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Cooperative Source Seeking with Distance-Based Formation Control and Single-Integrator Agents

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Abstract: In this paper and its companion paper (Ahmadi Barogh and Werner, 2017), the problem of source seeking with a group of agents in a distance-based formation is considered. Each agent is equipped with appropriate sensors to detect the distance between itself and its neighbors and to measure the strength of a field signal. The task is to find a maximum point of the scalar field in the area. Each agent estimates the gradient of the scalar field cooperatively and moves in the gradient direction, by maintaining the specified formation in movement. In this paper, a distributed controller for navigation of single integrator agents is proposed. We use a distance-based formation controller to maintain a predefined formation when moving with collision avoidance. The center of formation converges to the region near the source and agents do not need access to a common coordinate system. Conditions are provided for the asymptotic stability of the system and for the formation center to converge a bounded region near the source. Simulation results are presented to illustrate the effectiveness of the proposed approach.

Keywords: multi-agent systems, distance-based formation control, source seeking, cooperative control, gradient estimation, scalar field, single integrator, collision avoidance

1. INTRODUCTION

Source seeking means to localize the source of a scalar field in an unknown environment, using the fact that the value of the filed is larger near the source and decreases with increasing distance from the source. Localizing and detecting the source with a cooperative multi-agent system is effective in a wide range of applications nowadays. Cooperative source seeking can be used in environmental monitoring (Ogren et al., 2004), oil pollution management (Senga et al., 2007), environmental disaster monitoring, rescue cases sound source localization, chemical spill searching etc; cases when for a single agent estimating the gradient would be hard. Usually, all agents are equipped with relevant sensors to detect the concentration of the scalar field.

There are various proposed approaches to deal with the source seeking problem. Several approaches are focused on behavior-based source seeking methods (Farrell et al., 2005), (Spears et al., 2004). In behavior-based source seeking approaches, a set of behaviors and their combinations are defined. A group of behaviors is activated and caused to drive a single agent or a multi-agent system to the source direction. However, the main difficulty of behavior-based methods is ignoring the local dynamics of agents.

In (Rabbat and Nowak, 2004), sensors are distributed all over the testing environment in constant positions without moving. A centralized optimization algorithm uses the information obtained via sensors to solve the problem of source seeking. In (Cortes, 2005), (Biyik and Arcak, 2007), (Zhang and Leonard, 2010), a number of algorithms are proposed to deal with source seeking in multi-agent systems. In most of these algorithms all-to-all communication is required also the center of mass of formation is a key to estimating the gradient and steer the group of agents to the source. In (Wu and Zhang, 2012) not only immediate real-time measurements are used to estimate the gradient but also the off-time series of measurement data are included. In (Brinon-Arranz et al., 2016) the problem of source seeking is addressed for a group of agents via locating them in a circular formation. Agents are spinning around the center of mass of formation and the gradient is estimated at the center. In (Li et al., 2014) for both all-to-all and limited communication appropriate control laws are proposed. In the limited communication case, several consensus filters are assumed and therefore achieving consensus through those filters takes a long time, and the control laws act slowly. In (Rosero and Werner, 2014), Rosero and Werner proposed a new approach to locating the scalar field's unknown source. The communication is undirected and constrained. Each agent uses the neighbors' signal strength and absolute position to estimate the gradient direction, and with a distributed controller a specified formation shape is maintained. In (Moore and de Wit, 2010), (Brinon-Arranz et al., 2011), (Brinon-Arranz et al., 2014) a collaborative control law to lead a group of vehicles to the source by organizing a circular formation for vehicles is studied. Formation control laws to achieve and maintain a circular formation are investigated. In (Atanasov et al., 2014), two algorithms based on gradient estimation at the center of the formation are proposed; they use a stochastic approximation to achieve convergence in the estimation of the gradient at the center. The control laws are based on relative positioning with respect to neighbor agents and the center of mass of formation. In (Fabbiano et al., 2014) a circular formation and source seeking are combined as a two objectives. Both objectives are included in the control law and convergence is achieved. In (Fabbiano et al., 2016) a distributed algorithm to estimate the gradient of the scalar field is proposed

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and this information is used to drive a group of rotating sensors towards the source, without using full position information. Sensors are rotating around the circular formation.

