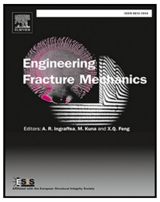
Engineering Fracture Mechanics 321 (2025) 111055

Contents lists available at ScienceDirect

Engineering Fracture Mechanics

journal homepage: www.elsevier.com/locate/engfracmech



Dynamic fracture in DCB specimen loaded by end moments

M. Ciavarella *

Politecnico di BARI. DMMM department., Viale Gentile 182, 70126 Bari., Italy
Hamburg University of Technology, Department of Mechanical Engineering, Am Schwarzenberg-Campus 1, 21073 Hamburg, Germany

ARTICLE INFO

Keywords:
Crack propagation
Inertia
Dynamic crack propagation
DCB specimen

ABSTRACT

Dynamic fracture depends on a delicate interplay of the dynamic Energy Release Rate (ERR) G_d which is the static value G_s as reduced by inertia terms depending on crack speed and sometimes acceleration, and the dynamic toughness Γ_d which generally varies with crack speed. We review some basic results on Double Cantilever Beam specimen, and make a simple analysis of a DCB under remote bending loads applied slowly. We comment that smooth propagation ensues here contrary to the case discussed in the literature where the loading the end by a constant displacement rate, a very complex non-linearly oscillating velocity results, possibly leading to crack arrest (stick–slip) and which is only partly captured by present models, making difficult the determination of the dynamic toughness.

1. Introduction

Delamination of composite materials, or failure of adhesive bonding of lightweight, high-performance materials give constraints to crashworthiness, improved fuel economy and reduced exhaust emissions in automotive or aircraft structures. Typically, adhesives are based on polymers exhibiting rate-dependent deformations, and this has prompted remarkable research into structural integrity as a function of loading rate [1–5]. Dynamic fracture mechanics is a relatively established field [6], but understanding fracture under dynamic loading requires profound knowledge and measurement of rate and transient effects in loading of a crack, and in resistance opposed by the material to this loading. Many transient effects emerge due to inertia, which depend also on loading mode and geometry.

1.1. General concepts in dynamic fracture

To write conservation of energy, the total energy in a specimen is the sum of the strain W, kinetic T, and dissipated Ω energies, and is equal to the external work U_{ext}. Thus, during crack propagation when size A of crack area changes $\partial/\partial A(W+T+\Omega-U_{ext})=0$, which is a generalization of the well known Griffith energy balance criterion. We can define $G_d = -\partial\left(W+T-U_{ext}\right)/\partial A$ as the dynamic energy release rate (ERR), whereas $\Gamma_d = \partial\Omega/\partial A$ is the dynamic critical fracture energy of the material (dissipated by various possible mechanisms including surface energy per unit area A created). Standard dynamic fracture mechanics [6] suggests that under dynamic conditions, for cracks in infinite systems, the dynamic Energy Release Rate (ERR) G_d for a Griffith crack of size $A=2Ba$ (where B is the transverse thickness of the specimen and a the halfwidth of the crack) at time t, propagating at speed $\dot{a}$

$$G_d\left(a,t,\dot{a}\right)=g\left(\dot{a}\right)G_s\left(a,t\right) \tag{1}$$

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* Correspondence to: Politecnico di BARI. DMMM department., Viale Gentile 182, 70126 Bari., Italy.
E-mail address: Mciava@poliba.it.

https://doi.org/10.1016/j.engfracmech.2025.111055
Received 10 December 2024; Received in revised form 3 March 2025; Accepted 18 March 2025

Nomenclature

W	strain energy
T	kinetic energy
Ω	dissipated energy
U_{ext}	external work
A	crack size
B	plane specimen width
a	crack width
G_d	dynamic Energy Release Rate
G_s	static Energy Release Rate
Γ	toughness of material
Γ_d	dynamic toughness of material
t	time
$g\left(\dot{a}\right)$	corrective function for dynamic ERR in Griffith crack
u	opening displacements of the beam
M_0	applied moment at the ends of the beams' arms
I	beam geometric section second moment
E	elastic modulus
ρ	density of the material
h	beam height
c_s	plane stress wave speed in the material

where $G_s = \frac{\pi\sigma_\infty^2 a}{E}$ is the equilibrium ERR for a Griffith crack, where σ_∞ is remote stress and E the Young's modulus, and $g(\dot{a})$ is a universal function of the crack tip speed (see Freund [6]) for infinite systems

$$g\left(\dot{a}\right) \approx 1 - \dot{a}/c_R \tag{2}$$

where c_R is Rayleigh's speed. The crack is sometimes called "inertialess" crack dynamics [7] because G_d does not depend on acceleration $\ddot{a}$. Hence, classical fracture mechanics analyses predict that in infinite systems tensile cracks in homogeneous isotropic solids under sustained loading for which G_s monotonically increases with crack size once they reach the condition for initiation will monotonically accelerate until they reach wave speeds (Rayleigh in this case)[1]. Indeed, for known $\Gamma_d\left(\dot{a}\right)$ (which can be also independently measured from curvature of crack tip), and Griffith crack with constant σ_∞, we have the differential equation $1 - \frac{E\Gamma_d\left(\dot{a}\right)}{\pi\sigma_\infty^2 a} = \dot{a}/c_R$ which has been shown to give correctly the evolution.

