whereby the interface elements are assumed as rigid to ensure the load transfer between the floating bearing, the flexible rod and the rigid crank. In Fig. 3, the motion of the rigid crank during a simulation time of 2 s is given. Using the approximate adjoint sensitivity analysis within an explicit update scheme, see also [4], a level-set-based topology optimization of the flexible rod is performed. In each optimization iteration, the approximated gradient is constructed using the exact gradient value of a set of design elements on a coarse mesh and around the active boundaries. In Fig. 4, the amount and distribution of these considered design elements along the optimization iterations are shown. In Fig 5, this irregular distribution is indicated for the initial rod design. Reducing the number of gradient points from the whole number of design elements to a proportion between 30% and 40%, the computational effort of shape function and volume integral sensitivity calculation and gradient evaluation are significantly reduced. The topology optimization using the approximate adjoint sensitivity analysis takes 274 min, whereas the same procedure with a sensitivity calculation for the whole design space takes 467 min. However, as it is shown in Figs. 5 and 6, both optimizations converge to similar final designs with a compliance of $\psi = 0.05$ Nmm. In summary, the presented

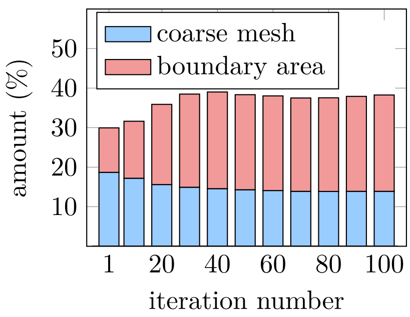

Fig. 4: Design elements

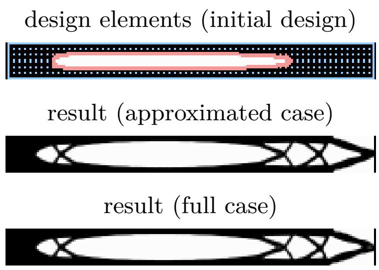

Fig. 5: Initial and final designs

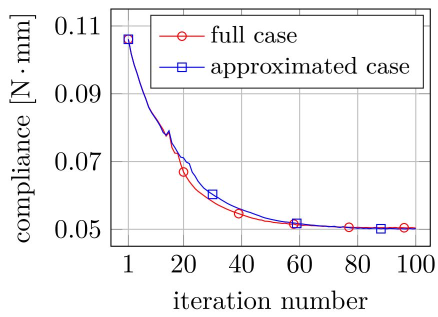

Fig. 6: Compliance of the flexible rod

idea of gradient approximation by radial basis functions can be seen as one possibility to reduce the tremendous effort producing exact gradients in large-scale topology optimization. However, the number and position of selected gradient points and the type of radial basis functions have an influence on the quality of the gradient approximation. Therefore, caution should be exercised when reducing the design space to avoid approximated gradients and, thus, optimization results of poor quality.

Acknowledgements Open access funding enabled and organized by Projekt DEAL.

References

[1] A. Held, S. Knüfer, and R. Seifried, Structural sensitivity analysis of flexible multibody systems modeled with the floating frame of reference approach using the adjoint variable method, Multibody System Dynamics **40**, 287-302 (2017).

[2] R. Seifried, Dynamics of Underactuated Multibody Systems - Modeling, Control and Optimal Design (Springer, Switzerland, 2014).

[3] Y. Wang, and Z. Kang, A velocity field level set method for shape and topology optimization, International Journal for Numerical Methods in Engineering **115**, 1315-1336 (2018).

[4] P. Wei, Z. Li, X. Li, and M. Y. Wang, An 88-line MATLAB code for the parameterized level set method based topology optimization using radial basis functions, Structural and Multidisciplinary Optimization **58**, 831-849 (2018).