Most of the above approaches except (Fabbiano et al., 2016), (Atanasov et al., 2014) need the absolute position with respect to a common coordinate system. In (Fabbiano et al., 2016) the position information is not necessary but sensors are constantly spinning in the circular formation. In (Atanasov et al., 2014) the relative localization with respect to center of formation is required.

Here we address the problem of source seeking using a formation of N identical agents with a distance-based formation method without access to a common coordinate system. We assume each agent uses its own local coordinate system and is equipped with appropriate sensors to detect the distance between itself and its neighbors. Each agent estimates the gradient direction with respect to its local coordinate system. To estimate the gradient direction each agent uses the neighbor's signal strength only and detects their position with respect to the local coordinate system of itself. A proposed distributed control law includes two parts: a formation control law and a source seeking trajectory control law. The first part of the control law guarantees that the group of agents achieves formation and maintains it in the presence of obstacles, while the second part enforces the source seeking behavior. In this work the control law neither needs the absolute position of agents nor to keep the agents in a specific circular rotating formation. Rotating agents in the circular geometric formation results in significant energy consumption in the system. When obstacles are in the area, the implementation of a circular rotating formation is difficult or might be impossible. In our approach the formation shape is not restricted to be circular and can be wide range of various shapes.

1.1 Outline of this paper

This work is divided into two parts. The first part (this paper) covers the source localization with a distance-based formation for single integrator models. The second part (the companion paper (Ahmadi Barogh and Werner, 2017)) is devoted to source localization for non-holonomic models.

The rest of this paper is arranged in five sections: Section 2 provides some mathematical background and a formulation of the formation control and source seeking problems. In Section 3, we propose a control law that achieves the source and the desired formation and present a stability proof for the closed-loop system in the presence of obstacles. Section 4 shows simulation results, and Section 5, concludes with a summary.

2. PRELIMINARIES

2.1 Notation

A formation of N mobile agents is described in terms of graph theory. Therefore some of the basic concepts in graph theory are reviewed briefly. A formation and source seeking undirected graph with N vertices and M_g edges is denoted by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ where $\mathcal{V} = \{1, \cdots, N\}$ and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$, $|\mathcal{E}| = M_g$. The neighbor agent set of agent i is denoted by $\mathcal{N}_i = \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$. An undirected edge satisfies $(i,j) \in \mathcal{E}$ if the distance between agent i and j is considered to be controlled. We let $p_i(t) \in \mathbb{R}^n$ represent the position of vertex (agent) i in n-dimensional space at time t for all $i \in \mathcal{V}$ and the vector $p = \begin{bmatrix} p_1^T & \cdots & p_N^T \end{bmatrix}^T \in \mathbb{R}^{nN}$ is the position vector of all agents. The framework $(\mathcal{G}, p)$ represents a graph $\mathcal{G}$ in $\mathbb{R}^n$. Two frameworks $(\mathcal{G}, p)$ and $(\mathcal{G}, p')$ from the same graph are equivalent if $\|p_i - p_j\| = \|p'_i - p'_j\|$ for all $(i,j) \in \mathcal{E}$ and are congruent if $\|p_i - p_j\| = \|p'_i - p'_j\|$ for all $i, j \in \mathcal{V}$. The graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ also represents the model of interaction among agents for gradient estimation. Every edge corresponds to bidirectional information exchange channel. An edge $(i,j) \in \mathcal{E}$ indicates that agent i and j exchange information concerning the gradient estimation. The adjacency matrix $A = [a_{ij}] \in \mathbb{R}^{N \times N}$ of a graph $\mathcal{G}$ with N nodes specifies the interconnection topology of the network. Here $a_{ij} = 1$ if $(i,j) \in \mathcal{E}$, else $a_{ij} = 0$ (Biggs, 1993). In the following, $\|.\|$ and $\|.\|_1$ denote the Euclidean norm and 1-norm respectively. The global coordinate system denoted by ${}^g\sum$ and the local coordinate system for agent i represents with ${}^i\sum$ and n denotes the dimension of space. In this paper we assume that global coordinate system and common coordinate system are the same.