For a finite system, experience shows that (1) is not the correct framework and the crack converges to final values which may be less than wave speeds. For example, in a infinite strip of height H loaded by initial uniform displacement where a guillotine suddenly creates a crack, deviation occurs when waves emanated from crack tip interact with near vertical boundaries i.e. typically when crack size becomes of the same order as H. In steady state propagation there is no change in kinetic energy and one can write energy balance in the form [7]

$$-\frac{\partial W}{\partial A} = \Gamma_d\left(\dot{a}\right) \tag{3}$$

where the dependence of Γ_d on velocity (if stable) means any velocity of crack can be reached, but typically lower than wave speed: if $\Gamma_d\left(\dot{a}\right)$ is decaying, the crack will accelerate to wave speed; if it is increasing as in the experiments with soft materials described in Goldman et al. [7], then the actual crack speed depends on the level of overstressing. Wave speed will be obtained only if the specimen is so "overstressed" that $-\frac{\partial W}{\partial A} = \Gamma_d\left(\dot{a}\right)$ for all $\dot{a}$. However, while the Griffith crack has no "inertia" (i.e. dependence on acceleration $\ddot{a}$), it is possible to write perturbative solution so that the effect of inertia $\ddot{a}$ appears [7], and it is seen that its effect is also exacerbated if speed is near wave speed. Therefore, there is no unique value of load for which there is "fast fracture", as standard quasi-static fracture mechanics tells us, and dynamic toughness Γ_d (which is the resistance opposed to G_d) can show both reductions [1,3], and increase with crack speed, see Goldman et al. [7].

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[1] Crack speeds greater than half the Rayleigh wave speed are rarely observed in such solids, because of crack branching [8]. The situation is different along weak interfaces, where the motion is constrained, see Svetlizky and Fineberg [9].

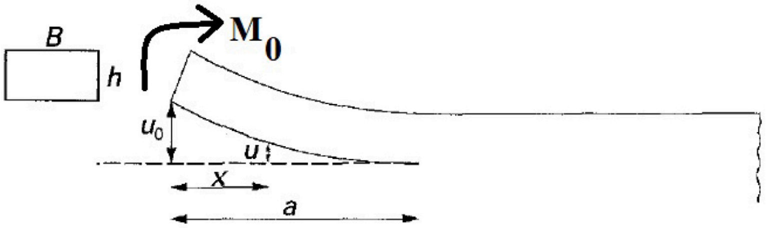

Fig. 1. Geometrical model considered.

1.2. DCB specimen

The case of the DCB has been studied in details in the literature (also because it is a ISO standard for testing of adhesives at least for quasi-static conditions using LEFM[2]) in particular in the configuration with end opening u_0 at constant rate $\dot{u}_0 = V$, where $G_s \sim u_0^2/a^4$ and hence a static analysis shows that the crack propagates (assuming a constant toughness $\Gamma = const$) such that $a(t) \sim \sqrt{u_0(t)} \sim \sqrt{Vt}$ and $\dot{a}(t) \sim \dot{u}_0(t)/\sqrt{u_0(t)} \sim \sqrt{\frac{V}{t}} \sim V/\sqrt{a(t)}$ (after an initiation time t_0) which obviously decreases with propagation, tending to zero asymptotically. This is what some authors [2] call "steady state" although the crack speed is not constant. Considering inertia in a simplified manner (the "Berry's method" which we shall describe and also adopt later), Blackman et al. [2] find dynamic expressions for ERR, G_d which reduce the static G_s by a term which depends only on the loading rate V which can be written also in the steady state as function of $\dot{a}(t), a(t)$ as shown above. Additionally, again using Berry's method, Blackman et al. [2] find also transient expression for dynamic ERR which show a dependence on $\ddot{a}(t)$ and lead to a true dynamic equation for the crack motion. Experimentally, depending on the loading rate, crack propagation can be slow (no significant inertia effects) or fast, and in both cases could be either 'stable' or 'unstable' (stick–slip cycles via short bursts interspaced by periods of crack arrest). In the fast mode, which is generally unstable, crack velocity depends also on details of the test geometry (stiff TDCB — Tapered DCB specimen tend to lead to a far higher crack velocity than less stiff DCB test geometry, where the TDCB is made to obtain a constant G_s during propagation). Blackman et al. [3] reports that crack velocity is linearly proportional to the loading rate speed, see also Karac et al. [4]. In TDCB case of Karac et al. [4], inertia reduces the static and constant applied ERR while loading rate and crack speed are in the same proportion, so crack speed was found to vary from few m/s to 400 m/s (which is one order of magnitude less than wave speed in Al alloys tested) changing the loading rate, corresponding to a decreasing dynamic toughness from 4000 to 2000 J/m^2.

