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Sensitivity Analysis for Ship Squat Predictions using Sobol' Indices

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ABSTRACT

Ship's sinkage, trim and resistance will increase when entering shallow waters. It is of great significance to investigate the squat of ship in shallow and confined navigation areas. However, there are many factors affecting the ship's squat, such as speed of the ship, block coefficient, water depth and draught of the ship. In this paper, the contribution of these factors to the squat is measured in empirical formula models using Sobol's indices method which is a useful method to quantify the respective effects of input random variables onto the variance of the response of a physical or mathematical model. Results are obtained from six empirical models, Barrass, Eryuzlu et al., Huuska/Guliev, Yoshimura, Romisch and Ankudinov, demonstrating that the speed of the ship has significant influence while the shape of the bow/stern, single/twin propeller have little effect on the squat of the hull. As for other parameters, they have different contribution to the output variance (squat) in different squat prediction models.

1 INTRODUCTION

Ship hydrodynamic performances experience great changes when entering shallow waters due to the hydrodynamic interaction between the hull and the seabed. In confined waters, the flow is restricted. This results in higher flow velocity of water and lower pressure under the hull. As the pressure decreases, the ship's sinkage, trim and resistance will increase. It is of great significance to investigate the behaviour of ship in shallow and confined navigation areas, such as rivers, harbors, dredged channels and other coastal areas. As we all know, there are many factors that affect the squat of the hull, like speed of the ship, block coefficient, water depth and draught of the ship et al. However, these factors have different effects on the squat of the hull. In order to minimize the risk of grounding, knowledge of the extent of the impact of each factor on the ship squat is necessary.

There are three main methods for studying the hydrodynamic characteristics of ships in restricted waters, experimental method, CFD method and empirical formula method. Experimental method is considered to be the most reliable method, but it costs a lot. CFD technique has been applied to numerical simulations of ship's manoeuvring performances. Many scholars have done a lot of research on a ship's squat using CFD method [1-5]. However, it is also not realistic to do a great deal of research by this method due to its requiring much time and effort. As for empirical formula method, it is simple and fast, which makes it possible to conduct large-scale calculations on the empirical models though it is not always valid for all kinds of ships and navigation areas.

The Permanent International Association of Navigation Congresses (PIANC) has recommended several empirical and physics-based formulas for the prediction of ship squat. Briggs et al detailed these empirical formulas [6] and made a comparison between the results obtained by empirical methods and by numerical methods [7]. In this paper, some of the most widely used formulas including Barrass [8], Eryuzlu et al. [9], Huuska/Guliev [10], Yoshimura [11], Romisch [12] and Ankudinov [13] are studied. There are many parameters in these empirical formulas, including ship speed, block coefficient, ship length between perpendiculars, beam, draft, shape of ship bow/stern, single/twin propeller, water depth, inverse side slope and bottom channel width, as listed in Table 1.

Table 1: Input variables of interest

Ship length between perpendiculars [m]	L_{pp}
Block coefficient [-]	C_B
Breadth of the ship [m]	B
Draught of the ship [m]	T
Ship speed [m/s]	V
Single/Twin propeller [-]	Prop
Bulbous Bow [-]	Bow
Stern transom [-]	Stern
Channel width [m]	W
Water depth [m]	h
Inverse bank slope [-]	n

In this paper, Sobol' indices method, one of global sensitivity analysis method, is applied to estimating the sensitivity of each factors affecting the squat of the ship. Global sensitivity analysis aims at quantifying the respective effects of input random variables onto the variance of the response of a physical or mathematical model. Among the abundant literature on sensitivity measures, the Sobol' indices have received much attention since they provide accurate information for most models. Sobol' indices method was proposed in 1993 [14] and developed by Sobol [15] and Saltelli et al [16, 17]. Nowadays, it has been applied to many fields of research [18-26].

The remainder of this paper is organized as follows. Section 2 introduces Sobol's indices method for sensitivity analysis. Section 3 details the empirical formulas used in this paper. Section 4 shows the Sobol' indices of all parameters of interest which measure the contribution to the output variance squat. Section 5 presents our conclusions.

