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METHODOLOGY FOR INTEGRATING BIOMIMETIC BEAMS IN ABSTRACTED TOPOLOGY OPTIMIZATION RESULTS

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ABSTRACT

This paper presents a five-step design methodology to generate designs of biomimetic structural components from topology optimization results. In step one, all material allocated by topology optimization is classified as either beam like structures or nodes to generate an auxiliary model consisting of preserved regions, cylindrical beams, and ball nodes, which is an abstraction of the original topology optimization result. In step two, the auxiliary model is exposed to the original boundary conditions in a finite element analysis. Then, internal forces, torsion, and bending moments in all beams of the auxiliary model are identified with respect to both of their ends. In step three, a database is used to find a suitable biomimetic beam for each previously analyzed beam in the auxiliary model. In step four, adapted nodes are designed to connect the biomimetic beams and preserved regions to generate an intermediate biomimetic component design. And in step five, a design iteration and a validation of the final design are performed. The design methodology allows for reproducible biomimetic component designs, a trackable and easily documentable component development process, and the possibility of automating the design process to ultimately save development costs when designing structural components.

Keywords: Topology optimization, FEM, beam structures, biomimetics, component design

1. INTRODUCTION

Topology optimization (TO) has become of great importance in the development of mechanical components in the last decades, especially for lightweight design in aerospace and automotive applications [1, 2]. Based on a design space and relevant boundary conditions, such as fixed surfaces, forces, and pressures, the topology optimization algorithm allocates material inside the design space such that the loads are well supported by the design. Using the minimum mass approach, only as much material as needed is allocated to support the loads without the material reaching a certain threshold, such as 80 % of the yield strength of the material at any point inside the component.

Previous works indicate that under certain load conditions, biomimetic beams may be beneficial in terms of structural integrity when being compared to cylindrical beams [3]. It is suggested that adding geometric complexity by integrating biomimetic beams into design concepts from topology optimizations may lead to designs with reduced mass. Considering components of up to 0.5 m length made from metals or plastics, production of these kinds of components can be done very effectively by additive manufacturing techniques, such as Powder Bed Fusion by Laser Beam of Metals (PBF-LB/M).

Although structural component designs incorporating biomimetic beams seem to have great potential, they are not widely used. The authors expect that this is due to absence of a corresponding design methodology. For this reason, a design methodology is proposed in this article in which topology optimization results are skeletonized and converted into new component designs by including biomimetic beams leading to designs of lower mass while sufficiently supporting the acting loads.

2. RELATED WORK

The methodology proposed in this article has parallels to and is partly built upon the methodologies from [4, 5]. Therefore, these methodologies are summarized in the following.

After a 3D topology optimization is performed, a smoothing algorithm is used to give the topology optimization a smoother surface. Then, a mesh contraction Laplacian-based algorithm is used to identify beam like structures by generating a curve skeleton. The curve skeleton consists of nodes, branches (consisting of one or multiple beams), and end points of branches. For each beam of the curve-skeleton, a comparative radius of the cross section at that section of the topology optimization is identified. Next, the curve skeleton is normalized by substituting each branch consisting of multiple beams by one beam. This beam has a ra-

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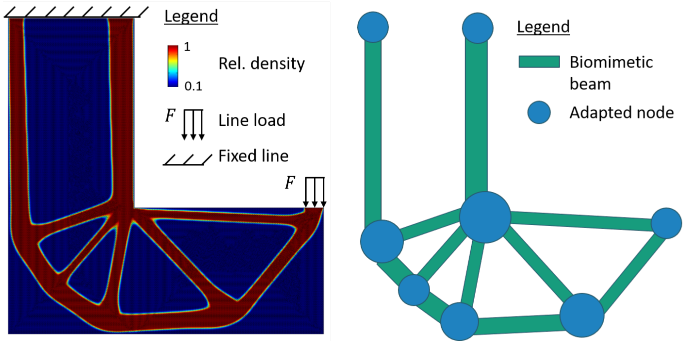