1.3. Overview of the present paper

In the present paper, we shall consider the case of a DCB with remote applied constant moment M_0, which is slowly applied up to a final value which is then kept constant, leading to initiation of the crack propagation. We shall find that with the same approximate method used by some authors to find the effect of inertia (Berry's method, see Blackman et al. [2]), we find no indication of stick–slip and other complex transient effects. This may be an interesting feature to produce alternative measurements of dynamic toughness. For our case, quasi-static standard Linear Elastic Fracture Mechanics (LEFM) shows $G_s = M_0^2/EI$ where I is the beam geometric section inertia, and therefore is independent on crack size. Therefore, for low crack speeds, the quasi static condition $G_s = \Gamma_d\left(\dot{a}\right)$ is the only condition which can determine the crack speed: in the theoretical case when toughness is independent on speed, we have that speed is undetermined, contrary to the DCB loaded by constant rate end opening $\dot{u}_0 = V$ studied in the references we cited where V determines a "quasi-static" crack motion $a(t) \sim \sqrt{Vt}$. However, while in the latter case it was found there is a very complex transient which involves wild non linear oscillations of speed, leading also to crack arrest and restart (stick–slip motion), which challenge the definition of dynamic toughness and require complex investigations, in our case the analysis will turn out much simpler.

2. DCB with end moments

Special fixtures exist to ensure the application of a pure moment M_0 in a DCB (Fig. 1), see Sørensen et al. [10] for details. The ERR is $G_s = M_0^2/EI$ and therefore, a quasi-static analysis seems to suggest that a steady state should exist, although the speed of propagation of the crack is undetermined. First of all, let us explore the conditions for a rigorous true steady state to exist including inertia terms.

The displacement for a beam under bending and with inertia must satisfy the equation in the separated region $0 < x < \dot{a}t$

$$EI\frac{\partial^4 u}{\partial x^4} = -\rho\frac{\partial^2 u}{\partial t^2} \tag{4}$$

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[2] ISO 25215. Determination of the mode I adhesive fracture energy GIC of structural adhesives using the double cantilever beam (DCB) and tapered double cantilever beam (TDCB) specimens; 2009.

where ρ is mass density. Steady states solutions should be of the form

$$u(x,t) = f\left(x - \dot{a}t\right) \tag{5}$$

where $\dot{a}$ is the travelling wave speed, and substitution in the equation of motion leads to a ODE

$$f^{IV}\left(x - \dot{a}t\right) + \lambda^2 f''\left(x - \dot{a}t\right) = 0 \text{ where } \lambda^2 = \frac{\rho\dot{a}^2 h}{EI} = \frac{h}{I}\left(\frac{\dot{a}}{c_s}\right)^2 \tag{6}$$

where $c_s = \sqrt{E/\rho}$ is the plane stress wave speed. The general solution is $f(x) = C_1 + C_2 x + C_3\cos(\lambda x) + C_4\sin(\lambda x)$, and imposing the boundary condition at the crack tip $f(0) = 0$, $f'(0) = \theta_0$, and $f''(0) = \frac{M_0}{EI}$. We consider also that there is no shear force at crack tip, and hence $f'''(0) = 0$ giving $C_4 = 0$. The bending moment along the separated region of the beam is therefore computed and in particular at the loaded end $x = 0$ we would need a non constant moment and shear force

$$M(0,t) = EI\frac{\partial^2 u}{\partial x^2} = M_0\cos\left(\sqrt{12}\frac{\dot{a}}{c_s}\frac{\dot{a}t}{h}\right) \tag{7}$$

$$F(0,t) = \frac{\partial M}{\partial x}(0,t) = \frac{\dot{a}}{c_s}\frac{M_0}{h}\sin\left(\sqrt{12}\frac{\dot{a}}{c_s}\frac{\dot{a}t}{h}\right) \tag{8}$$

Clearly, this is a good approximation to the loading $M = M_0$, and $F = 0$ only if

$$\sqrt{12}\frac{a}{h} \ll \frac{c_s}{\dot{a}} \tag{9}$$

and therefore for not be too long cracks propagating at low speeds, this is reasonable. However, remark that even if we applied these time-varying end loadings, the dynamic ERR at the crack tip which is here $G_d = G_s = M_0^2/EI$ could not fix the rate of propagation $\dot{a}$. The situation is similar to the infinite strip of height H loaded by initial uniform displacement: the actual velocity depends on the shape of the function $\Gamma_d\left(\dot{a}\right)$.

3. Dynamic effects with Berry's method

Berry's method suggested by Blackman et al. [2] consists of solving the quasi-static problem for the beam $\frac{\partial^4 u}{\partial x^4} = 0$ with the usual b.c. $EI\frac{\partial^2 u}{\partial x^2}(0) = M_0$ and $\frac{\partial^3 u}{\partial x^3}(0) = 0$, while $u(a) = u'(a) = 0$. This leads obviously to

$$u(x) = \frac{M_0}{2EI}(x-a)^2 = u_0(1-\xi)^2 \tag{10}$$

where $\xi = x/a$, and $u_0 = \frac{M_0}{2EI}a^2$.