Remark 1. In this paper, we assume graph $\mathcal{G}$ is undirected, connected and $|\mathcal{N}_i| \geq n \forall i = 1 \cdots N$, $N \geq n+1$ and agent i and their neighbors are not collinear.

2.2 Agent models

Consider N identical agents in n-dimensional space. Each agent follows the integrator model represented by

$$\dot{p}_i(t) = u_i(t), i \in \{1 \cdots N\}, \tag{1}$$

where $p_i \in \mathbb{R}^n$ denotes the coordinates of the position of agent i in n-dimensional space and $u_i \in \mathbb{R}^n$ is the control input velocity vector for steering the agent. All quantities are calculated with respect to ${}^g\sum$. We assume that the agents do not necessarily use a common coordinate system, but each agent maintains its own local coordinate system. Superscripts on p_i^i, u_i^i, v_i^i are references to the local coordinate system of agent i.

By adopting this notation, the dynamics of an agent can be written as

$$\dot{p}_i^i = u_i^i, \tag{2}$$

The detecting topology to achieve a formation is represented by an undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Each agent uses its own local sensors to detect the relative position of its neighbors with respect to its own local coordinate system. Then for agent i, p_j^i is the position of agent j with respect to ${}^i\sum$. To analyze the stability of the system, it is better to represent the dynamics of all agents in a common coordinate system. Regarding the results of (Oh and Ahn, 2014) with an appropriate rotation and translation, all results from a common coordinate system are converted to the local coordinate system of agent i. In this paper to prove the stability and convergence, all superscripts are eliminated, and the control laws and the agent dynamics are represented in a common coordinate system.

2.3 Scalar field

The scalar field is defined as $\mu(p_i(t)) : \mathbb{R}^n \to \mathbb{R}^+$. Assume μ is a twice continuously differentiable function with a global maximum $\bar{\mu}(p_s)$, when $p_s \in \mathbb{R}^n$

Remark 2. Agents detect the instantaneous values of the scalar field with their appropriate sensors. Considering the fact that each agent uses its local coordinate system so they detect the different values of the same function, μ in different positions with respect to their local coordinate. Note in this paper the scalar field μ is unknown analytically. The scalar field value that is detected with agent i is denoted by μ_i and is bounded, $|\bar{\mu}(p_s)| < \gamma$ where $\gamma > 0$

2.4 Distributed gradient estimation

Consider a group of mobile agents with the communication graph $\mathcal{G}$. Agent i measures the signal strength $\mu(p_i^i)$. In this paper, we use the approach that was proposed in (Rosero and Werner, 2014) to estimate the gradient for each agent. An estimation of the gradient that is proposed in (Rosero and Werner, 2014) as

$$R_i = \begin{bmatrix} (p_j^i - p_i^i)^T \\ \vdots \end{bmatrix}_{|\mathcal{N}_i| \times n},$$

$$b_i = \begin{bmatrix} \mu_j(p_j^i) - \mu_i(p_i^i) \\ \vdots \end{bmatrix}_{|\mathcal{N}_i| \times n}, j \in \mathcal{N}_i,$$

$$\hat{g}_i^i(p_i^i) = (R_i^T R_i)^{-1} R_i^T b_i, \tag{3}$$

where $\hat{g}_i^i$ is the estimated gradient in agent i with respect to its own local coordinate system. The matrix R_i must have full column rank and thus the rank($R_i = n$) and this is satisfied if and only if agent i and its neighbors are not collinear. Since $R_i \in \mathbb{R}^{|\mathcal{N}_i| \times n}$, we thus need $|\mathcal{N}_i| \geq n$.

Assumption 1. According to the results of (Li et al., 2014), the Hessian matrix of $\mu(p_i)$ satisfies $\lambda_{max} \leq \eta_1$ and $\lambda_{min} \geq \eta_2$ where $\eta_1, \eta_2 < 0$. The scalar field such as temperature has a maximum at the p_s and with the increase of distance, it declines and this Assumption simplifies this fact.

Assumption 2. The estimated gradient that is calculated by (3) has a bounded error. $\left\|\hat{g}_i^i - g_i^i\right\| \leq e_g$, where e_g is a positive constant and g_i^i is the true value of the gradient in the location of agent i with respect to ${}^i\sum$.