(a) Exemplary input **(b) Schematic output**

FIGURE 1: SCHEMATIC OF THE PROPOSED APPROACH TO CONVERT A RESULT FROM DENSITY-BASED TOPOLOGY OPTIMIZATION INTO A COMPONENT COMPOSED OF BIOMIMETIC BEAMS AND ADAPTED NODES

dius of the average of all comparative radii of the cross sections of all shorter beams of the respective branch. By that, a beam like CAD model can be generated consisting of cylindrical beams and preserved regions from the original topology optimization. Finally, a validation is done by creating a mixed dimensional finite element model. The normalized beams are simulated as 1D finite element beams while preserved regions are simulated using 3D finite elements. By applying the original load case to this model, internal stresses in all (normalized) beams can be evaluated. Based on the occurring stresses and the yield strength of the material, it is decided whether the design can support the loads or not.

In case the design does not fulfill the stress objective, two solutions are proposed. The first one is to modify the beams in which the stress objective is not achieved. The respective beams can be optimized such that they are supporting the loads sufficiently. The second option is to rerun the whole methodology with a topology optimization with an increased volume fraction. In case the newly generated design achieves the stress objective, the design is accepted as a final design. In case the newly generated design does not achieve the stress objective, another iteration is gone through with an even higher volume fraction. The number of iterations and therefore also the volume fraction are increased until the generated design is accepted based on the stress objective.

As the two methodologies [4, 5] are based on a skeletonization of a topology optimization result, a literature review was done in the fields of computer graphics, computational geometry, and visualization to derive so-called curve skeletons from two dimensional and three dimensional bodies. Cornea *et al.* [6] classified numerous methodologies for curve skeleton extraction into the four groups: thinning and boundary propagation, distance field based, as well as geometric and general field functions. The authors compared the four general approaches and found that the general field function approach gives the cleanest and smoothest results. At the same time, they also identified that this method resulted in the highest computational cost.

Because the methodologies [4, 5] use cylindrical beams and spherical nodes, the authors of this article expect that the methodology presented in this work can generate component designs of reduced mass while supporting loads sufficiently by introducing biomimetic beams and topology-optimized nodes.

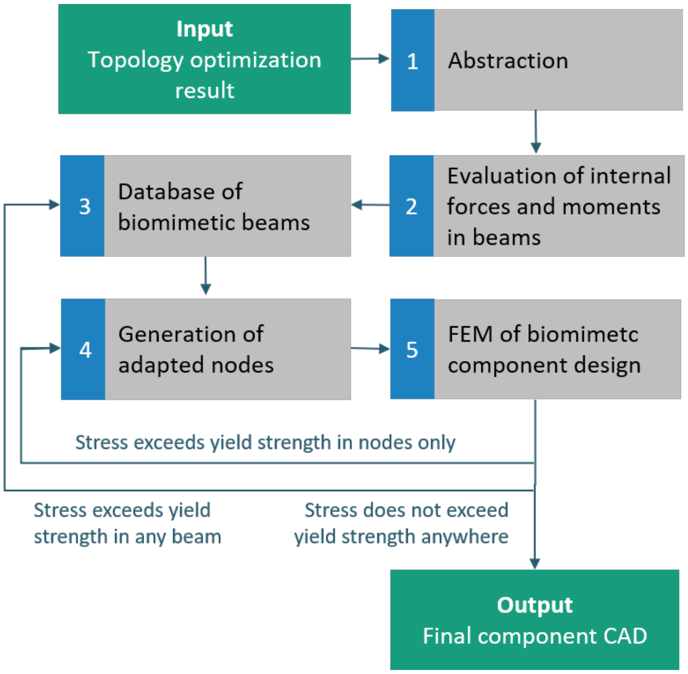
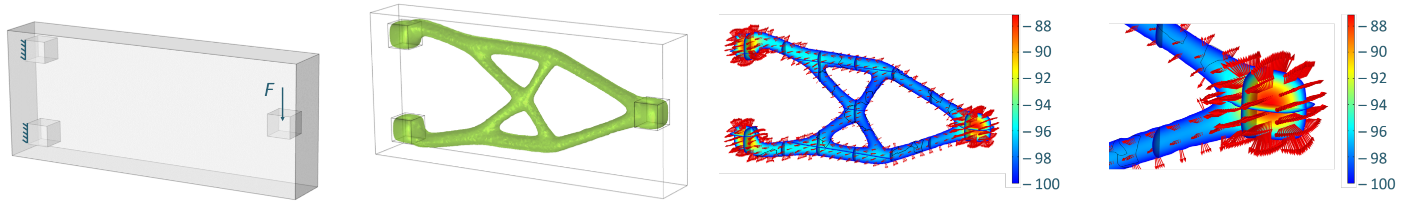