Before the crack is moving, we have necessarily to increase the load and hence $\dot{u}_0 = \frac{\dot{M}_0}{2EI}a^2 = V$. The kinetic energy is then

$$T = \frac{1}{2}\rho BhaV^2\int_0^1(1-\xi)^2 d\xi = \frac{1}{6}\rho BhaV^2 \tag{11}$$

Hence,

$$\frac{1}{B}\frac{dT}{da} = \frac{1}{6}Eh\left(\frac{V}{c_s}\right)^2 \tag{12}$$

which differs only for a prefactor from that of Blackman et al. [2] due to the different displacement function. It is clear that the kinetic term makes the static ERR G_s needed for initiation greater than $\Gamma = \Gamma_d(0)$. However, dynamic effects are present only if we have a significant rate of loading: suppose instead that we are increasing the end moment by a very slow rate, dynamic effects are negligible prior to initiation, differently from Blackman et al. [2] where loading the end point is with fixed velocity prior to initiation, at initiation, and at propagation.

Now, for crack propagation (when the crack has initiated, which Blackman et al. [2] calls "steady-state"), we consider the case where the moment is constant and

$$\dot{u} = \frac{du}{dt} + \dot{a}\frac{du}{da} = \frac{du_0}{dt} + \dot{a}\frac{du}{da} = \frac{M_0}{EI}a\dot{a}(2-\xi) \tag{13}$$

and in this case the kinetic energy is

$$T = \frac{1}{2}\rho Bha\left(\dot{a}\frac{M_0}{EI}a\right)^2\int_0^1(2-\xi)\,d\xi = \frac{3}{4}\rho Bha^3\left(\dot{a}\frac{M_0}{EI}\right)^2 \tag{14}$$

Hence as

$$\frac{1}{B}\frac{dT}{da} = \frac{3}{4}\frac{E}{c_s^2}\left(\frac{M_0}{EI}\right)^2 ha^2\left(3\dot{a}^2 + 2a\ddot{a}\right) \tag{15}$$

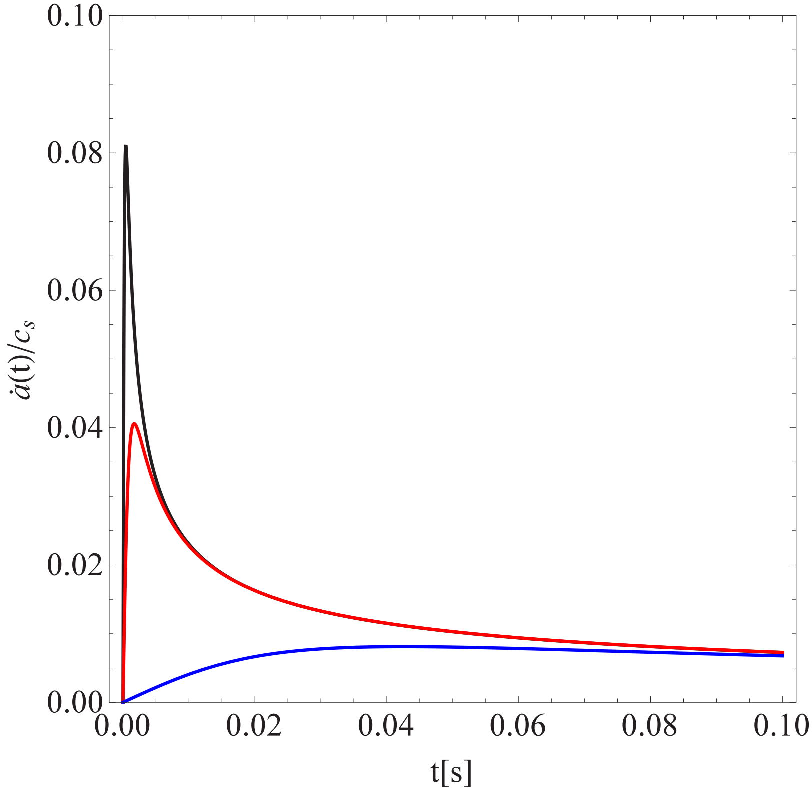

Fig. 2. Results of integration of the dynamic crack motion equation Eq. (18) under slowly applied moment M_0 for $\Gamma/G_s = 0.9$, $a(0)/h = 0.5, 1, 5$ (respectively black, red and blue curve). Here, we assume $c_s = 1000$ m/s.

we obtain the effect of crack acceleration. It is clear that the bigger the crack, the bigger is the dynamic correction to ERR. For the usual rectangular cross section, $I = h^3/12$ and

$$G_d = 12\frac{M_0^2}{Eh^3}\left[1-\frac{9}{c_s^2}\frac{a^2}{h^2}\left(3\dot{a}^2+2a\ddot{a}\right)\right] \tag{16}$$

Notice that when the crack initiates, we can assume $\dot{a}(0) = 0$ and we have that

$$G_d = 12\frac{M_0^2}{Eh^3}\left[1-\frac{18}{c_s^2}\frac{a^3}{h^2}\ddot{a}\right] \tag{17}$$

so the crack strictly speaking will not initiate for the quasi static condition $G_s = \Gamma$. We need a small overstress $G_s = 12\frac{M_0^2}{Eh^3} > \Gamma$ to get some acceleration. Naturally, as in many cases (see Karac et al. [4]) $\Gamma_d\left(\dot{a}\right)$ is a decaying function, crack initiation can occur even for $G_s = 12\frac{M_0^2}{Eh^3}$.

