This paper was presented at the 18th Wind Integration Workshop and published in the workshop's proceedings

18th Wind Integration Workshop, October 16-18, 2019, Dublin

Making Use of Analytical Wake Models for Large Scale Power System Models by Generation of Generic Efficiency Fields

Gerrit Erichsen
Institute of Energy Systems
Hamburg University of Technology
Hamburg, Germany
https://orcid.org/0000-0002-2951-2750

Oliver Schülting
Institute of Energy Systems
Hamburg University of Technology
Hamburg, Germany
oliver.schuelting@tuhh.de

Alfons Kather
Institute of Energy Systems
Hamburg University of Technology
Hamburg, Germany
kather@tuhh.de

***Abstract*—Hereby a modelling approach for wind power in large scale power system models is presented. Therefor a generic wind farm layout is implemented and an analytical wake model is applied to it. The generic layout is organized as a square grid in resolution of the rotor diameter matching the given area. The wind turbines are placed in a fixed pattern matching the total given power and aiming to match rules of ideal placement. From that an efficiency field in dependence of wind speed and wind direction is obtained. The efficiency is defined as the total power of the wind farm considering wake losses for a certain wind speed and direction divided by the power a single turbine would produce given that respective wind speed times the number of turbines. This efficiency field is then introduced into a power system model. The power system model is therefore able to model the wind power generation not only in dependence of wind speed, but also the wind direction with comparatively low computational effort. This allows maintaining a physically sensible interpretation of a wind power plant and keeping the performance factor to availability and uncertainty losses only.**

For the generation of the efficiency field the CharL model is used. As an exemplary power system model the TransiEnt Library is used. For the validation of the approach the German wind power generation at a municipal level is modelled using the characteristic lines of a small and a large wind turbine. The efficiency fields are then generated for each turbine with a generic farm layout for three power densities (30, 60 and 90 MW/km²). The area of the wind farms is set to accompany 40 turbines at 60 MW/km². The resulting power profiles are scaled according to the installed power in each municipality, whereby the choice of turbine and efficiency field is made appropriately. For all farms the main wind direction is set to western. The wind power feed – as published by the TSOs – is used as reference for the validation.

With this approach the wind generation profile can be modelled closely and the impact of the wind direction can be shown as an additional parameter by successfully modelling power generation drops that relate to changes in wind direction.

Keywords—wake model, large energy system model, efficiency field

I. INTRODUCTION

With the Paris Agreement 186 states aim to reduce CO_2 emissions [1]. This leads to a transition towards using more renewable energy resources to generate electrical power in many participating countries. In addition to economic measures, it is often tried to control the current expansion of renewables by national laws [2]. For long-term perspectives, scientific investigations are carried out to find economic and ecological sensible energy system configurations [3]. An inevitable part of these investigations is the prognosis of the potential future electrical feed by wind power plants.

As large energy system modelling is computationally quite expensive, there is need for complexity reduction. For VRE (volatile renewable energy) producers such as wind power plants, a commonly used approach is to use capacity factors to scale the total installed power of wind power plants. These can be generated from data of TSOs (Transmission System Operator) or using models [4] such as the one presented here. An advantage of scaled TSO data is that many factors are taken into consideration, such as e.g. wake losses, multiple wind turbine types or curtailment. The one big disadvantage of TSO data is, that – besides installed power – nothing can be adjusted [4].

Therefore, most future energy system projections use weather data to generate a wind power profile. Depending on data availability and model approach often the wind speed is the only variable parameter in these models, which then model wake effects using a constant performance or loss factor [5, 6]. A recent alternative is to use a non-constant wake loss factor depending on wind speed alone as demonstrated in [7].

Alternatively, stochastic and machine learning approaches are suggested for wind power prediction for current energy systems [8, 9]. These models are able to include large amount of data to make accurate power feed predictions [9]. However, these approaches are not able to vary locations, turbine types and other important parameters for future energy system scenarios [10].