2 SOBOL'S INDICES METHOD

Sobol' Indices method is a global sensitivity analysis method, which transforms the model into a function containing single parameter and parameter groups. In this paper, the empirical formula for ship squat prediction is defined as an objective function.

$$Y = f\left(X\right) = f\left(x_1, x_2, \cdots, x_k\right) \tag{1}$$

where Y, the value of ship squat is the output variable of interest; k is the number of input parameters; x_i is the uncertain input parameter which is assumed to be uniformly distributed according to realistic problems. The objective function can be expanded into a series of orthogonal terms.

$$f(x) = f_0 + \sum_{i=1} f_i\left(x_i\right) + \sum_{i<j} f_{ij}\left(x_i, x_j\right) + \cdots + f_{1,2,\cdots,n}\left(x_1, x_2, \cdots, x_k\right) \tag{2}$$

where each term belongs to L_2 space, and f_0 is constant. Then, the total non-conditional variance of the model output is defined as:

$$V\left(Y\right) = \sum_{i=1} V_i + \sum_i \sum_{j>i} V_{ij} + \cdots + V_{1,2,\cdots,k} \tag{3}$$

$$\begin{aligned} V_i &= V_{x_i}\left(E_{x\sim i}\left(Y \middle| x_i\right)\right) \\ V_{ij} &= V_{x_i x_j}\left(E_{x\sim ij}\left(Y \middle| x_i, x_j\right)\right) - V_{x_i} - V_{x_j} \end{aligned} \tag{4}$$

where $V(\cdot)$ is a variance operator, while $E(\cdot)$ is an expected value operator. V_i is the partial variance that is obtained when parameter x_i is fixed, and V_{ij} is the covariance that is obtained when parameters x_i and x_j are fixed.

Then normalize the Equation (3):

$$S_{i_1,\cdots,i_n} = \frac{V_{i_1,\cdots,i_n}}{V(Y)} \tag{5}$$

After each value is divided by $V(Y)$, Equation (3) can be rewritten as follows:

$$1 = \sum_{i=1} S_i + \sum_{i=1}\sum_{i<j} S_{ij} + \sum_{i}\sum_{i<j}\sum_{j<k} S_{ijk} + \cdots + S_{1,2,\cdots,k} \tag{6}$$

The corresponding Sobol' sensitivity analysis indices are defined as:

$$S_i = \frac{V_i}{V(Y)} \tag{7}$$

$$S_{ij} = \frac{V_{ij}}{V(Y)} \tag{8}$$

$$S_{Ti} = S_i + \sum_j S_{ij} + \sum_j \sum_k S_{ijk} + \cdots \tag{9}$$

where S_i is first order sensitivity index; S_{ij} is second order sensitivity index; and S_{Ti} is total sensitivity index. S_i is the fraction of the model variance explained by single parameter x_i, which measures the contribution to the output variance by a single model input alone and can be regarded as the main effect. S_{ij} measures the contribution to the output variance caused by the interaction of two model inputs. S_{Ti} is the mean output variance for keeping the parameter x_i fixed, which measures the contribution to the output variance caused by a model input, including both its first-order effects and all higher-order interactions. In other words, it implies the main effect of parameter x_i and its interactions with other parameters. Equation (9) can be rewritten as follows:

$$S_{Ti} = 1 - \frac{V_{x\sim i}}{V(Y)} \tag{10}$$

where $\sim i$ means all input parameters except for the i-th parameter. $V_{x\sim i}$ is the variance contribution due to the effects of all parameters except for parameter x_i. In comparison with S_i, S_{Ti} takes into account of the impact of parameter x_i's interactions with other parameters.

3 EMPIRICAL SQUAT FORMULAS

In this section, six different empirical formulas are introduced. It should be noted that no one formula is considered to be universally better for all ship types and channel shapes. It is necessary to pay attention to the parameter constraints for each squat formula. Table 2 lists the parameter constraints. As for Ankudinov empirical formula, the restriction is $F_{nh} \leq 0.6$.