We have therefore obtained the governing equation for crack dynamics from (16) imposing $G_d = \Gamma_d\left(\dot{a}\right)$ as

$$a^2\left[3\dot{a}^2+2a\ddot{a}\right] = \frac{c_s^2h^2}{9}\left(1-\frac{\Gamma_d\left(\dot{a}\right)}{G_s}\right) \tag{18}$$

In particular, a real steady state with $\ddot{a} = 0$ is only obtained in the trivial state for infinite times when speed is zero.

We integrate numerically Eq. (18) with the function NDSolve in Mathematica, and provide some example results next.

Fig. 2 shows the crack speed as a function of time for $\Gamma/G_s = 0.9$, $a(0)/h = 0.5, 1, 5$ (respectively black, red and blue curve). It is clear that there is great sensitivity to the initial crack size only in the very beginning, while the speed assumes a common slowly decaying curve just after a fraction of seconds. However, if speed is large even a fraction of seconds means the crack has travelled a large distance: in our example at $c_s = 1000$ m/s the end of the Figure occurs at 0.1 s is certainly outside the reach of a practical size of the specimen. Starting from a small crack we reach the highest speeds, but the analysis here would need some refinements because the beam theory itself may be inaccurate. However, what is interesting is that we do not find the complex behaviour reported in

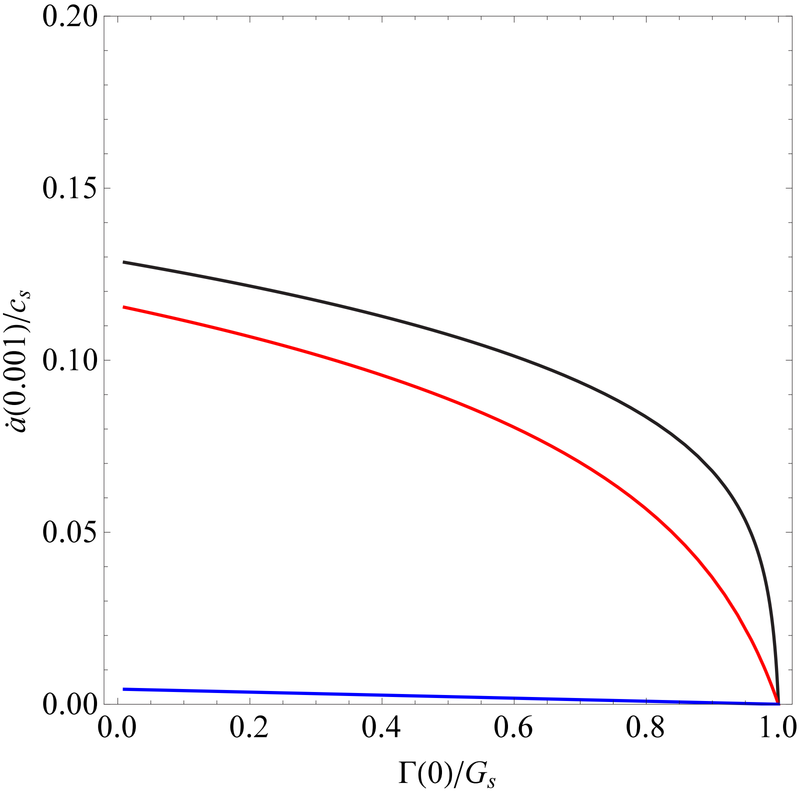

Fig. 3. Results of integration of the dynamic crack motion equation Eq. (18) under slowly applied moment M_0, $a(0)/h = 0.5, 1, 5$ (black, red and blue curves) and velocity at 0.001 s as a function of Γ/G. $c_s = 1000$ m/s.

Blackman et al. [2] where a complex transient turns out to lead to possibly wild oscillations in velocity which may even result in stick–slip of the crack, complicating the measurements of the dynamic toughness. Here, the picture instead seems a lot simpler, and we refer to the Discussion paragraph for more reasoning about the experimental setup.

Fig. 3 shows the velocity after 0.001 s of propagation as a function of Γ/G, where black, red and blue curves indicate increasing size of starting crack. As it can be seen only for small initial cracks high speeds are reached, and overstressing is required.

Further, we now consider the effect of possibly decaying dynamic toughness. Fig. 4 has the same conditions as Fig. 2, namely $\Gamma_d(0)/G_s = 0.9$, but a dynamic toughness which, as an example, is taken to be $\Gamma_d\left(\dot{a}\right) = 1 - \dot{a}/c_s$. As it can be seen, the velocities are higher due to the reduced resistance to crack energy. Similarly, Fig. 5 shows the velocity of the crack after 0.001 s for conditions like in Fig. 3, but now as a function of $\Gamma_d(0)/G_s$, for $\Gamma_d\left(\dot{a}\right) = 1 - \dot{a}/c_s$.