This paper aims to present an alternative method by using an efficiency field. The efficiency of the farm is supposed to be dependent on both wind speed and direction, to model wake loss more accurately. This makes the orientation of a wind power plant an additional parameter, that is also easily physically interpretable. Additionally, we present a method to generate such efficiency field using analytical wake models and generic turbine placement. These fields will then be used in an energy system model that will be introduced subsequently. Given the energy system configuration two scenarios, namely modelling using a constant wake loss factor and modelling using the proposed approach, will be the basis for method validation. Based on the results a short discussion with a recommendation for future modelling will be given.

II. WIND FARM MODELLING

A. *Wake Modelling*

Various approaches exist for modelling wake in a single wind power plant. They range from highly detailed CFD simulation to machine learned parameterization of linear correlation models. For the paper at hand we use the model CharL[1], in which various analytical wake models for efficiency field generation are implemented. These are the model approaches by e.g. Jensen [11] or Frandsen [12], among others. For the paper at hand we use the Jensen model, which is implemented using the early top hat approach for expansion of the wake diameter

$$D_{wake} = D(1 + 2 \cdot k \cdot x) \tag{1}$$

with $k = 0.075$ as a typical value for onshore wind power plants and x being the distance between the rotor planes of the wake inducing and affected turbine. The affected area of a wind turbine rotor is calculated using a simplified estimation

$$A_{\mathrm{wake}} = A \cdot \frac{D_{\mathrm{affected}}(\varphi)}{D} \tag{2}$$

with $D_{\mathrm{affected}}(\varphi)$ being the length of the rotor, that is affected by the wake diameter $\mathrm{D}_{\mathrm{wake}}$ of the wake inducing turbine. For the effective wind speed at the turbine the wake wind speed

$$v_{wake} = v \left(1 - \frac{2 \cdot a}{(1 + 2 \cdot k \cdot x/D)^2}\right) \tag{3}$$

with a as the axial induction factor

$$a = \frac{1 - \sqrt{1 - c_{\mathrm{t}}}}{2} \tag{4}$$

is needed. Therefor the c_{t} curve for the respective turbine linearly interpolated for v and the wind speed of the wake inducing turbine is needed. The effective wind speed is then the wake wind speed v_{wake} proportionally to the wake affected rotor area and the free wind speed

$$v_{\mathrm{eff}} = v_{\mathrm{wake}} \cdot \frac{A_{\mathrm{wake}}}{A} + v_{\infty} \cdot \left(1 - \frac{A_{\mathrm{wake}}}{A}\right). \tag{5}$$

Depending on the superposition of wakes, the Jensen model is considered as either optimistic [13] or pessimistic [14], whereby the presented approach, without superposition but only relating to previous wake inducing turbine, is considered as pessimistic [13]. In this paper the pessimistic approach is chosen as it is implemented more easily and some inaccuracy is to be expected anyway.

B. *Generation of Efficiency Fields*

Calculating the wake losses using the analytic approach presented above is computationally expensive. Hence, it is aimed for a computationally less expensive method. This is done by reducing the amount of data from turbine individual wind speeds, their inner relations and the resulting turbine individual powers, to a simple efficiency. Efficiency defined as the power the entire wind power plant would deliver, considering wake, divided by the power the wind power plant could deliver, if all turbines were to receive the free wind speed v_{∞}. The field is generated for each wind speed of the wind turbine's power curve and each wind direction φ at 1° resolution.

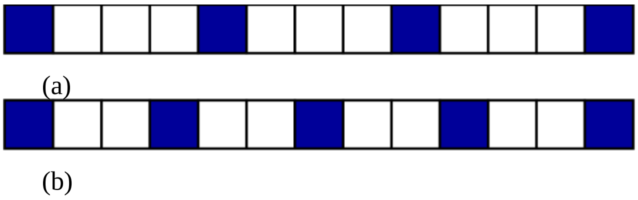

Fig. 1: Placement of turbines in an example row (a) and example column (b) according to heuristic placement distance. Filled boxes indicate a turbine.