FIGURE 2: FIVE-STEP METHODOLOGY FOR THE INTEGRATION OF BIOMIMETIC BEAMS IN ABSTRACTED TOPOLOGY OPTIMIZATION RESULTS

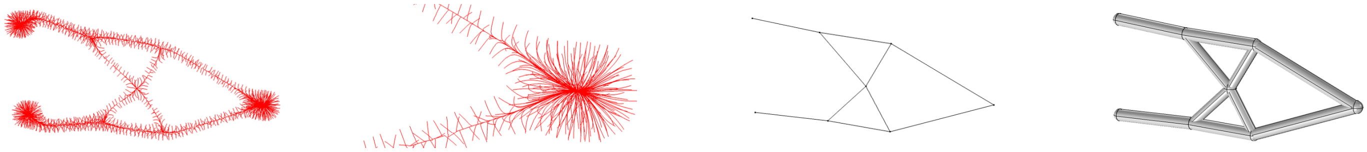
3. PROPOSED METHODOLOGY

A schematic of the proposed approach is shown in Fig. 1. It is assumed that all regions of a mechanical topology optimization result can be classified into one of two groups, namely, 'beam-like structures' and 'nodes'. The methodology creates an auxiliary abstraction of the topology optimization result, identifies internal forces and moments in each beam like structure in the auxiliary model, to then substitute the beam like structures by biomimetic beams in order to obtain a more efficient design in terms of mass and structural integrity. One of the strengths of the proposed methodology compared to [4, 5] is that for each beam, internal forces (normal force and shear force), internal torsion, and internal bending moments are identified such that each beam can be substituted by a biomimetic beam suitable for the corresponding load case. The authors suggest that the proposed methodology leads to designs of lower mass while supporting defined load cases sufficiently.

Figure 2 gives an overview over the five main steps of the methodology. In step 1, all material allocated by the topology optimization is classified as being part of either nodes or beams, which is denoted as skeletonization. Location and orientation of these entities are derived from the topology optimization result. In step 2, the internal forces and moments in all beams are identified with respect to both of their ends. In step 3, the load case of each beam is given as an input to a database of biomimetic beams. The result is a suitable biomimetic beam as an output for every input load case. In step 4, adapted nodes are designed to



(a) Schematic of design space and boundary conditions
(b) TO result showing relative densities above 0.7
(c) Electrical field within the TO result
(d) Detail view of electrical field within the TO result



(e) Streamline plot of trajectories of particles in the electrical field
(f) Detail view of streamline plot from (e)
(g) Abstraction of the TO result by nodes and lines
(h) Auxiliary model for the proposed methodology

FIGURE 3: EXEMPLARY CASE FOR THE ABSTRACTION STEP OF THE PROPOSED METHODOLOGY (IN (C) AND (D), THE ELECTRICAL POTENTIAL IN VOLTS IS GIVEN BY SCALE AND ARROWS SHOW FIELD DIRECTIONS)

connect the biomimetic beams and preserved regions to generate an intermediate biomimetic component design. And in step 5, a design iteration and a validation of the final design are performed. Finally, a CAD file is generated for the overall design of the component.

Each step of the methodology is discussed in detail in the following sections. It is noted that the methodology is not restricted to 2D problems as sketched in Fig. 1 but is also valid in 3D.

3.1 Abstraction

The goal of the input topology optimization result abstraction is to identify 'beams' and 'nodes' in the optimization result and generate an auxiliary design using spheres (nodes) and cylinders (beams). For the presented methodology, it is assumed that all regions of high density of the results can be assigned to one of the two groups. Relevant information to be identified is:

(a) coordinates of nodes,
(b) information about connections of nodes (also initial abstraction of beams),
(c) average diameter of each beam.