Fig. 6 indicates the maximum velocity reached during the transient for the two conditions of constant toughness (solid lines), or decaying toughness (dashed lines), for initial cracks of size $a(0)/h = 0.5, 1, 5$ (black, red and blue curves). It is confirmed that with decaying toughness one has non zero crack speed even with $G_s = \Gamma_d(0)$, whereas with large overstressing, but also with large initial crack sizes, the results do not differ significantly.

4. Discussion

Freund's (1998, chapt.7) treats also a case of DCB, this time for constant opening of the crack $u_0 = const$, whereas we have a constant moment M_0. His analysis starts more general, but eventually uses Berry's method to find a dynamic equation for the crack. He also suggests that we need an initial $G > G_s$ to initiate the crack, and he attributes this to the fact that crack is initially blunt so that one has to amplify G with what he calls nG with $n > 1$ to obtain initiation. This confirms our analysis on one hand, but also suggests that it is not impossible to overstretch the specimen without adding inertia. In his case, Freund [6, chapt.7] finds that final equilibrium crack is $a = a_0 n^{0.25}$, so this confirms that if $n = 1$ really there is no propagation: however, this is a difference with our case where there is no equilibrium (final) crack size. The validity of Berry's method, on the other hand, is confirmed by experimental investigations by Blackman et al. [2]. Although the details of transient waves emission and reflection from the boundaries are clearly not simulated, the main effect of inertia, including the transient effects which were even much more pronounced in the case of Blackman et al. [2], is captured by this method. However, the main result of this paper is that the crack dynamics in our case is a lot smoother and simpler than that of Blackman et al. [2]. This is not really due to the fact that we do not have inertia effects

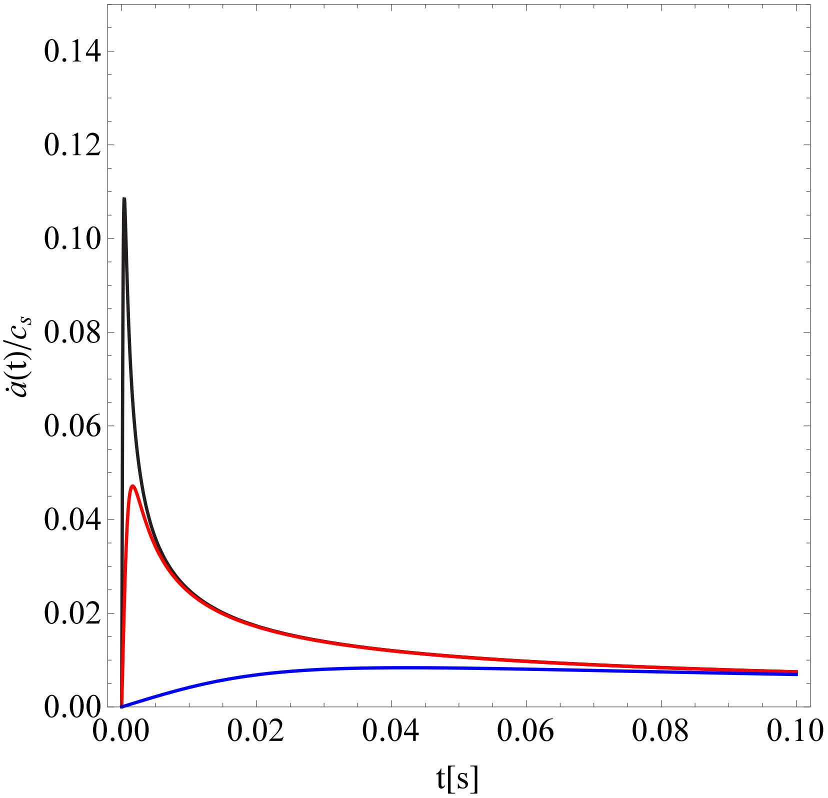

Fig. 4. As Fig. 2, namely $\Gamma_d(0)/G_s = 0.9$, but now $\Gamma_d\left(\dot{a}\right) = 1 - \dot{a}/c_s$.

before initiation (which differs from that during propagation), but rather by the simpler equation of motion which results from the constant end moment M_0.

We obviously do not claim that testing with constant end moment is a universal alternative to the various possible high rate delamination/adhesive testing of materials. Overstressing the specimen as we have shown makes it possible to reach very high crack speed. As we said, we do not think in general we need to devise as in the infinite strip testing described in Goldman et al. [7] to use a guillotine to introduce the crack after strain energy is loaded in the specimen, and this may not be always possible. Also, we rely on the fact that $\Gamma_d\left(\dot{a}\right)$ is decaying to find significant crack acceleration even if we make initiation at equilibrium when we reach $G_s = \Gamma_d(0)$. A precise planning of testing should be made in order to find significant crack speeds in a realistic testing setup. The limitations of our testing, from the examples shown, are clear: high crack speeds seem only obtained with short initial cracks (which may invalidate the simple Euler–Bernoulli type of beam investigation), or for very long specimen, which may be impractical. Clearly further investigation, including a full experimental one, would be needed, but are outside the scopes of the present small note.