$$\eta(v_{\infty}, \varphi) = \frac{\sum P_{\mathrm{turbine}}(v_{\mathrm{eff}}(\varphi))}{n \cdot P_{\mathrm{turbine}}(v_{\infty})} \tag{6}$$

The effective wind speed is calculated in dependence of wind direction, whereby the influence of the wind direction results from wind turbine placement within a rectangular grid. The length/width of each cell within the grid is equal to the rotor diameter of the wind turbine. For simplicity and universal applicability an optimized turbine placing as in [15] will not be pursued, but rather a heuristic rule such as five rotor diameters distance in main wind direction and four rotor diameters distance in secondary wind direction [16] is implemented. Accordingly, the rows and columns will be populated to match this heuristically optimal distance. As a key assumption the three diameters distance placing is used within a column, therefore representing north-south distribution, and the four diameter distance is used within a row, representing east-west distribution. Hence follows, that the main wind direction of a generically placed wind power plant is 315° or NW. It is assumed, that rotating the efficiency field should be enough to accommodate for other main wind directions. This rotation of the efficiency field can be done by offsetting the wind direction relative to the main wind direction of 315°. Given a particular wind turbine, an available area for the wind power plant, and a target power density Φ in W/m² a sample row as in Fig. 1(a) and a sample column (Fig. 1(b)) will be generated. From that a grid is populated placing a sample column at each element of the sample row with a turbine. The resulting grid for the sample placements of Fig. 1(a) and Fig. 1(b) can be

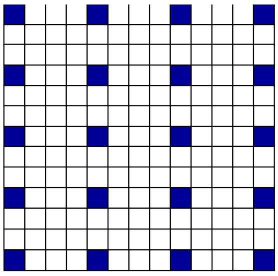

Fig. 2: Resulting grid from Fig. 1(a) and (b)

---

[1] https://collaborating.tuhh.de/iet/CharL at sha1: `699588ec504cb7e25798a16e98b5150d5436f3eb`

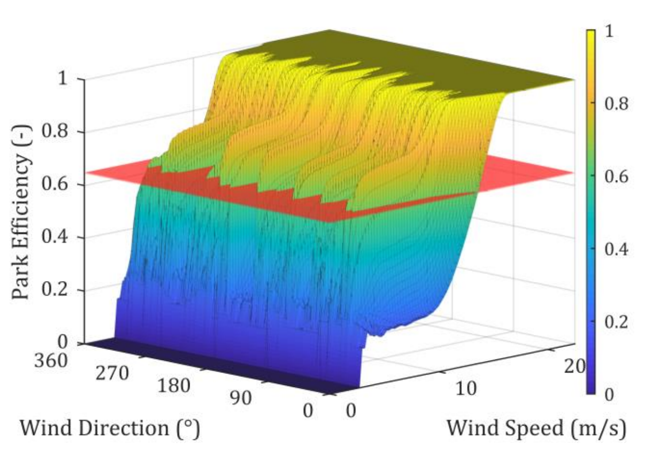

Fig. 3: An efficiency field for the Vestas V126-3.45 at 60 MW/km² placed over an area of 2.3 km². Additionally, a representation of the Simple approach is displayed as a horizontal red plane.

seen in Fig. 2. Should the sample placements result in large variances in distribution, these are equalized and any larger mismatch in grid power density and targeted power density will be addressed by deletion of turbines from the second populated column. This is explained in detail in [17].

As a visual example the resulting efficiency field for the Vestas V126-3.45 at 60 MW/km² placed over an area of 2.3 km² can be seen in Fig. 3. To illustrate the difference of the approach at hand, the simplified approach – which is a constant performance factor (Park Efficiency in Fig. 3) for any wind speed and direction – is shown as a red plane.

III. ENERGY SYSTEM MODEL

To show the advantages of the presented method the wind power feed for the Federal Republic of Germany for the year 2015 is calculated in two ways where the methods differ in the way of calculation of losses. To evaluate the calculated results of the electrical feed by wind power plants resulting from these different computational methods the electrical feed as published by the TSOs in Germany $P_{\text{Ger,ref}}(t)$ – as presented in [18] – is used as a reference.