Table 2: Parameter constraints for squat formulas

	Constraints				
Formulas	C_B	B/T	h/T	L_{pp}/B	L_{pp}/T
Barrass	0.5-0.9	-	1.1-1.4	-	-
Eryuzlu et al	⩾0.8	2.4-2.9	1.1-2.5	6.7-6.8	-
Huuska/Guliev	0.6-0.8	2.19-3.5	1.1-2.0	5.5-8.5	16.1-20.2
Yoshimura	0.55-0.8	2.5-5.5	⩾1.2	3.7-6.0	-
Romisch	-	2.6	1.19-2.25	8.7	22.9

Ankudinov	-	-	-	-	-

3.1 Barrass

In 1981 Barrass proposed the following formula for bow squat S_b based on validation with full-scale measurements:

$$S_b = \frac{C_B S_2^{2/3} V_k^{2.08}}{30} \tag{11}$$

where S_2 refers to the velocity return factor, which is the ratio between the ship's cross-sectional area and the net cross-sectional area. S_2 can be defined as follows:

$$S_2 = \frac{S}{1-S} \tag{12}$$

where S refers to the blockage factor, which is the ratio between the ship's underwater midships cross-section and the cross-sectional area of the waterway. S varies from 0.33 to 0.50 in confined waters and is less than 0.05 in open waters.

3.2 Eryuzlu et al.

Eryuzlu et al. conducted a series of model tests in both confined and open waters. Based on the experimental data, a formula was developed for cargo ships and bulk carriers with bulbous bows.

$$S_b = 0.298 \frac{h^2}{T} \left(\frac{V_s}{\sqrt{gT}} \right)^{2.289} \left(\frac{h}{T} \right)^{-2.972} K_b \tag{13}$$

where K_b is a correction factor, which takes the effect of channel width into account.

$$K_b = \begin{cases} \dfrac{3.1}{\sqrt{W/B}} & \dfrac{W}{B} < 9.61 \\ 1 & \dfrac{W}{B} \ge 9.61 \end{cases} \tag{14}$$

3.3 Huuska/Guliev

Huuska developed an empirical formula for unrestricted and restricted channels and canals by adding a correction factor K_s which was developed by Guliev. It is defined as:

$$S_b = \frac{2.4 C_B B T}{L_{pp}} \frac{F_{nh}^2}{\sqrt{1-F_{nh}^2}} K_s \tag{15}$$

where K_s is the channel width correction factor, which is defined as follows:

$$K_S = \begin{cases} 7.45 s_1 + 0.76 & s_1 \ge 0.03 \\ 1.0 & s_1 \le 0.03 \end{cases} \tag{16}$$

where the corrected blockage factor $s_1 = S$ for canal areas. As for unrestricted/restricted channels, the values for s1 are defined by Huuska. In this paper, we only focus on canal channels.

3.4 Yoshimura

The squat formula developed by Yoshimura is for fairways in Japan. It is defined as

$$S_b = \left[\left(0.7 + \frac{1.5T}{h} \right) \left(\frac{B C_B}{L_{pp}} \right) + \frac{15T}{h} \left(\frac{B C_B}{L_{pp}} \right)^3 \right] \frac{V_e^2}{g} \tag{17}$$

where V_e, the enhanced ship speed term, is the ratio between V_s, ship speed in m/s and 1-S:

$$V_e = \frac{V_s}{1-S} \tag{18}$$

3.5 Romisch

The values of S_b and S_s can be both obtained by the formulas developed by Romisch. The formula for canals is defined as follows:

$$S_b, S_s = C_V C_F K_{\Delta T} T \tag{19}$$

where C_V and C_F are correction factors for ship speed and ship shape respectively; $K_{\Delta T}$ refers to squat at critical speed.

$$C_V = 8\left(\frac{V_S}{V_{CR}}\right)^2 \left[\left(\frac{V_S}{V_{CR}} - 0.5\right)^4 + 0.0625\right] \tag{20}$$

$$C_F = \begin{cases} \left(\dfrac{10BC_B}{L_{pp}}\right)^2 & \text{BOW} \\ 1.0 & \text{Stern} \end{cases} \tag{21}$$

$$K_{\Delta T} = 0.155\sqrt{h/T} \tag{22}$$

As for canals, the critical ship speed V_{cr} is the wave celerity and the product of channel shape correction factor K_C.

$$V_{cr} = CK_C \tag{23}$$

$$C = \sqrt{gh} \tag{24}$$

$$K_C = 0.2306\ln\left(\frac{1}{S}\right) + 0.0447 \tag{25}$$

3.6 Ankudinov

The Ankudinov formula is considered to be one of the most complicated formulas for predicting ship squat because it takes account of many factors including bulbous bow, stern transom, single/twin propeller and channel type.