Inspired by the methodologies from [4, 5] as well as the curve skeletonization methods from [7, 8], a methodology for abstraction of the topology optimization result is proposed. It is an adapted version of the potential field approach and, therefore, is assumed to have better accuracy in terms of identification of nodes than the methodologies from [4, 5].

In Fig. 3(a), the design space and mechanical boundary conditions of an exemplary case are depicted. Figure 3(b) shows the respective 3D topology optimization result, which is a possible input for the proposed abstraction methodology. In a first step, a negative electric potential is added to the surface of the volume and a positive space charge density is added to the volume. This results in an electric field with gradients oriented from the center of nodes to the surface of the model, Figs. 3(c) and (d).

In a second step, particles of negative charge are spread on the surface of the model. In a time-dependent study, the particles are exposed to the electric field and an additional friction force is added such that the particles move along within the electric field and accumulate in the various nodes of the volume. The friction force helps to achieve a steady state of the particles faster by reducing oscillations around the nodes. Figures 3(e) and (f) show streamline plots of the paths that the particles moved along the electric field. The final locations of all particles are very close to each other in the center of each node.

In a third step, an algorithm is run to identify the nodes. For the final position of each particle, it is evaluated which other particles are within a certain maximum distance. Particles that are within this distance are assigned to the same node. Average values for the x, y, and z coordinates of all particles assigned to one node are defined as the coordinates of the node. Then, connections between nodes based on the original model are identified using another algorithm. For every particle in its initial position, neighboring particles are identified by a maximum distance. If two neighboring particles move into separate nodes during step 2, the corresponding nodes are considered connected by a beam. Thus, by using the obtained information, an abstraction of the initial 3D volume model can be generated using lines and nodes, Fig. 3(g). Algorithms discussed in this paragraph were developed by the authors and implemented in COMSOL Multiphysics using Java code.

To obtain data on the thickness of the beam like structures of the input topology optimization, trajectories of the particles can be evaluated. Looking at the beam like structures, particles are initially at the surface of the topology optimization result and move from there along the electric field toward the center of the beam like structure. After they reached the center of the beam like structure, they move toward the nodes of the topology optimization result along the center curve of the beam like structures. It is noted that this is an idealized model regarding the trajectories of the particles. As the trajectories show a sudden change of direction of the movement after they reached the center of the beam like structure, this characteristic can be used to distinguish each trajectory into two parts:

1. path from starting point of particle to center of beam like structure,
2. path from center of beam like structure (end of path 1) until node is reached.

Based on common type 2 paths of particles and knowledge about interconnection of nodes, particles of the same beam like structure can be identified. Using this information based on paths 1 of each particle, an approximation on the average thickness of each of the beam like structures can be obtained. It is noted that this methodology has a certain similarity with the method used for evaluation of thickness of beam like structures used in [4]. Particles' paths close to nodes are excluded from evaluation using a distance filter. Although this approach uses an idealized model of the trajectories, it is expected that the quality of the results is sufficient for the overall methodology.

Next, an auxiliary 3D model is generated as an abstraction of the original topology optimization result by modeling nodes as spheres and beams as cylindrical beams. The radii of the cylindrical beams are based on the previously identified average thicknesses of beam like structures of the topology optimization result. Finally, preserved regions (if existing) that were defined for the input topology optimization are added as copies to the model and are united by a Boolean operation to obtain the auxiliary model, Fig. 3(h). Partial implementation of the methodology as shown in Fig. 3 for presentation in this article was done in COMSOL Multiphysics 5.4.

3.2 Evaluation of internal forces and moments in beams

For evaluation of internal forces and moments in the beams, the auxiliary model is exposed to the original boundary conditions in a finite element analysis (FEA). Figure 3(h) shows the auxiliary model that can be used. Based on the FEA results, the internal forces and moments at both ends of each of the beams are evaluated.

It can be assumed that for each beam, normal forces, shear forces, and torsion (moment around the beam's main axis) are constant along the beam's length, as forces, torsion, and bending moments only act at the ends of the beams where the nodes are. Bending moments in the beams are assumed to be composed of two parts. The first part is constant and, due to a bending moment, acting at the ends of the beam. The second part is a bending moment due to shear forces at the ends of the beams that increases with increasing distance from the node.