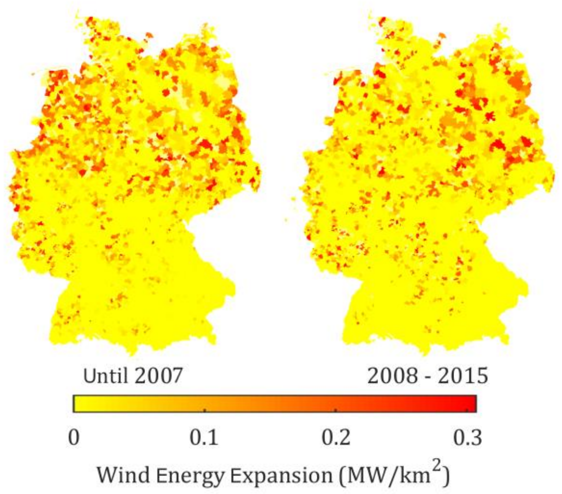

Fig. 4: Expansion of onshore wind power plants in Germany

In both computational methods, the calculated feed is regionally based on municipalities, the smallest administrative division in Germany. For each of the 11,168 municipalities the theoretically possible feed by wind power plants is calculated using regional weather data. The weather data was obtained from the assimilation analysis of the COSMO-DE model with hourly resolution [19]. The electrical feed is calculated in Modelica using the TransiEnt Library [20]. To account for differences of local installed wind turbines, power curves of two different wind turbines are used. For wind turbines installed before 2007 (18.47 GW in total) the characteristics of the Vestas V80 turbine with a hub height of 78 m are used. For newer turbines, which were built between 2008 and 2015 (18.57 GW in total), the characteristics of the Vestas V126 turbine with a hub height of 117 m are used. This way the total installed capacity is split into two approximately equal parts. Fig. 4 shows the specific expansion of onshore wind power plants in Germany for both time periods. It becomes obvious that for both time periods the main expansion takes place in the north of Germany. Still, there are differences in the spatial distribution of the expansion of both time periods.

For each municipality *i* the feed is scaled to 1 Watt ($P_{\text{scaled},i}(t)$), such that the final feed without losses for each municipality $P_i(t)$ results from scaling $P_{\text{scaled},i}(t)$ by the installed capacity of each municipality $P_{\text{inst},i}$. The installed capacity for each municipality in 2015 is based on [21].

$$P_i(t) = P_{scaled,i}(t) \cdot P_{inst,i} \tag{7}$$

A summation over all *n* municipalities in Germany thus results in the feed of whole Germany without losses.

$$P_{Ger}(t) = \sum_{i=1}^{n} P_i(t) \tag{8}$$

The first computational method, from now on referred to as 'Simple method', uses a constant performance factor PF_{Simple} to account for wake losses and any further losses, independent of wind speed and direction and thus constant over time and constant for each municipality. For the sake of comparability with the feed $P_{\text{Ger,ref}}(t)$ the performance factor is defined as:

$$PF_{Simple} = \frac{\sum_{t=1\,h}^{8760\,h} P_{Ger,ref}\ (t)}{\sum_{t=1\,h}^{8760\,h} P_{Ger}(t)} \tag{9}$$

This results in an overall performance factor of PF_{Simple} = 63.62 %.

The second computational method, from now on referred to as 'New method', uses a combination of two losses. On the one hand, a representation of wake losses via the presented model from Jensen with an efficiency $\eta_i(v_i(t), \varphi_i(t))$ for each municipality *i*, depending on the time dependent wind speed *v(t)* and wind direction $\varphi_i(t)$, is used. The efficiency fields for both wind turbines are generated by the CharL model. To represent a distribution of the installed power density Φ three different efficiency fields with different power densities are combined for each turbine. Power densities of 30, 60 and 90 MW/km^2 are used, whereby the resulting efficiency field used to calculate $\eta_i(v_i(t), \varphi_i(t))$corresponds to an averaging of these three fields. The share of each efficiency field depends on the respective federal state of the municipality. Based on [22] the shares stated in Table 1 are used.