The maximum ship squat is defined as

$$S_{\max} = L_{pp}\left(S_m \mp 0.5Tr\right) \tag{26}$$

The maximum ship squat can be at the bow or stern depending on the trim. Normally, if the ship is even keel when stationary, ships with $C_B \geq 0.7$ will trim by head and ones with $C_B < 0.7$ will trim by astern.

The midpoint sinkage S_m is defined as

$$S_m = \left(1 + K_P^S\right) P_{Hu} P_{Fnh} P_{+h/T} P_{Ch1} \tag{27}$$

The propeller parameter K_P^S is given by

$$K_P^S = \begin{cases} 0.15 & \text{single propeller} \\ 0.13 & \text{twin propellers} \end{cases} \tag{28}$$

The ship hull parameter P_{Hu} is defined as

$$P_{Hu} = 1.7C_B\left(\frac{BT}{L_{pp}^2}\right) + 0.04C_B^2 \tag{29}$$

The term P_{Fnh} refers to the ship forward speed parameter.

$$P_{F_{nh}} = F_{nh}^{\left(1.8+0.4F_{nh}\right)} \tag{30}$$

The water depth effects parameter $P_{+h/T}$ is given by

$$P_{+h/T} = 1.0 + \frac{0.35}{\left(h/T\right)^2} \tag{31}$$

The channel effects parameter P_{Ch1} for canals is defined as

$$P_{Ch1} = 1.0 + 10S_h - 1.5\left(1.0 + S_h\right)\sqrt{S_h} \tag{32}$$

where the term S_h refers to the channel depth factor.

$$S_h = C_B\left(\frac{S}{h/T}\right) \tag{33}$$

As for vessel trim, *Tr* is given by

$$Tr = -1.7P_{Hu}P_{F_{nh}}P_{h/T}K_{Tr}P_{Ch2} \tag{34}$$

The term $P_{h/T}$ is the vessel trim parameter, which stands for the reduction in trim due to the propeller and shallow water effects.

$$P_{h/T} = 1 - e^{\left[\frac{2.5(1-h/T)}{F_{nh}}\right]} \tag{35}$$

The trim coefficient K_{Tr} is given by

$$K_{Tr} = C_B{}^{n_{Tr}} - \left(0.15K_P^S + K_P^T\right) - \left(K_B^T + K_{Tr}^T + K_{T1}^T\right) \tag{36}$$

The power of the term C_B, n_{Tr} is defined as

$$n_{Tr} = 2.0 + 0.8\frac{P_{Ch1}}{C_B} \tag{37}$$

The term K_P^T accounts for the propeller effect on the vessel trim.

$$K_P^T = \begin{cases} 0.15 & \text{single propeller} \\ 0.20 & \text{twin propellers} \end{cases} \tag{38}$$

The term K_b^T accounts for the bulbous bow effect on the vessel trim.

$$K_b^T = \begin{cases} 0.1 & \text{bulbous bow} \\ 0.0 & \text{no bulbous bow} \end{cases} \tag{39}$$

The term K_{Tr}^T accounts for the stern transom effect on the vessel trim.

$$K_{Tr}^T = \begin{cases} 0.04 & \text{stern transom} \\ 0.00 & \text{no stern transom} \end{cases} \tag{40}$$

The term K_{T1}^T accounts for the initial trim effect on the vessel trim.

$$K_{T1}^T = \frac{\left(T_{ap} - T_{fp}\right)}{\left(T_{ap} + T_{fp}\right)} \tag{41}$$

where T_{ap} and T_{fp} are the static drafts at the stern/bow, respectively.

The channel effect trim correction parameter P_{Ch2} for canals is given by

$$P_{Ch2} = 1.0 - 5S_h \tag{42}$$

4 RESULTS

In this section, the respective contribution of all factors on ship squat in confined canals is quantified using Sobol' Indices method. It can be seen from the empirical formulas that the effects of some factors are obvious in some models. For example, the magnitude of a ship's squat is approximately proportional to the square of the ship's speed, as shown in Equation (11) in Barrass model. However, the effects of some factors, such as propeller, shape of bow/stern, cannot be measured directly and precisely in Ankudinov model. It should be noted that the first-order indices are used to demonstrate the contribution to the squat by a single model input alone instead of total-order indices because total-order indices include the effects of higher-order interactions, which differs in each empirical model. For example, the total order index of *h* in Ankudinov model measures the contribution of *h* - Propeller, *h* - Bulbous bow, *h* - Stern et al interactions while the one in Eryuzlu et al model not.