Cut surfaces are generated at 20 % and 80 % of the length of each beam. At these cross sections, internal forces, torsion, and bending moments are evaluated. Bending moments at 20 % and 80 % of the lengths of the beams are used for extrapolation such that bending moments at 0 % and 100 % of the lengths of the beams can be obtained. The authors expect that cut surfaces should have a certain distance from nodes such that complex stress distributions at nodes do not interfere with the evaluation of internal forces and moments based on the assumptions. At the same time, the two cut surfaces in a beam should have a certain distance between them to ensure high quality results for extrapolation toward the ends of the beams. Evaluation at 20 % and 80 % was chosen as a compromise for the presented methodology. For all other obtained quantities, it is checked whether the following is true (for the nomenclature, see Fig. 4):

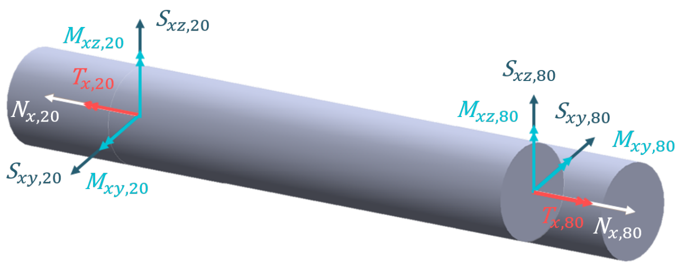


FIGURE 4: INTERNAL FORCES, TORSION, AND BENDING MOMENTS AT CUT SURFACES AT 20 % AND 80 % OF THE LENGTH OF THE BEAM WITH INTERNAL NORMAL FORCES ($N_{x,20}$, $N_{x,80}$), SHEAR FORCES ($S_{xy,20}$, $S_{xz,20}$, $S_{xy,80}$, $S_{xz,80}$), TORSIONS ($T_{x,20}$, $T_{x,80}$), AND BENDING MOMENTS ($M_{xy,20}$, $M_{xz,20}$, $M_{xy,80}$, $M_{xz,80}$)

$$N_{x,20} \approx N_{x,80} \ , \tag{1}$$

$$S_{xy,20} \approx S_{xy,80} \ , \tag{2}$$

$$S_{xz,20} \approx -S_{xz,80} \ , \tag{3}$$

$$T_{x,20} \approx T_{x,80} \ . \tag{4}$$

If the above is true, the calculated values are used. A schematic of a beam with internal forces, moments and torques is shown in Fig. 4.

Therefore, a normal force (N_x), two shear forces components (S_{xy} and S_{xz}), a torque (T_x) and two bending moment components (M_{xy} and M_{xz}) have to be evaluated at numerous cross sections of beams in the auxiliary Finite Element Method (FEM) model. The expressions for evaluation of internal forces and moments in the cut surfaces are derived from basic mechanics.

3.3 Database of biomimetic beams

Based on the loading case, each beam may be substituted by a biomimetic beam. It is expected that biomimetic beams may increase the stiffness of the structure or, in turn, reach a similar stiffness at a lower mass. Emmelmann *et al.* [9] suggests that use of biomimetic beams in structural parts can improve the mechanical performance of the overall component. Ma *et al.* [3] showed that biomimetic beams based on geometric characteristics of bamboo can have a considerably increased load bearing capacity for certain load cases.

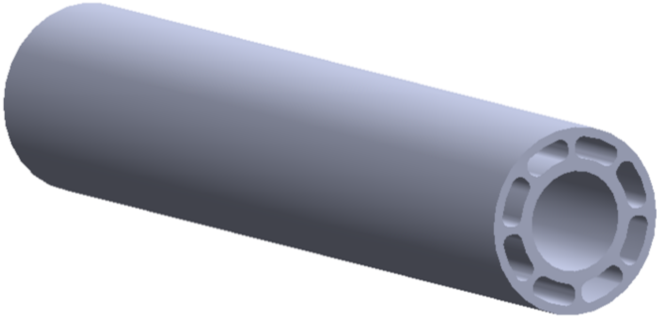
An exemplary biomimetic beam is shown in Fig. 5. In this case, the cut surface of a trunk of a banana plant was abstracted, Fig. 5(b). Extrusion of the abstraction leads to a biomimetic beam, Fig. 5(c). Depending on its length, the beam can support certain load cases.