Table 1: Chosen distribution of wind power plant configurations among federal states

Federal State	30	60 [MW/km^2]	90
SH, HH, NI, HB, NRW, HE, RP, BW, BY, MV, SN, TH	25 %	50 %	25 %
SL	0 %	50 %	50 %
BE, BB	50 %	50 %	0 %
ST	25 %	75 %	0 %

The fields are rotated to the assumed main wind direction of the wind park in each municipality. Therefor the mean wind direction of each municipality in the years 2010 – 2016 based on the weather model COSMO-DE [19] is used. Additionally to the wake losses according to Jensen, a constant performance factor PF_{New} for all municipalities is used to account for further losses. Analogously to the Simple method, here as well the performance factor is defined such that the annual energy that is produced by wind turbines corresponds to the actual energy that was fed in.

$$PF_{New} = \frac{\sum_{t=1\,h}^{8760\,h} P_{Ger,ref}(t)}{\sum_{i=1}^{n} \sum_{t=1\,h}^{8760\,h} P_i(t) \cdot \eta_i(v_i(t), \varphi_i(t))} \tag{10}$$

This results in an overall performance factor of PF_{New} = 95.51%.

From this results the calculated feed of whole Germany via two methods with an identical annual electrical energy, matching the actually fed in electrical energy by onshore wind power plants according to [18]. Hereby the two methods differ in the way of calculation of wake losses such that they have a different qualitative progress over time. To match the annual feed of $P_{\text{Ger,ref}}$, the performance factors of (9) and (10) are used for (11) and (12) respectively.

$$P_{Ger,Simple}(t) = \sum_{i=1}^{n} P_i(t) \cdot PF_{Simple} \tag{11}$$

$$P_{Ger,New}(t) = \sum_{i=1}^{n} P_i(t) \cdot \eta_i(v_i(t), \varphi_i(t)) \cdot PF_{New} \tag{12}$$

Fig. 5 shows the calculated feed via the two presented methods as well as the actual feed according to the TSOs as a reference for the first two weeks of the year 2015.

IV. RESULTS

Comparing the two used methods shows clear similarities of the feeds, but also noticeable differences, especially looking at high electrical powers. Fig. 6 shows a sorted histogram of the calculated electrical power as well as the reference value according to the TSO. Since the losses in the Simple method are constant, the actual nominal power of the wind power plants of 37.04 GW can never be reached. Using the New method, the wake losses decrease for high wind speeds up to no wake losses such that the maximum calculated electrical power is higher.

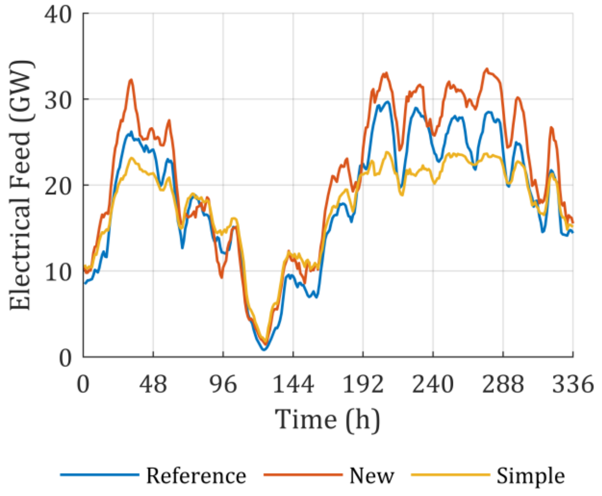

Fig. 5: Calculated power feeds by wind power plants of the two different computational methods and actual feed given by the TSO

Additionally, in Fig. 7 the power gradients are displayed, which show no clearly visible differences for both approaches.