Remark 3. $\hat{g}_i^i$ denotes the estimation of the gradient of agent i with respect to ${}^i\sum$. To prove the stability of proposed control law, we assume all gradient vectors are transformed to ${}^g\sum$ with an appropriate rotation and transformation and denoted by $\hat{g}_i$. We define $\hat{g}_c = \frac{1}{N}\sum_{i=1}^{N} \hat{g}_i$ as an estimated gradient of a scalar field in the center of formation. In the following control laws, each agent only needs to calculate the $\hat{g}_i^i$ and the gradient in the center is not required for agents. We use $\hat{g}_c$ only to prove the stability and convergence. From Assumption 2 one can conclude that $\|\hat{g}_c - g_c\| \leq e_g$

2.5 Problem Statement

Given $p^* \in \mathbb{R}^{nN}$ as a desired representation of $\mathcal{G}$ in n-dimensional space. The communication graph to estimate the gradient in each agent is also represented via $\mathcal{G}$. Design an algorithm and propose a control law to enforce the group of the agents to achieve the desired formation based on local information about the distance and to drive the center of the formation to the source p_s based on the estimated gradient in each agent. The desired formation of a group of single integrator agents is represented by the set

$$E_p = \left\{p \in \mathbb{R}^{nN} : \|p_j - p_i\| = \left\|p_j^* - p_i^*\right\|, \forall (i,j) \in \mathcal{E}\right\}. \tag{4}$$

Therefore E_p is the sets of all formations equivalent to p^*.

2.6 Distance-based formation control

In our previous work (Ahmadi Barogh and Werner, 2016) we proposed an approach for distance-based formation control by combining the distance and angle that is called distance-angle-based formation. A combination of angles and distances can provide additional benefits to formation control in distance-based approaches (Ahmadi Barogh and Werner, 2016). Regarding the complexity of applying a distance-angle-based formation to source seeking problem, in this work we use the formal distance-based formation control law that has been proposed in (Baillieul and Suri, 2003), (Krick et al., 2008) and extended with the full stability conditions in (Oh and Ahn, 2014). In the journal version of this paper, we will apply the distance-angle-based formation with source seeking. In this work, the collision avoidance and source seeking gradient based tracking terms are combined with the distance-based formation control law.

The desired distance between agent i and agent k is defined by $d_{ik}^* = \|p_i^* - p_k^*\|$ and the real distance between those agents is denoted by d_{ik}.

To achieve a desired formation the following function is defined as a formation error.

$$\psi_{ik}(t) = \frac{1}{2}(d_{ik}^2(t) - d_{ik}^{*2})^2, \forall k \in \mathcal{N}_i,$$
$$\psi_i(t) = \sum_{k \in \mathcal{N}_i} \psi_{ik}(t), \tag{5}$$

where k is the index of the neighbors for agent i. Then agent i achieves the desired formation if $\psi_i = 0$. By taking the derivative of ψ_i with respect to p_i^i, the following result is obtained (Oh and Ahn, 2014)

$$\Psi_i^i(t) = -\frac{1}{2}\nabla_{p_i^i}\psi_i(t),$$
$$= -\sum_{k \in \mathcal{N}_i}(\left\|p_i^i - p_k^i\right\|^2 - \|p_i^* - p_k^*\|^2)(p_i^i - p_k^i). \tag{6}$$

From (5) one can conclude that

$$\nabla_{p_k}\psi_{ik} = \nabla_{p_k}\psi_{ki}. \tag{7}$$

Remark 4. In this paper, we assume that the wireless communication among agents applies to gathering the information on scalar field for each agent. Each agent receives real-time detected scalar field values from its neighbors and shares its detected value with them. But to achieve a formation the wireless communication is not required, agents do not share the information about their position because each agent localizes itself with respect to its local coordinate system. We assume each agent can distinguish its neighbors and can detect the distance between itself and them via local sensors (camera, laser scanner, sonar sensor, etc.). Further, each agent calculates the position of neighbors with respect to ${}^i\sum$ and uses wireless communication to receive a value of the scalar field from the corresponding neighbor.