For the presented design methodology, a database of biomimetic beams is proposed. Figure 6 shows the function of the database. Its inputs are the length of the beam and its normal forces, shear forces, torsions, and bending moments at both ends of the beam.

Various parametric biomimetic beams are stored in the database. For each of the parametric beams, numerous versions with different parameters are included (lines in simplified table in Fig. 6). Furthermore, a function is provided with respect to

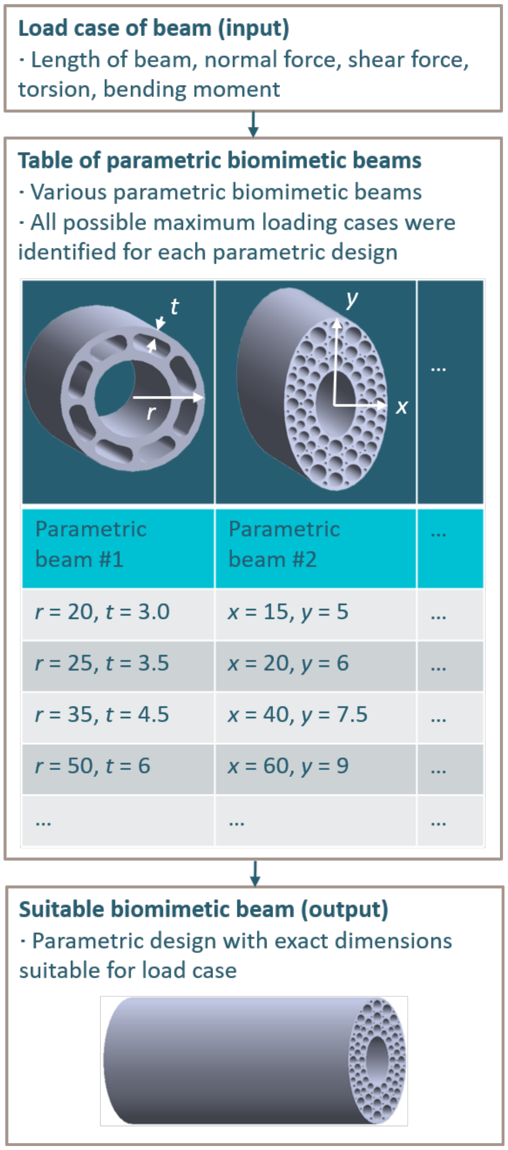
(a) Photograph of a banana tree trunk cut surface [10]

(b) Cross section view of the abstracted biomimetic beam



(c) Isometric view of the biomimetic beam

FIGURE 5: EXEMPLARY BIOMIMETIC BEAM BASED ON A BANANA TREE TRUNK



Load case of beam (input)
- Length of beam, normal force, shear force, torsion, bending moment

Table of parametric biomimetic beams
- Various parametric biomimetic beams
- All possible maximum loading cases were identified for each parametric design

Parametric beam #1	Parametric beam #2	...
$r = 20$, $t = 3.0$	$x = 15$, $y = 5$	...
$r = 25$, $t = 3.5$	$x = 20$, $y = 6$	...
$r = 35$, $t = 4.5$	$x = 40$, $y = 7.5$	...
$r = 50$, $t = 6$	$x = 60$, $y = 9$	...
...	...	...

Suitable biomimetic beam (output)
- Parametric design with exact dimensions suitable for load case

FIGURE 6: SCHEMATIC OF THE BIOMIMETIC BEAM DATABASE UTILIZATION

normal forces, shear forces, torsions, and bending moments evaluating the maximum load cases that this specific type of beam can successfully support.