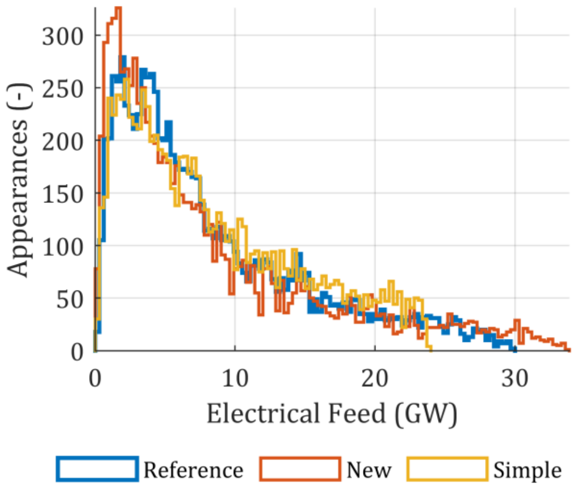

Fig. 6: Histogram of electrical power resulting from the two different computational methods as well as from the actual reference feed given by the TSO

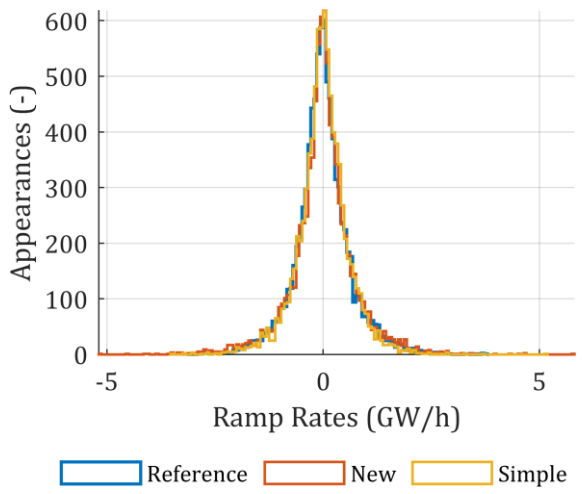

Fig. 7: Histogram of electrical power ramp rates resulting from the two different computational methods as well as from the actual reference feed given by the TSO

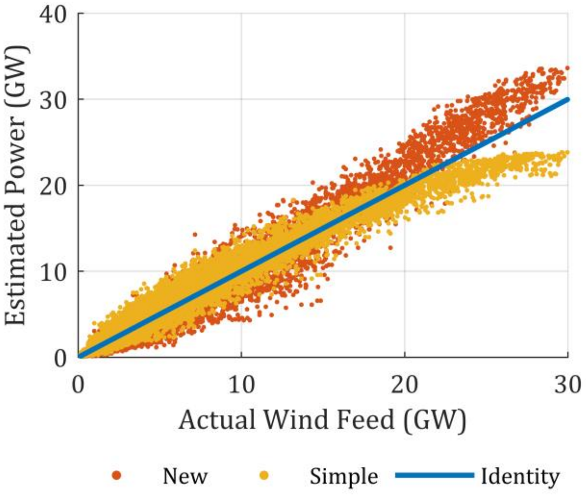

Fig. 8: Model comparison of proposed New method (red) and Simple method (yellow) to actual feed data (blue).

A. *Correlation Factors*

Some of the common correlation coefficients, such as the Pearson Coefficient, normalized root mean square error (NRMSE, normalized by difference of maximum and minimum value of the reference data), mean absolute error (MAE), and R^2 were calculated, to investigate the claim, that the New method matches the data more closely. Due to the methodology described above, the mean bias error is not applicable to this approach, since the performance factor scaling intentionally leads to values close to zero.

Table 2: Observed correlation coefficients.

Coefficient	New	Simple
Pearson Coefficient	0.9781	0.9761
NRMSE	0.0446	0.0353
MAE	1.33 GW	1.05 GW
R^2	0.9579	0.9560

To make better sense from the table values the distribution of estimated wind power feeds is shown in Fig. 8.

V. DISCUSSION

The performance factor of the New method with $PF_{New} = 95.51$ % is rather close to the value supposed by [23, p. 628] for availability of a wind power plant (98 %), while the scaled performance factor of the Simple method with $PF_{Simple} = 63.63$ % is rather low, compared to the suggestion of [23, p. 916] considering a uniform value for wake losses. This is viewed at as a benefit of the presented method, as the parameterization can be done with literature data more accurately.