Our analysis assumes the end moment is constant and we have shown in the paragraph “DCB with end moments”, that this cannot lead to a true steady state: oscillating shear and moments would be needed ((7),(8)) which take into account of inertia effect at large speeds and/or at large cracks. Naturally, the type of loading which is realistic of typical engineering impacts or transient is matter of debate. Alternative testing include the use of the Wedge-Double cantilever Beam (WDCB) test [5,11,12] where the wedge is pushed perhaps using a Split Hopkinson Pressure Bar (SHPB) set-up to induce a constant velocity of propagation. However also this type of tests presents its challenges, and for example the amount of friction loss in the wedge is much greater than the energy spent in the delamination. Our model, for its simplicity, is also useful for understanding the possible mechanisms of dynamic fracture in laminated structures.

Pure moment (PM) has been probably been suggested experimentally on a DCB specimen by Freiman et al. [13]. It is a quite robust and simple specimen fixture which has perhaps the disadvantage of inducing stress concentrations. An extension is Sørensen et al. [10], where testing was done on ceramic material like Freiman et al. [13], but with the innovation of applying the load by wires by means of pulleys. However, it remains problematic for testing composite materials, with some possible spurious effects in the measurement induces by stiffness and locking of the wires. A different rig was designed by Lindhagen and Berglund [14] to be mounted on a conventional axial testing machine. Here, the moment was transferred to the specimen by a pair of forces, and a variant has been proposed more recently by Pappas and Botsis [15].

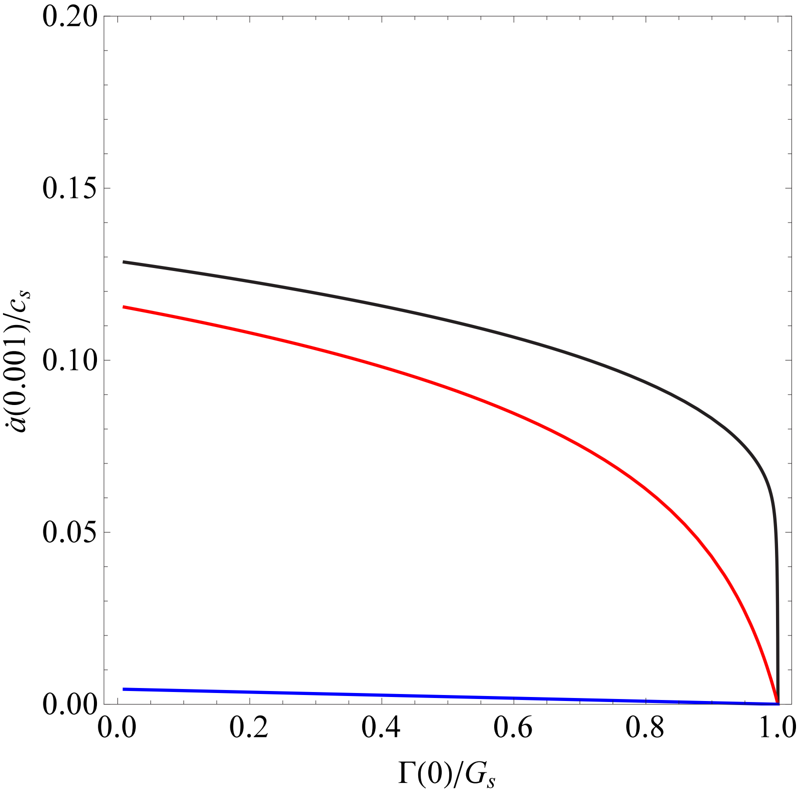

Fig. 5. As Fig. 3, but now velocity of crack as a function of $\Gamma_d(0)/G_s$, for $\Gamma_d\left(\dot{a}\right) = 1 - \dot{a}/c_s$.

5. Conclusions

We have discussed crack propagation in the dynamic regime for a simple Double Cantilever Beam (DCB) under constant pure moment. We have shown that with a simplified analysis based on Berry's method it is possible to obtain the dynamic equation governing the crack motion, which depends non linearly on crack speed and acceleration. The energy equilibrium alone (in the absence of dynamic effects) does not permit to find the crack speed, contrary to the case of moving the end of the DCB at constant speed, that has been investigated more at length in the literature. Some level of overstressing is needed to obtain some acceleration of the crack, and then the speed becomes a transient variable, which however eventually decays to zero at very long crack length. This model however permits a simple discussion of elementary dynamic fracture mechanics, which may inspire some new experimental investigations.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Acknowledgements

MC acknowledges support from the Italian Ministry of Education, University and Research (MIUR) under the program "Departments of Excellence" (L.232/2016). Also, discussion with Prof.JR Barber from U.of Michigan.

Data availability

No data was used for the research described in the article.