With a given load condition of a beam from the auxiliary model, the following steps have to be carried out. First, the best suitable specific design of the parametric beam #1 is identified. Starting with the specific design of the smallest cross sectional area and carrying on with the specific designs that have greater cross sectional areas, it is checked whether the design can support the given load. The first specific design that is found to successfully support the loads is identified as the best suitable specific design of parametric beam #1. Analogously, for parametric beams #2, #3, etc., the most suitable specific types of designs are identified. Finally, the mass of all identified specific designs is compared and the design with the smallest mass is chosen. This specific design is the output of the digital database for the given input load condition.

3.4 Generation of adapted nodes

Based on the abstraction of the original topology optimization results by lines and nodes [Fig. 3(g)] and the output from the database of biomimetic beams, the orientations and locations of biomimetic beams in 3D space are known. With this information, adapted nodes can be generated to connect the biomimetic beams with each other as well as the biomimetic beams and the preserved regions of the input topology optimization. In the following, two approaches are presented for the generation of these adapted nodes.

In the first approach, all biomimetic beams and connecting surfaces are positioned in a topology optimization model. The bounding box of all components in the model is used as a design space for the topology optimization. All connecting surfaces and biomimetic beams are fixed at a relative density of one. All mechanical boundary conditions of the original input topology optimization are added to the new model. The optimization objective is defined using the minimum mass approach in which only as much material as needed is allocated to support the loads without the material reaching a certain threshold, such as 80 % of the yield strength of the material at any point inside the component.

As the biomimetic beams can be expected to support loads that act at their ends, material should be placed by the algorithm at nodes only. Figure 7 shows a conceptual representation of nodes generated by the presented approach in a 2D example. To generate the shape of the nodes and also the whole biomimetic model, a filter can be used to only export all volume where the relative density exceeds a certain threshold (e. g., 0.7).

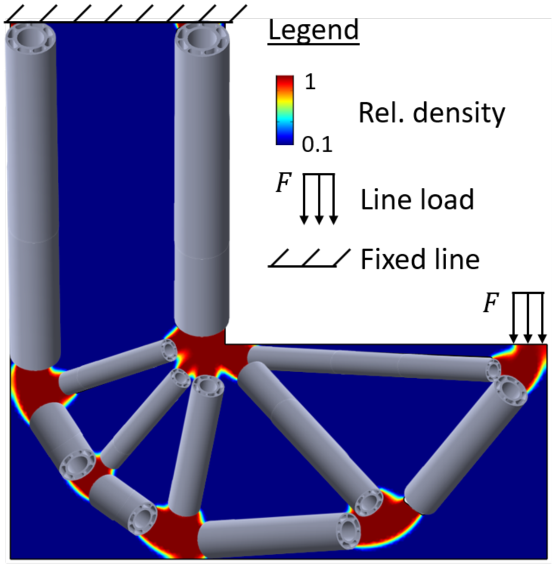


FIGURE 7: CONCEPTUAL REPRESENTATION OF GENERATION OF NODES FOLLOWING THE FIRST APPROACH FOR AN EXEMPLARY 2D COMPONENT (RELATIVE DENSITIES OF THE RESPECTIVE DESIGN SPACE ARE GIVEN BY SCALE)

In the second approach, individual topology optimization models are generated for each node. Based on the abstraction of the original topology optimization results by lines and nodes [Fig. 3(g)] and the output from the database of biomimetic beams, a topology optimization model such as the one in Figure 8 can be generated. The design space contains the ends of all biomimetic beams that are to be connected by the node.

The topology optimization model is set up the following way: Biomimetic beams that are to be connected by the node are placed into the model. However, only the ends of the biomimetic beams are added rather that the whole biomimetic beams. Internal forces and moments identified with the auxiliary model are used to introduce forces and moments as boundary conditions to the ends of the biomimetic beams in the new topology optimization. In case preserved regions are within a certain distance to the node, they are added to the model as well as loads as defined in the input topology optimization of the overall methodology.

Analogously to the first approach, all preserved regions and biomimetic beams are fixed at a relative density of one. Furthermore, also as in the first approach, the minimum mass approach is used (see above). To generate the shape of the node, all volume in which the relative density exceeds a certain threshold (see above) can be exported from each of the models. Using this methodology, shapes for all nodes can be generated.