Considering the almost parity in statistical values of both the presented method and the Simple approach however, the argument for the presented approach is hard to make. Given the rough parameterization on the other hand, by use of only two wind turbine types and three respective wind power plant densities, the authors like to focus on the additional information the presented method provides as exemplified by Fig. 5. The presented method considers wind direction in a meaningful, interpretable and parameterizable way.

VI. CONCLUSION

A two-step process for modelling wind power in large energy system models was presented. The first step is the generation of an efficiency field using analytical wake models. The second step is setting the orientation of that efficiency field and the respective power curve and power scaling factor in an energy system model, for which the TransiEnt Library was used.

The results show, that the presented method calculates more accurately high power feeds and is able to model effects related to changes in wind direction. Other than that, the improvements to more common approaches were low. The authors suggest to use this method for future energy system scenarios, as the “sudden” power decreases due to change in wind direction might affect the need for energy storages.

As an outlook the current wake modelling might be improved by sizing the placing grid independently from turbine diameter and implementing a more recent iteration of an analytical model, e.g. [24]. For the energy system model a more detailed parameterization might improve the results. Especially regarding the power density distribution taken from [22] or wind turbine types and parameters. For the latter [25] will present a method at the same workshop.

ACKNOWLEDGMENT

This research was funded by the BMWi.

Supported by:

Federal Ministry
for Economic Affairs
and Energy

on the basis of a decision
by the German Bundestag

REFERENCES

[1] United Nations, Paris Agreement, 2015.

[2] Bundesministerium für Wirtschaft und Energie (BMWi), "Gesetz für den Ausbau erneuerbarer Energien": EEG . [Online] Available: https://www.gesetze-im-internet.de/eeg_2014/. Accessed on: Aug. 20 2019.

[3] M. Peter, D. Bertschmann, and H. Lückge, “Metastudie nationale Energieszenarien und deutsche Energiepolitik,” Dessau-Roßlau, 2017 . [Online] Available: https://www.umweltbundesamt.de/sites/default/files/medien/1410/publikationen/2017-11-06_climate-change_27-2017_metastudie-energieszenarien.pdf. Accessed on: Aug. 26 2019.

[4] I. González-Aparicio *et al.*, “Simulating European wind power generation applying statistical downscaling to reanalysis data,” Applied Energy, vol. 199, pp. 155–168, 2017,doi: *10.1016/j.apenergy.2017.04.066*.

[5] P. Elsner, B. Erlach, M. Fischedick, B. Lunz, and D. U. Sauer, Eds., Flexibilitätskonzepte für die Stromversorgung 2050: Technologien - Szenarien - Systemzusammenhänge. München: acatech - Deutsche Akademie der Technikwissenschaften e.V, 2015.

[6] D. Stetter, “Enhancement of the REMix energy system model: Global renewable energy potentials, optimized power plant siting and scenario validation,” Dissertation, Institute of Thermodynamics and Thermal Engineering, University of Stuttgart, Stuttgart, 2014.

[7] Sabine Haas, “Implementation and validation of an open source model for generating wind feed-in time series,” Master thesis, Reiner Lemoine Institut, Technische Universität Berlin, Berlin, 2019.

[8] G. Dany, Kraftwerksreserve in elektrischen Verbundsystemen mit hohem Windenergieanteil. Zugl.: Aachen, Techn. Hochsch., Diss., 2000, 1st ed. Aachen: Klinkenberg-Verl., 2000.

[9] I. Colak, S. Sagiroglu, and M. Yesilbudak, “Data mining and wind power prediction: A literature review,” Renewable Energy, vol. 46, pp. 241–247, 2012,doi: *10.1016/j.renene.2012.02.015*.

[10] J. Yan, Y. Liu, S. Han, Y. Wang, and S. Feng, “Reviews on uncertainty analysis of wind power forecasting,” Renewable and Sustainable Energy Reviews, vol. 52, pp. 1322–1330, 2015,doi: *10.1016/j.rser.2015.07.197*.