2.7 Collision avoidance

To achieve obstacle collision avoidance, usually an avoidance function is employed. In this work, the following avoidance function is employed (G. Leitmann and Skowronski, 1977).

$$V_{aij}(t) = \begin{cases} \left(\min\left\{0, \frac{d_{ij}^2(t)-R^2}{d_{ij}^2(t)-r^2}\right\}\right)^2 & d_{ij} > r \\ \infty & d_{ij} \le r \end{cases}, \tag{8}$$

where $R > r > 0$. Define $V_{ai}(t) = \sum_{j=1}^{N_{ai}} V_{aij}(t)$, where $N_{ai}(t)$ is the number of all agents or obstacles that are in the vicinity of agent i at a given point in time and d_{ij} is a distance between agent i and obstacle j. R and r are the radii of protection and detection areas, respectively, around the agent.

Assumption 3. If the tracking force has opposite direction of the vector for avoiding a collision but equal magnitude of it, the sum of the forces is equal to zero. In this case, we assume that a small perturbation is applied to make it nonzero.

3. CONTROL LAW

In this section, a control law to achieve a formation and source seeking will be introduced. In addition, asymptotic stability of the formation is proved. The control law for agent i can be designed as

$$u_i^i(t) = k_v\Psi_i^i - k_o\frac{\partial V_{ai}}{\partial p_i^i} + k_t\hat{g}_i^i(p_i^i), -k_s\,\mathrm{sgn}(u_i^i) \tag{9}$$

where $k_v > 0, k_o > 0, k_t > 0, k_s > 0$, Ψ_i^i is obtained from (6), and $\hat{g}_i^i$ comes from (3). The function, sgn(.) equals to 1, -1, 0 for positive, negative and zero inputs respectively. The control law (9) is completely distributed in the sense that each agent can implement it in its local coordinate system by using only local measurements. As mentioned already, to study the stability it is more convenient to represent the control law and the dynamics of agents with respect to ${}^g\sum$. The control law (9) with respect to ${}^g\sum$ is represented as

$$u_i(t) = k_v\Psi_i - k_o\frac{\partial V_{ai}}{\partial p_i} + k_t\hat{g}_i(p_i) - k_s\,\mathrm{sgn}(u_i), \tag{10}$$

where $\Psi_i = -\frac{1}{2}\nabla_{p_i}\psi_i$. In (9), $k_v\Psi_i^i$ drives the agent to the desired formation, $k_o\frac{\partial V_{ai}}{\partial p_I^i}$ guarantees collision avoidance and $k_t\hat{g}_i(p_i)$ generates the gradient tracking. The proposed control law with the distance-based formation is stated in Algorithm 1.

Algorithm 1 Source seeking algorithm for agent i with distance-based formation

1: **repeat**
2: $\mu_i, p_i^i \Leftarrow$ Sensor readings.
3: $\forall\mu_j \Leftarrow$ Communication with neighbors $\forall j \in \mathcal{N}_i$
4: $p_j^i \Leftarrow$ Detecting the position of neighbors $\forall j \in \mathcal{N}_i$
5: Distributed gradient estimation $\hat{g}_i^i(p_i^i) \Leftarrow$ Equation (3)
6: Distributed control law $u_i^i(p_i^i) \Leftarrow$ Equation control law
7: **until** $\left\|\hat{g}_i^i(p_i^i)\right\| < \epsilon$, where ϵ is a predefined positive constant.

The following theorem states the convergence of the proposed control law to the source.

Theorem 1. Given a multi-agent system with N single Integrator agents (2), and the distributed control law (9) under Algorithm 1. Suppose that Assumptions 1,2,3 are fulfilled. Then, for all $p_i(0) \in \mathbb{R}^n$ and $t \ge 0$, the agents achieve a desired formation that is equivalent with p^*, without collision and in the formation center($p_c(t)$) satisfies $\hat{g}_c \to 0$ as $t \to \infty$ when $k_s \ge k_t\sqrt{n}e_g$. Then the center of formation is stabilized in the region with a bounded distance from the source p_s as $\|(p_s - p_c)\| \le \frac{-2e_g}{\eta_1}$.