In a final step, biomimetic beams, generated nodes, and preserved regions can be positioned and oriented in 3D space and united in a single model.

In the following, the two approaches are briefly discussed. The first approach is a holistic approach. It can be expected that the result of this approach supports all loads of the original mechanical problem. However, to follow this approach, relatively large stiffness matrices need to be set up and solved, as the geometry of the biomimetic beams is complex. Furthermore, when applying topology optimization algorithms, usually fine meshes are needed to generate designs of high mechanical performance. Therefore, rather high computational costs are expected for this approach.

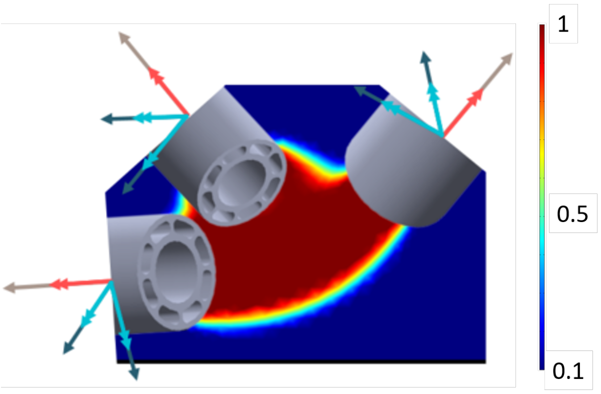


FIGURE 8: CONCEPTUAL REPRESENTATION OF GENERATION OF NODES FOLLOWING THE SECOND APPROACH FOR AN EXEMPLARY 2D COMPONENT (RELATIVE DENSITIES OF THE RESPECTIVE DESIGN SPACE ARE GIVEN BY SCALE)

The second approach subdivides the problem of node generation in multiple problems. This reduces the size of the stiffness matrices to be used tremendously or, in turn, increases the quality of the result as the volume at the node can be discretized by a much finer mesh. The downside of this approach is that the forces and moments acting inside the biomimetic beams can be expected to have a certain error. This is due to the fact that substitution of the beams in the auxiliary model by biomimetic beams leads to changed force paths and moments in the structure. Therefore, the nodes generated using this approach may be oversized or undersized. Future research has to be done to investigate advantages and disadvantages of the two approaches.

3.5 FEM of biomimetic component design and output

Based on the previously described methodologies, volumetric models of new biomimetic component designs containing biomimetic beams can be generated. In this step, the original boundary conditions are applied to the new design in an FEA. In post processing of the FEA results, all areas in which the von Mises stress exceeds the yield strength of the material are identified. According to Fig. 2, three different cases are to be distinguished:

- If there are no areas in which the yield strength is exceeded by the von Mises stress, the component design is accepted as the final design.
- If the yield strength is exceeded by the von Mises stress in any of the nodes, the step of node generation is carried out again, using either smaller finite elements or adapting the threshold of the optimization objective (only as much material as needed is allocated to support the loads without the material reaching, e. g., 70 % instead of 80 % of the yield strength of the material at any point inside the component). With the achieved design, the workflow is carried on from step 5 ‘FEM of biomimetic component design’.

- If the yield strength is exceeded by the von Mises stress in any of the beams, the step utilizing the database of biomimetic beams is partly carried out again. For all beams in which the yield strength is exceeded, previously chosen biomimetic beams are substituted by the type of this parametric beam that is next in terms of structural integrity. With the achieved results, the workflow is carried on from step 4 'Generation of adapted nodes.'

Using this methodology, a final biomimetic component design can be generated that meets the defined stress criterion.

4. CONCLUSION AND OUTLOOK

Although structural component designs combining topology optimization and incorporation of biomimetic beams seem to have great potential, they are not widely used. For this reason, a five step design methodology was proposed in this article in which topology optimization results are converted into new component designs including biomimetic beams leading to designs of lower mass while sufficiently supporting the acting loads. The methodology has high potential to allow for reproducible biomimetic component designs, a trackable and easily documentable component development process, and the possibility of automating the design process to ultimately save development costs when designing structural components.

As the methodology was developed on a conceptual level, detailed implementation, testing, and comparison to multiscale topology optimization approaches is still to be done and will be addressed by the authors in future works.

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