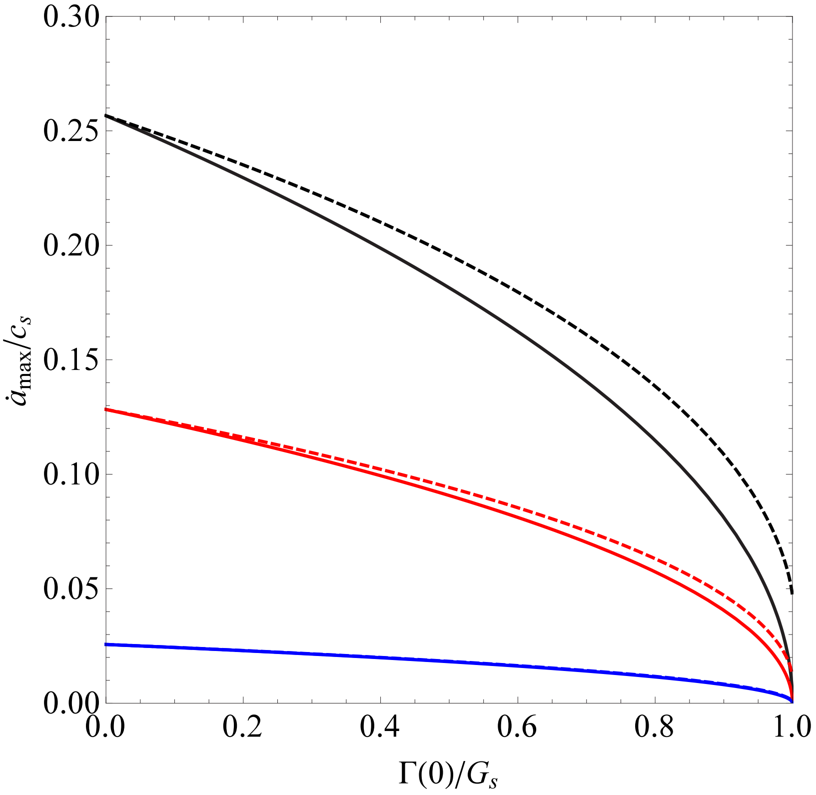

Fig. 6. Maximum velocity of crack as a function of $\Gamma_d(0)/G_s$, for $\Gamma_d = \Gamma$ (solid lines) and $\Gamma_d\left(\dot{a}\right) = 1 - \dot{a}/c_s$ (dashed lines). $a(0)/h = 0.5, 1, 5$ (black, red and blue curves).

References

[1] Blackman BRK, Dear JP, Kinloch AJ, Macgillivray H, Wang Y, Williams JG, Yayla PAŞA. The failure of fibre composites and adhesively bonded fibre composites under high rates of test: Part I mode I loading-experimental studies. J Mater Sci 1995;30:5885–900.
[2] Blackman BRK, Kinloch AJ, Wang Y, Williams JG. The failure of fibre composites and adhesively bonded fibre composites under high rates of test: Part II mode I loading—dynamic effects. J Mater Sci 1996;31:4451–66.
[3] Blackman BRK, Kinloch AJ, Sanchez FR, Teo WS, Williams JG. The fracture behaviour of structural adhesives under high rates of testing. Eng Fract Mech 2009;76(18):2868–89.
[4] Karac A, Blackman BRK, Cooper V, Kinloch AJ, Sanchez SR, Teo WS, Ivankovic A. Modelling the fracture behaviour of adhesively-bonded joints as a function of test rate. Eng Fract Mech 2011;78(6):973–89.
[5] Sridhar N, Massabò R, Cox B, et al. Delamination dynamics in through-thickness reinforced laminates with application to DCB specimen. Int J Fract 2002;118:119–44. http://dx.doi.org/10.1023/A:1022884410968.
[6] Freund LB. Dynamic fracture mechanics. Cambridge University Press; 1998.
[7] Goldman T, Livne A, Fineberg J. Acquisition of inertia by a moving crack. Phys Rev Lett 2010;104(11):114301.
[8] Sharon E, Gross SP, Fineberg J. Energy dissipation in dynamic fracture. Phys Rev Lett 1996;76(12):2117.
[9] Svetlizky I, Fineberg J. Classical shear cracks drive the onset of dry frictional motion. Nature 2014;509(7499):205–8.
[10] Sørensen BF, Brethe P, Skov-Hansen P. Controlled crack growth in ceramics: the DCB specimen loaded with pure moments. J Eur Ceram Soc 1996;16(9):1021–5.
[11] Blackman B, Kinloch A, Taylor A, Wang Y. The impact wedge-peel performance of structural adhesives. J Mater Sci 2000;35(8):1867–84.
[12] Ponnusami SA, Cui H, Erice B, Lißner M, Pathan M, Petrinic N. An integrated inverse numerical–experimental approach to determine the dynamic Mode-I interlaminar fracture toughness of fibre composites. Compos Struct 2022;293:115734.
[13] Freiman SW, Mulville DR, Mast PW. Crack propagation studies in brittle materials. J Mater Sci 1973;8:1527–33.
[14] Lindhagen JE, Berglund LA. Application of bridging-law concepts to short-fibre compositespart 1: DCB test procedures for bridging law and fracture energy. Compos Sci Technol 2000;60(6):871–83.
[15] Pappas GA, Botsis J. Variations on R-curves and traction-separation relations in DCB specimens loaded under end opening forces or pure moments. Int J Solids Struct 2020;191:42–55.