Proof 1. We will present the proof in two steps. First, the stability is analyzed. Then the source seeking is investigated. To show asymptotic stability we choose the Lyapunov function as a candidate

$$V(p_i) = \sum_{i=1}^{N}\left(\frac{1}{4}k_v\psi_i + k_oV_{ai} + k_t(\bar{\mu}(p_s) - \mu(p_i))\right). \tag{11}$$

Taking the derivative of (11) yields

$$\dot{V} = \sum_{i=1}^{N}\left(\frac{1}{4}k_v\sum_{k\in\mathcal{N}_i}\left(\frac{\partial\psi_{ik}^T}{\partial p_i}\dot{p}_i + \frac{\partial\psi_{ik}^T}{\partial p_k}\dot{p}_k\right) + k_o\frac{\partial V_{ai}^T}{\partial p_i}\dot{p}_i - k_tg_i^T\dot{p}_i\right). \tag{12}$$

The graph $\mathcal{G}$ is connected and undirected and therefore from (7) we can obtain

$$\dot{V} = -\sum_{i=1}^{N}\left(\left(k_v\Psi_i - k_o\frac{\partial V_{ai}}{\partial p_i}\right)^T\dot{p}_i + k_tg_i^T\dot{p}_i\right). \tag{13}$$

Let define $g_i = \hat{g}_i + \varepsilon_i$, $\varepsilon_i \in \mathbb{R}^n$ and by substituting from (10) and (1) in (13) yields

$$\dot{V} = \sum_{i=1}^{N} -u_i^T\left(u_i + k_t\varepsilon_i + k_s\,\mathrm{sgn}(u_i)\right). \tag{14}$$

Then we have

$$\dot{V} = -\sum_{i=1}^{N}\|u_i\|^2 - k_t\sum_{i=1}^{N}u_i^T\varepsilon_i - k_s\sum_{i=1}^{N}\|u_i\|_1. \tag{15}$$

Note the norm inequalities imply that

$$-u_i^T\varepsilon_i \le \|u_i\|_1\|\varepsilon_i\|_1. \tag{16}$$

Thus in (15) using (16), we have

$$\dot{V} \le -\sum_{i=1}^{N}\|u_i\|^2 + k_t\sum_{i=1}^{N}\|u_i\|_1\|\varepsilon_i\|_1 - k_s\sum_{i=1}^{N}\|u_i\|_1. \tag{17}$$

As $\varepsilon_i = g_i - \hat{g}_i$, from the norm properties we have $\|\varepsilon_i\|_1 \le \sqrt{n}\|g_i - \hat{g}_i\|$ and therefore from Assumption 2 we have $\|\varepsilon_i\|_1 \le \sqrt{n}e_g$. By substituting in (17) we obtain

$$\dot{V} \le -\sum_{i=1}^{N}\left(\|u_i\|^2 + \|u_i\|_1(k_s - k_t\sqrt{n}e_g)\right). \tag{18}$$

In (18), by applying the condition that $k_s \ge k_t\sqrt{n}e_g$, we have $\dot{V} \le 0$ and stability is proved. To show asymptotic stability, the equilibrium point of $\dot{V} = 0$ is analyzed. From $k_s \ge k_t\sqrt{n}e_g$ and (15) we have $\dot{V} = 0$ implies that $u_i \to 0$ as time goes to infinity. Then from Assumption 3 and (10) it is concluded that in the equilibrium $\frac{\partial V_{ai}}{\partial p_i} = 0$ and the collision avoidance is guaranteed. Therefore $\Psi_i \to 0$ as $t \to \infty$. Then from $\Psi_i \to 0$ it is concluded that $(d_{ik}^{*2} - d_{ik}^2) \to 0$ and therefore $d_{ik} \to d_{ik}^*$ for all $k \in \mathcal{N}_i$ and asymptotic stability of system is concluded. Thus in the equilibrium points of $\dot{V} = 0$ we have $\psi_{ik} = 0$. Therefore $p \in E_p$ when $t \to \infty$. From Remark 3 is concluded that $\hat{g}_c \to 0$ as $t \to \infty$. Finally we will show that the distance between p_c and p_s is bounded. The first two terms of the Taylor expansion of $\bar{\mu}(p_s)$ around p_c is

$$\bar{\mu}(p_s) = \mu(p_c) + (p_s - p_c)^T\nabla\mu(p_c) + \frac{1}{2}(p_s - p_c)^T\nabla^2\mu(p_c)(p_s - p_c). \tag{19}$$