The sensitivity analysis in this section follows four steps:

1) Determine the squat model input parameters and their sample ranges.
2) Generate the model inputs using sampling method.
3) Evaluate the value the squat using the generated inputs and save the model outputs.
4) Compute the sensitivity indices using Sobol's method.

Figure 1 shows the first-order indices obtained from different squat models. It indicates the speed of the ship has significant influence while the shape of the bow/stern, single/twin propeller have little effect on the squat of the hull. Four of these models includes the factor L_{pp}. It seems that L_{pp} has significant effect on ship's squat in Romisch model and Yoshimura model. In Huuska model and Ankudinov model, the effect of this factor L_{pp} cannot be ignored. In all of these models, the value of T is in a small range because of the constraints of h/T, as shown in Table 2. Hence, the value of first-order index of T is very small in Figure 1 and the value of the first-order index of h can measure the contribution of h/T to the ship squat. The factor B is one of the three most important factors affecting the ship squat except Eryuzlu model. The factor C_B has less influence on the ship squat, compared with other factors.

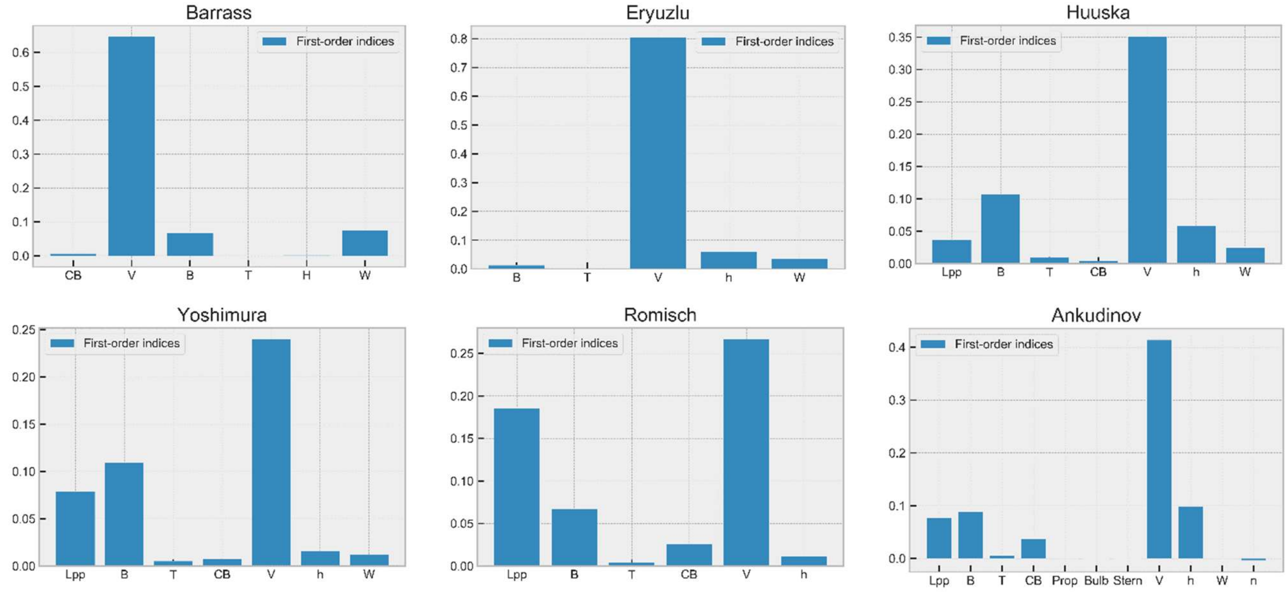

Figure 1: First-order indices

5 CONCLUSIONS

We introduce a sensitivity analysis method based on Sobol's indices and apply it to the problem of identifying the most influential factors in the empirical model for ship squat. The results of this study indicate that the speed of the ship has most significant influence while the shape of the bow/stern, single/twin propeller have little effect on the squat of the hull. As for L_{pp}, B and h/T, they have different contribution to the ship squat in different models. The influence of C_B is little, compared with other factors.

This study is a preliminary to ship hydrodynamics problems. Sensitivity analysis is promising in terms of ship hydrodynamic performance prediction, for example, the study of how the uncertainty in the output of the modular model or the Abkowitz model can be apportioned to different sources of uncertainty in hydrodynamic derivatives.

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