[11] Jensen and Niels Otto, “A note on wind generator interaction,” Physics Department, 1983.

[12] S. Frandsen *et al.*, “Analytical modelling of wind speed deficit in large offshore wind farms,” Wind Energ., vol. 9, no. 1-2, pp. 39–53, 2006,doi: *10.1002/we.189*.

[13] T. Göçmen *et al.*, “Wind turbine wake models developed at the technical university of Denmark: A review,” Renewable and Sustainable Energy Reviews, vol. 60, pp. 752–769, 2016,doi: *10.1016/j.rser.2016.01.113*.

[14] N. Charhouni, Arbaoui Abdelaziz, and M. Sallaou, “Qualification of Three Analytical Wake Models,” in S12 Fiabilité et robustesse des systèmes mécaniques, Association Française de Mécanique, Ed., 2015.

[15] G. Mosetti, C. Poloni, and B. Diviacco, “Optimization of wind turbine positioning in large windfarms by means of a genetic algorithm,” Journal of Wind Engineering and Industrial Aerodynamics, vol. 51, no. 1, pp. 105–116, 1994,doi: *10.1016/0167-6105(94)90080-9*.

[16] B. Neddermann and T. Schorer, “Land Requirement Values for Wind Farms,” in Proceedings - 19th to 20th May 2015 in the Congress Centrum Bremen : 19th to 20th May 2015 in the Congress Centrum Bremen, Bremen, 2015.

[17] D. Gelder, “Bewertung einer Windparkausbaustrategie unter Verwendung eines analytischen Verlustmodells,” Master Thesis, Institute of Energy Systems, Hamburg University of Technology, Hamburg, 2017,doi: *10.15480/882.2378*.

[18] G. Erichsen, T. Zimmermann, and A. Kather, “Effect of Different Interval Lengths in a Rolling Horizon MILP Unit Commitment with Non-Linear Control Model for a Small Energy System,” Energies, vol. 12, no. 6, p. 1003, 2019,doi: *10.3390/en12061003*.

[19] Deutscher Wetterdienst, "Regionalmodell COSMO-DE" . [Online] Available: https://www.dwd.de/DE/forschung/wettervorhersage/num_modellierung/01_num_vorhersagemodelle/regionalmodell_cosmo_de.html. Accessed on: Aug. 19 2019.

[20] Hamburg University of Technology, "TransiEnt Library" . [Online] Available: https://www.tuhh.de/transient-ee/. Accessed on: Aug. 20 2019.

[21] I. Schlecht and M. Simic, "Renewable power plants",doi: *10.25832/RENEWABLE_POWER_PLANTS/2019-04-05*.

[22] Claudia Konetschny, Tobias Schmid, Fabian Jetter, “Potenzielle Leistungsdichte und Stromerzeugung von Windparks: Anteil der regionalen Windstromerzeugung am Verbrauch für ein "2 %-Szenario",” Energiewirtschaftliche Tagesfragen, vol. 67, 2017.

[23] E. Hau, Windkraftanlagen: Grundlagen, Technik, Einsatz, Wirtschaftlichkeit, 5th ed. Heidelberg: Springer Vieweg, 2014.

[24] L. Tian, W. Zhu, W. Shen, N. Zhao, and Z. Shen, “Development and validation of a new two-dimensional wake model for wind turbine wakes,” Journal of Wind Engineering and Industrial Aerodynamics, vol. 137, pp. 90–99, 2015,doi: *10.1016/j.jweia.2014.12.001*.

[25] M. J. Koivisto, K. Plakas, and P. E. Sørensen, “Large-Scale Wind Generation Simulations: Estimating Missing Technical Parameters Using Random Forest,” in 18th Wind Integration Workshop: International Workshop on Large-Scale Integration of Wind Power into Power Systems as well as on Transmission Networks for Offshore Wind Power Plants : 16-18 October 2019, Dublin, Ireland : proceedings, Energynautics GmbH, Ed.: Energynautics GmbH, 2019.