Regarding Assumption 1, we have

$$\bar{\mu}(p_s) \leq \mu(p_c) + (p_s - p_c)^T \nabla \mu(p_c) + \frac{1}{2} \|(p_s - p_c)\|^2 \eta_1,$$

and with respect to the norm inequality

$$\bar{\mu}(p_s) - \mu(p_c) \leq \left\|(p_s - p_c)^T \nabla \mu(p_c)\right\| + \frac{1}{2} \|(p_s - p_c)\|^2 \eta_1.$$

From Remark 3 and considering the fact that $\hat{g}_c \to 0$ can be obtained that $\|\nabla \mu(p_c)\| \leq e_g$ as $t \to \infty$. Note that according to the fact that $\bar{\mu}(p_s)$ is a global maximum then we have $\bar{\mu}(p_s) - \mu(p_c) \geq 0$, and from Assumption 1 $\eta_1 < 0$ therefore $\|(p_s - p_c)\| \leq \frac{-2e_g}{\eta_1}$.

Remark 5. In this paper, all results in all Theorems are based on Assumption 3. Assumption 3 in some circumstances is not established. For example, if obstacles are located very close to the source. In that case, an incorrect equilibrium point is created, and the formation errors do not converge to zero. Hence, our results on asymptotic stability only in a local sense are valid.

4. SIMULATION RESULTS

In this section, simulation results are presented to illustrate the functionality and effectiveness of the proposed control laws in the previous section. Experimental results on real robots are omitted in this paper and will be provided in the journal version of this paper. Consider five agents in a $2D$ plane and $n = 2$. The aim is to achieve a desired formation and to steer the center of formation to the source. The desired formation is a pentagon shape with five sides equal to 15 and angles equal to $3\pi/5$ and $p^* = [12.13\ 0\ 0\ 8.81\ -12.13\ 0\ -7.5\ -10.81\ 7.5\ -10.81]$. The initial positions of agents are defined as $p_1 = [25\ 10]^T$, $p_2 = [12\ \ -10]^T$, $p_3 = [-10\ 40]^T$, $p_4 = [-40\ 10]^T$, $p_5 = [-10\ 10]^T$. The adjacency matrix of communication undirected graph $\mathcal{G}$ is defined as

$$A = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 & 0 \end{bmatrix}.$$

The scalar field is defined as

$$\mu(p_i) = A_0 e^{-\left((p_i - p_s)^T H_1 (p_i - p_s)\right)} + A_0 e^{-\left((p_i - p_s)^T H_2 (p_i - p_s)\right)}$$

, where $A_0 = 50$, $H_1 = \begin{bmatrix} \frac{1}{2\sigma_{x1}^2} & 0 \\ 0 & \frac{1}{2\sigma_{y1}^2} \end{bmatrix}$, $H_2 = \begin{bmatrix} \frac{1}{2\sigma_{x2}^2} & 0 \\ 0 & \frac{1}{2\sigma_{y2}^2} \end{bmatrix}$, $\sigma_{x1} = 30$, $\sigma_{x2} = 90$, $\sigma_{y1} = 75$, $\sigma_{y2} = 25$. The maximum of scalar field is located at $p_s = [70\ 70]^T$. In all subsequent simulations the tuning parameters are set to $k_v = 0.0004$, $k_o = 0.001$, $k_t = 0.1$, $k_s = 0.001$, $R = 8$, $r = 4$.

We apply control law (9). Results are shown in Fig. 1. As depicted in Fig. 1, agents achieve a desired formation and maintain it and locate the maximum of scalar field p_s as well. Fig. 2 shows the formation errors considering the formation. Note that when one agent or an obstacle are inside the detection area of another agent, the avoidance function is activated. As a consequence, the agents change the trajectories to avoid the collisions and the formation errors increase or decrease temporarily, depending on the direction of the repulsive force. In Fig. 3 the distance between center of formation p_c and the source p_s is shown and as depicted the distance is decreasing close to zero.

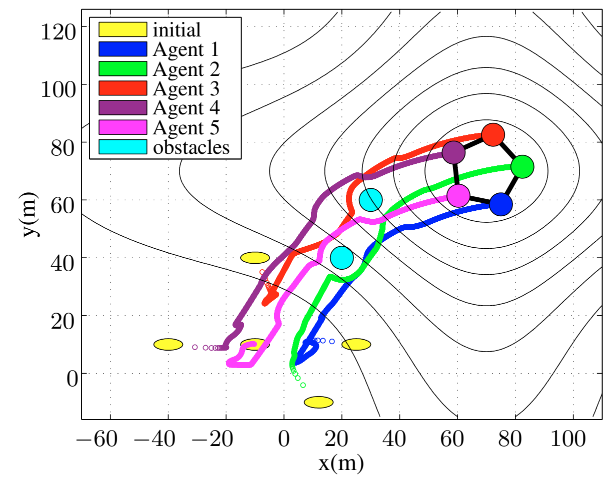

Fig. 1. Formation and source seeking with collision avoidance

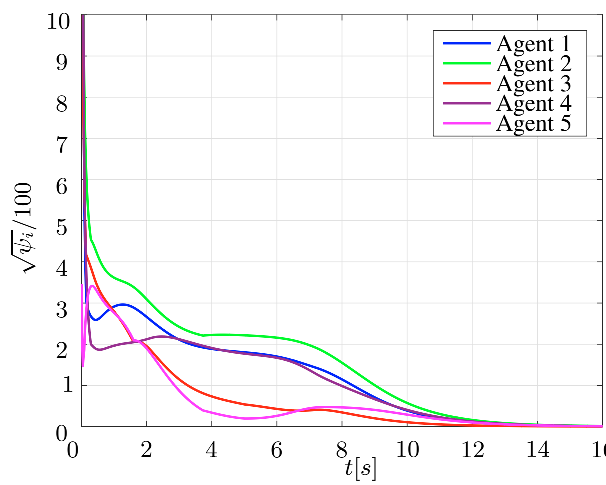

Fig. 2. Formation (ψ_i) errors

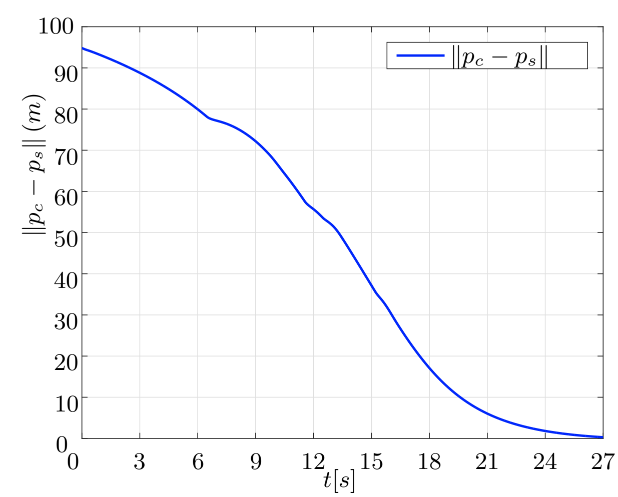

Fig. 3. Distance between source (p_s) and center of formation (p_c)

5. CONCLUSION

In this paper, a new method for source seeking with distance-based formation control is proposed. In this approach, a distributed source seeking algorithm is combined with distance-based formation control and is used to design a control law. In this method, the absolute position is not required. In our approach, each agent can use its own local coordinate system. That would be helpful in an environment where a common coordinate is not available. Also the communication among agents is limited and all-to-all communication is not required. In our approach agents do not need to choose a particular formation shape such as circular and different formation shapes are allowed. We prove asymptotic stability of the system by using Lyapunov-like functions for single integrator agents in the presence of obstacles and we show the convergence of the center of formation to the source. The control law has the capability of preventing collision between agents, or agents and static obstacles and steer the center of the formation to the source. Simulations confirm the effectiveness of the proposed algorithms.

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